HARALD BOHR

22 April 1887 — 22 January 1951.

Memorial address given at a meeting of Danish mathematicians on 6 April 1951 at the University of Copenhagen.[1]

By BØRGE JESSEN.

The Mathematical Society, the Society of Mathematics Teachers, and the club Parentesen have desired that we should meet here today in memory of Harald Bohr, through whose passing we have lost the one who more than anybody else tied us together.

We should have liked to hold this meeting at the Mathematical Institute, where he had his work; however, space would not permit. But this hall too has

[1] The Danish text has been printed in *Matematisk Tidsskrift*. Certain omissions and minor changes have been made in the present translation.

its associations. It was here that he, not yet 23 years old, defended his doctoral dissertation. If we can believe the newspapers, it was, however, not mathematicians who filled the hall but for the most part football enthusiasts who had come to see one of their favourites in this role, so unusual for a star athlete. The proceedings received a special imprint through Zeuthen's opposition. This was his last official act, and those present understood that it was an heir to his place in Danish mathematics who was now embarking on his academic career. And it was in this hall that four years ago we celebrated Harald Bohr's 60th birthday. His friends and pupils rejoiced on this day at saying what one does not say in words every day. But even on this day it was he who contributed most through an unforgettable lecture, in which he looked back over his life and work and also gave a vivid description of mathematical life at home and elsewhere, especially in his youth. Through the kindly, yet well-considered characterizations of Danish and foreign mathematicians whom he had met, he gave at the same time a picture of himself. This lecture, which was printed in *Matematisk Tidsskrift*, will be often read.

I have been asked today to speak about Harald Bohr and his life work, and I feel it as the dearest duty one could assign to me, in this way to contribute to preserving his memory. But I have found the task a difficult one. I understand so well the reaction of another of his closest friends whom I had asked if he could write me something about a time which I have not known myself. This friend answered that he had tried again and again but had found nothing which could be mentioned in a larger circle. And he adds: "He was so fundamentally human that one cannot abstract from the most personal elements without destroying the essential." Nor have I found it easy to describe Bohr's work. His own radiant exposition has been constantly in my mind. This must be my excuse for not going very deeply into his works. I have restricted myself to pointing out the most important results and their mutual connection.

Harald Bohr was born in Copenhagen in 1887, the son of the distinguished physiologist Christian Bohr; his mother was a daughter of the prominent financier, politician, and philanthropist D. B. Adler, and Bohr throughout his life maintained the closest ties with the Adler family circle. In his family home, which many of the most distinguished Danish men of science and letters of the time frequented as friends, he imbibed, together with his elder brother Niels Bohr, a deep love for science; and he learned also helpfulness and sympathy for others, as well as the uncommon thoughtfulness which was so strong a trait in his character, the more to be admired since it was coupled with an impulsive temperament.

His older cousin, Miss Rigmor Adler, has told me that he was the most lovable and attractive child one can imagine, full of bubbling life, thoroughly kind and helpful, quick at everything, intelligent and full of information, musical, and highly amusing. He was everyone's favourite, but even then he was quite unspoiled by so much admiration. He loved to tease those around him, but never in a malicious way. To be sure, he knew also how to be impertinent. His parents said that he reminded them of the irrepressible Lavinia in *Our Mutual Friend.* Even as a child, he read widely and matured rapidly. Then, as throughout his later life, his relations with his brother formed a central influence in his being. Each was the other's closest friend; and they shared everything between them, thoughts, interests, and wordly goods.

When only 17 years of age, he entered the University of Copenhagen and chose mathematics as his field of study. Richly gifted as he was, there can be no doubt that, even had he chosen a different subject, he would have distinguished himself in it. For example, anyone who knew him could hardly doubt that his sure judgment and profound human understanding would have made him an outstanding jurist. We mathematicians may be grateful that he chose mathematics.

Nevertheless, it was in another field that he won his first laurels. From early boyhood, he had been a keen football player. At the age of eleven, he joined the Copenhagen Football Club, and the next year he transferred to the Academic Football Club, on whose team he played in matches in Denmark and abroad throughout his student days, and occasionally even later, the last time as a young professor. He also played on the national team. Denmark was among the first continental countries where the game became popular, and "little Bohr", as he was called in football circles, played on the team which won second place for Denmark at the Olympic Games in London in 1908; Britain, of course, held an unassailable first place. The following story will illustrate his great popularity. One day, he had taken his mother to the streetcar. A boy with a large bundle of newspapers under his arm followed her into the car and took a seat next to her. He nudged her with his elbow to attract her attention: "Do you know who that was who helped you onto the car?" Mrs. Bohr, who did not wish to betray herself, replied, "What do you mean, my boy?" "That was Harald Bohr, our greatest football player." Throughout his life, Bohr maintained a close interest in the game, and was often a spectator, especially when his old club was playing. One would hardly be in error if one connected his great ease in meeting men of all classes with his athlete's life as a young man.

As a student, Bohr attended lectures by Zeuthen, Thiele, and Niels Nielsen, among others. Among these very different persons, Bohr felt the closest kinship to Zeuthen. The strong impression which his teachers made on him showed itself in how often and with what pleasure he would tell of them. The most decisive factor, however, in his development as a mathematician was the excellent works which he studied; among these, he himself emphasized Jordan's *Cours d'Analyse* and Dirichlet's *Vorlesungen über Zahlentheorie* with Dedekind's supplements.

In his latter student years, his interests centered on analysis, and he was led to the study of divergent series. His first comprehensive investigation was concerned with the application of Cesàro summability to Dirichlet series of the ordinary type

$$\sum_{n=1}^{\infty} \frac{a_n}{n^s},$$

where the coefficients a_n are complex numbers and $s=\sigma+it$ is a complex variable.

Generalizing the well-known theorem of Jensen, according to which every such series possesses an abscissa of convergence γ_0 (which may be $+\infty$ or $-\infty$) such that the series converges in the half plane $\sigma>\gamma_0$ and diverges in the half plane $\sigma<\gamma_0$, Bohr showed that, corresponding to every integral order of summability $n\,(\geqq 0)$, the series possesses an abscissa of summability γ_n such that the series is summable of the nth order in the half plane $\sigma>\gamma_n$ but not at any point of the half plane $\sigma<\gamma_n$. These abscissae form a decreasing sequence, and thus the vertical lines through the points $\gamma_0, \gamma_1, \gamma_2, \ldots$ form the boundaries of a sequence of vertical strips. Bohr showed that the abscissae of summability satisfy certain inequalities which mean, in geometrical terms, that the widths of these strips are all $\leqq 1$ and form a decreasing sequence. He further showed that this is all that can be said regarding the distribution of the abscissae of summability, since for every sequence $\gamma_0, \gamma_1, \gamma_2, \ldots$ satisfying these conditions, he was able to construct a Dirichlet series having exactly these numbers as its abscissae of summability. Still another noteworthy result was obtained by Bohr in this study, by considering the limit γ of the sequence of abscissae of summability. Unlike the abscissa of convergence γ_0, which has no simple function-theoretic significance, the number γ is intimately connected with the function represented by the series. Indeed, the half plane $\sigma>\gamma$ is the largest half plane into which the function can be continued analytically while remaining of finite order with respect to the ordinate t.

Almost at the same time as Bohr, and independently of him, Marcel Riesz also studied Cesàro summability of Dirichlet series, which he generalized so as to apply also to Dirichlet series of the general type

$$\sum_{n=1}^{\infty} a_n e^{\lambda_n s}.$$

Such a generalization was also envisaged by Bohr, but since Riesz's method was technically simpler (*e.g.* it was immediately applicable to summability of non-integral order), Bohr limited himself to working out his theory for ordinary Dirichlet series, as set forth here. Meanwhile, in the spring of 1909, he had received his master's degree; and he used the work described above for his doctoral dissertation, which he defended in the following winter.

The problems treated in his doctoral dissertation are not comparable in difficulty to his later work. I have reviewed the dissertation in such detail because in this first of his longer papers, we already see the main characteristics of Bohr as a mathematician: a clearly defined and attractive problem and the execution of the investigation leading to conclusive results. The dissertation also shows the mastery of style which distinguishes all of his writings: his rare gift for bringing out the large lines of the investigations while at the same time exposing every detail; the gift of making difficult matters appear simple through explanations inserted at the right places.

Having completed the dissertation, Bohr left this subject and returned to it only in his latter years, when he considered, among other things, the connection between the order of magnitude of the function and the summability abscissae, now for arbitrary non-negative orders of summability. In this topic, he obtained some complete results and also pointed out interesting open problems.

Bohr's investigations as a student had led him into correspondence with Edmund Landau, with the result that Landau had proposed that Bohr should come to study with him. Immediately after his master's examination, therefore, Bohr set off for Göttingen, where Landau had just been called. This center of mathematics, to which the best young men of all countries were attracted, became for Bohr almost like a second home; and he returned there again and again in after years for longer or shorter visits. He loved to talk of the rich life which flourished there, and many a young mathematician has thereby formed a lively impression of the glorious period in the history of mathematics which is centered foremost on Hilbert's name. With a number of the mathematicians whom he met in Göttingen, Bohr formed warm friendships. With Landau, he came into an extremely fruitful collaboration, primarily on the theory of the Riemann zeta-function.

As is well known, the zeta-function $\zeta(s)$ is defined in the half plane $\sigma > 1$ by the Dirichlet series

$$\sum_{n=1}^{\infty} \frac{1}{n^s};$$

it can be extended analytically over the whole complex plane and is regular except for a pole at the point 1. In the half plane $\sigma > 1$ it can also be represented by the Euler product

$$\prod_{n=1}^{\infty} \frac{1}{1 - \frac{1}{p_n^s}},$$

where the p_n run through the prime numbers. In view of this representation, the function vanishes nowhere in the half plane $\sigma > 1$. In the critical strip $0 \leqq \sigma \leqq 1$ are found the zeros, so important for the distribution of the prime numbers. The celebrated Riemann hypothesis asserts that they all lie on the vertical center line $\sigma = \frac{1}{2}$.

Among the results which came out of the collaboration with Landau, I shall content myself with describing the theorem in which it culminated, namely the so-called Bohr—Landau theorem, dating from 1914, regarding the distribution of the zeros. It is known that the zeros in the critical strip lie symmetrically both with respect to the real axis and with respect to the center line of the strip, and further that the number $N(T)$ of zeros with ordinates between 0 and T is asymptotically $\frac{1}{2\pi} T \log T$. The first result which they obtained was that in every strip bounded by vertical lines to the right of the center line, the number $n(T)$ of zeros with ordinates between 0 and T is smaller than a constant times T and hence is of lower order than the total number of zeros $N(T)$. Together with the symmetry mentioned above, this showed that in any case the overwhelming majority of the zeros in the critical strip lie in the immediate neighbourhood of or on the center line. Their proof showed that, more generally, for every value of a the number $n_a(T)$ of a-points of $\zeta(s)$ lying in a vertical strip to the right of the center line and having ordinates between 0 and T is smaller than a constant times T. For the number of zeros they succeeded shortly afterwards in proving the stronger result that

$$\lim_{T \to \infty} \frac{n_0(T)}{T} = 0.$$

At about the same time, Hardy proved that actually infinitely many of the zeros are situated on the center line. Both results have since been sharpened by other authors.

Side by side with the collaboration with Landau, Bohr also carried through, in these years before the first war, a number of investigations on Dirichlet series.

Most of these investigations are concerned with a method for the treatment of the distribution of the values of functions represented by Dirichlet series, in particular the zeta-function. This method consists in a combination of arithmetic, geometric, and function-theoretic considerations. In the original form of the method, its arithmetical part depended on Kronecker's theorem on diophantine approximations, through which, with the use of the Euler product, the zeta-function was brought into connection with functions of infinitely many variables. The method showed that when the point s in the complex plane traces out a vertical line to the right of the point 1, then $\zeta(s)$ will trace out a curve whose closure is a certain figure which can be described. Moreover, this closure is identical with the set of values attained by $\zeta(s)$ at points lying arbitrarily near to the vertical line. A discussion of how the figure changes when the line varies led in particular to the remarkable result that the zeta-function assumes every value except 0 in the half plane $\sigma > 1$, and in fact infinitely often.

It seemed natural to try to apply this method in the critical strip also. Though the Euler product is divergent here, one might hope to succeed because of a certain mean convergence of the product to the right of the center line of the strip. But at first the method failed. Then, as Bohr has told me, he happened to be in Göttingen when Hermann Weyl presented his famous generalization of Kronecker's theorem to the mathematical society. Bohr saw immediately that this refinement was just what was needed to make his method work. In a preliminary exposition of the method, written in collaboration with Richard Courant, it was shown that the values of the zeta-function on a vertical line in the right half of the critical strip are everywhere dense in the whole plane. The final exposition was given by Bohr in a paper in *Acta mathematica* in 1915. The main result is a counterpart of the Bohr—Landau theorem, to the effect that for every substrip, however thin, of the right half of the critical strip, the above mentioned number $n_a(T)$ of a-points in the strip having ordinates between 0 and T exceeds a positive constant times T, for all sufficiently large T, when $a \neq 0$. Through this result, it was proved for the first time that whether or not the value 0 is assumed by the zeta-function to the right of the center line, this value plays an exceptional role. Indeed, according to the Bohr—Landau theorem, the number of zeros is infinitely small compared with T whereas for every $a \neq 0$ the number of a-points is exactly of the order of magnitude of T.

This result, however, did not exhaust the possibilities of the method. By elaborating it further, Bohr proved some years later that the zeta-function possesses an asymptotic distribution function on every vertical line to the right of the center line and also that the limit

$$\lim_{T\to\infty} \frac{n_a(T)}{T}$$

exists for every strip to the right of the center line, not only for $a=0$ but for every a. He was interrupted in the exposition of these results by his discovery of almost periodic functions. When he returned to the subject at the end of the 1920's, he invited me, who was then among his students, to help him with it. From this collaboration, continued through the years, developed our close friendship which has been so decisive a factor in my life.

During the years before the first war, Bohr also came into scientific contact with Hardy and Littlewood and formed close friendships with them. He often went to Cambridge and Oxford to study. In collaboration with Littlewood, he wrote a book on the theory of the zeta-function, which, however, was never sent to a printer. Later, when the theory had been developed further, their manuscript became the basis of the two excellent Cambridge tracts by Ingham and Titchmarsh. Bohr felt deeply attracted by life in the old English universities, especially by the spirit of freedom and tolerance prevalent there. Just before the outbreak of the war, he also spent a few months in Paris, where in particular he came into contact with Lebesgue.

Bohr's investigations on Dirichlet series and the zeta-function won him an early reputation, which found expression when he was invited to write, together with Cramér, the article on the recent development in analytic number theory in the *Encyklopädie der mathematischen Wissenschaften*.

Immediately after obtaining his doctor's degree, Bohr had joined the faculty of the University of Copenhagen. In 1915, he was appointed professor at the Polyteknisk Læreanstalt (the technical university of Denmark), where at that time also the university students received their introductory courses in mathematics. He retained this position until returning in 1930 to the University of Copenhagen, where through a gift of the Carlsberg Foundation, a mathematical institute was founded with him as leader. It was a real joy to him that it became possible to erect this institute directly adjacent to his brother's institute for theoretical physics. Bohr's work as a teacher has left a deep imprint on Danish mathematics. Together with Mollerup, his colleague at the Polyteknisk Læreanstalt, he wrote a comprehensive textbook on analysis, which has contributed greatly to raising the level of mathematics in Denmark. This occupied him for several years, during which period his own scientific work lay almost dormant.

After the completion of the textbook, Bohr returned to his scientific work; and it was in the following years, in the beginning of the 1920's, that he performed his main achievement, the establishment of the theory of almost periodic functions. The starting point was the problem of characterizing those functions $f(s)$ which can be represented by a Dirichlet series

$$\sum_{n=1}^{\infty} a_n e^{\lambda_n s}.$$

If the series is considered on a vertical line, *e. g.* the imaginary axis, it reduces to a trigonometric series

$$\sum_{n=1}^{\infty} a_n e^{i\lambda_n t}.$$

It was therefore natural to consider more generally the problem of which functions $F(t)$ of a real variable can be represented by such a series, *i.e.* can be formed by superposition of pure oscillations. In the special case where the frequencies λ_n are integers, the answer is given in the classical theory of Fourier series. The functions represented by such series are in essence all periodic functions of period 2π. Whereas hitherto in the theory of Dirichlet series one had always worked with frequencies forming a monotonic sequence, Bohr discovered that, in order to obtain an answer to the problem, one would have to consider series with quite arbitrary frequencies λ_n. The answer was obtained by introducing the notion of almost periodicity.

Restricting myself for the present to functions $F(t)$ of a real variable, I shall briefly state the main results of the theory. The number τ is called a translation number of $F(t)$ belonging to $\varepsilon > 0$ if

$$|F(t+\tau) - F(t)| \leqq \varepsilon \text{ for all } t.$$

The function $F(t)$ is called almost periodic if for every $\varepsilon > 0$ such translation numbers τ exist and form a relatively dense set, *i.e.*, a set with the property that any sufficiently long interval on the real axis contains at least one number of the set. This relative density is the crucial point of the definition.

Having got the idea that this was the desired definition, Bohr developed the theory of these functions systematically. Without undue trouble, it was proved that functions obtained from almost periodic functions by simple operations are again almost periodic. Further it was proved that every almost periodic function possesses a mean value $M\{F(t)\}$ obtained as the limit of the mean value over an interval when the length of the interval tends to infinity. This made it possible to copy the classical theory of Fourier series. For an arbitrary frequency λ, the corresponding Fourier constant was defined as the mean value

$$a(\lambda) = M\{F(t)e^{-i\lambda t}\}.$$

The use of a classical argument showed that this Fourier constant differs from 0 for only a countable set of frequencies $\lambda_1, \lambda_2, \ldots$, and Bohr now attached to the function $F(t)$ the series

$$\sum_{n=1}^{\infty} a(\lambda_n)\, e^{i\lambda_n t},$$

which he called its Fourier series.

So far everything had gone smoothly. Now the essential difficulty was encountered: namely, to prove that this series actually represents the function in a certain sense, more precisely, that it converges in the mean to $F(t)$, which amounts to the validity of the Parseval equation

$$M\{|F(t)|^2\} = \sum_{n=1}^{\infty} |a(\lambda_n)|^2.$$

This was the decisive criterion, that the class of functions considered was actually the right one. It was during a summer vacation, spent in idyllic surroundings in the country near Copenhagen, that Bohr overcame this difficulty. His proof for this fundamental theorem is a climax in his work. In the printed version, it fills nearly forty pages. Bohr has often said that he worked with his bare hands. That holds true for this proof more than for anything else he has done. The doctoral dissertation showed that when necessary he could work with a formal apparatus; but in his later work, formal manipulations played for the most part only a subordinate role, and he worked best when he could tackle problems directly. The fact that he nevertheless always reached conclusive results shows his strength as a mathematician. Simpler proofs have since been found for his fundamental theorem and few, unhappily, will now read his own long proof.

Having established the Parseval equation, Bohr, by a skilful extension of his old method of passing to functions of infinitely many variables by use of Kronecker's theorem, arrived at another theorem, which may be considered the main result of the theory. It generalizes the classical theorem of Weierstrass on trigonometric approximation of periodic functions by stating that the class of almost periodic functions is identical with the class of those functions which can be uniformly approximated by means of trigonometric polynomials with arbitrary frequencies.

On the basis of the theory of almost periodic functions of a real variable, it was easy to develop a corresponding theory of almost periodic functions of a complex variable and their representation by Dirichlet series with quite arbitrary exponents λ_n. The further development of this theory led to interesting problems of a function-theoretic nature.

Bohr published the theory in 1924—26 in three extensive papers in *Acta mathematica*, dedicated to his teacher and friend Edmund Landau, and it created a great sensation. Numerous mathematicians joined in the work on its extension, and pupils from many countries found their way to Copenhagen in the following years to study with him. Soon there appeared new treatments of the fundamental results of the theory. Thus, Weyl and Wiener connected it with the classical theories of integral equations and Fourier integrals. De la Vallée Poussin gave a simpler proof for the Parseval equation. Bochner developed a summation method for Fourier series of almost periodic functions, generalizing Fejér's theorem, and also gave a new definition of almost periodicity. Stepanoff, Wiener, and Besicovitch studied generalizations depending on the Lebesgue integral. Favard considered differential equations with almost periodic coefficients, and Wintner introduced statistical methods into the study of the asymptotic distribution of the values of almost periodic functions. Many more could be mentioned. Bohr rejoiced at every advance, and not the least at those investigations which simplified his own exposition. When in the 1930's von Neumann, starting from Weyl's treatment and using Bochner's way of defining almost periodicity, succeeded in extending the theory to functions on arbitrary groups, it found a central place in contemporary mathematics, as a step in the unification of different mathematical theories which is such an essential feature in the modern development of our science. I shall not here go into the further development of the theory, in which Bohr always maintained a leading part. His Danish pupils have found in this theory a rich field of study.

As visiting professor in Göttingen and later in America, where he made new friends, Bohr gave a revised exposition of the fundamental parts of the theory, making use of the simplifications which had been obtained. This he published as a little book in the series *Ergebnisse der Mathematik.* The popularity which this work attained was a source of much pleasure to him, and he used it often in his teaching, primarily as a basis for seminars.

As an academic teacher, Bohr was greatly loved. He always prepared his lectures in minute detail; but this was hardly noticeable to the audience, spellbound as it was by his dynamic delivery. On special occasions when he wanted to cover a great deal of material in a short time, he filled the blackboard in advance, making lavish use of coloured chalk, so that it shone in all the hues of the rainbow. His lectures made an extraordinary impression on all who attended them. One of his students from Göttingen once said to me almost reproachfully: "Er zerreisst sich ja für die Studenten". His courses for advanced students alternated between function theory and number theory. He did not place so much emphasis on covering a large amount of material as he did on having the students really understand the subject. Early in a course, he liked to point to some great theorem as a goal to be aimed at in order that the students might feel from hour to hour their progress towards this goal.

His collaborators and students in the mathematical institute have enjoyed fruitful and happy years under his leadership. Certain days stand out, among them in particular his 60th birthday, when the students at the morning chocolate served on this occasion presented a cantata in his praise. How he delighted in its facetious words! In the afternoon, he gave here at the university the lecture to which I have already referred. Reluctantly he permitted us on this occasion to set up a plaque portraying him, in the library of the institute. For the youth who work there in the future, it will be a modest outward expression of the unique place he holds in Danish mathematics.

Nobody who met Harald Bohr could help coming under the influence of his rich and many-sided character. With his deep humanity and radiant spirit, he made every encounter with him an event. In 1919, he had married Ulla Borregaard, and in their hospitable home, Danish mathematicians have spent many happy hours, often with guests from other lands. When their home is mentioned, a smaller circle will also think of many summers spent on the island of Als in the mathematical colony founded by Jakob Nielsen and Bohr.

Harald Bohr had the rare gift of being for many the closest confidante, the first person to whom they would come with their troubles. One never turned to him in vain. Indeed, so active was his helpfulness that one almost felt it was doing him a service to call on him.

To those who had the good fortune to become his close collaborators, scientific or not, Bohr's warm interest was a unique encouragement; and he knew how to stimulate his collaborators to achieve their best. He liked to say that he "sponged on youth", and indeed he allowed his associates to do a great deal; but his influence was so strong that a joint work always bore his stamp. "I have long ago bitten off the head of all shame", was a Danish saying which he often used when he had a task in mind for an associate.

Bohr's enthusiastic interest in their work has been a great encouragement for numerous mathematicians, among them many whose achievements are of such excellence that one would not expect them to need it, and has helped to give them confidence that their work was worth the effort. He also loved to praise his friends to others. Once, when his children had reached the age at which one can begin to tease in an amiable way, he told me that one of them had asked him: "Father, why is it that your friends in other countries are all among the very foremost mathematicians?" He smiled at being teased about his tendency to praise, rejoicing at the same time in its being so.

From his childhood, Bohr was well acquainted with the works of Goethe and Schiller, and he often quoted them. Two quotations, both from *Die Wahlverwandtschaften*, he used so often that perhaps I will repeat them here: "Die angenehmsten Gesellschaften sind die, in welchen eine heitere Ehrerbietung der Glieder gegeneinander obwaltet" — a characterization which applies so aptly to his own circle — and "Gegen grosse Vorzüge eines anderen gibt es kein Rettungsmittel als die Liebe". Another favourite author of his was Dickens, whose works he had devoured as a child. He read much, in recent as well as classic literature, and liked to make his associates participate in what thus moved him.

When disaster struck Germany in 1933 and hit academic circles, among others, so hard, Bohr was among the first to offer help. His close personal relations with colleagues in many countries enabled him to help in finding new homes for those scientists who were either forced to or who chose to leave Germany, and he turned all his energies to this task. In the summer of 1933, he made several visits to Germany, which were a great encouragement to his friends in their distress. His correspondence on their behalf was enormous; and he and Mrs. Bohr never tired of inviting German mathematicians to come up and stay with them to talk things over, since the exchange of letters was difficult. The problem was further complicated by the fact that American universities had been severely hit by the coincident economic crisis. But through his efforts, and those of his friends in other countries, all obstacles were overcome. Many prominent mathematicians, to whom his help in these years was decisive, now hold positions throughout the world, and in particular at American universities.

During these years, he also participated energetically in the endeavors of a Danish committee to alleviate the conditions of those people in other academic fields who took refuge in our country, and through this work he formed new friendships with others who had dedicated themselves to this task.

He himself did not escape the experience of exile in the latter years of the last war, being compelled to take refuge in Sweden. Here he was warmly received by his Swedish colleagues. Soon he became a member of the university and school committees formed by Danish refugees, and through his gift for encouragement and his resourcefulness, he was able to do much for the Danish youth in Sweden.

After the war, as chairman of a committee for help to Poland, he again undertook a great humanitarian work.

In international mathematical circles and in the academic life of his own country, wherever he moved, Harald Bohr exerted an extraordinary influence. When he had something close to his heart he would bring his whole force to bear on it, and he was irresistible in effecting it. He did not wish to take on any regular administrative duties, though with his sharp eye for essentials and deep understanding of the human factor in every question, he would have been excellently suited for such a task. However, when after the war the position of Provost of Regensen (an old student collegium of the university) became vacant, he felt a wish to assume this post. He made his home here among the students and his warm interest in their lives brought the students very close to him.

When one came into the old courtyard to see him, one found him in his study, worthy of the traditions of the place, puffing on his eternal cigar and often in dressing gown late in the day. The reason for the latter, was to be sure a sad one, namely that he often did not feel well. From his youth, he suffered intermittently from an internal malady, for which he sought a cure through the years in vain. When he said, "I have been a little tired and unwell", one knew what that meant. Yet he always returned refreshed and strengthened from his stays in hospitals and convalescent homes. His spirits were not at all affected by his illness, and only those nearest to him actually understood it.

In the last year or so, it was clear to him that his illness was becoming critical; but even last year when he took part in the International Congress of Mathematicians in America, everyone had the impression that he was in full vigour. However, just after New Year's he had to enter a hospital to undergo an operation, which he did not survive. He remained himself to the last, and those who came to see him in the hospital as always went away cheered. A great and irreparable loss we have suffered through his passing.

The Publications of Harald Bohr[1]

A. Dirichlet Series

1. Sur la série de Dirichlet. C. R. Acad. Sci. Paris, v. 148, pp. 75—80. 1909
2. Ueber die Summabilität Dirichletscher Reihen. Nachr. Ges. Wiss. Göttingen 1909, pp. 247—262. 1909
3. Bidrag til de Dirichlet'ske Rækkers Theori. Afhandling for den filosofiske Doktorgrad. G. E. C. Gad, København 1910. IX, 136 pp. 1910
4. Sur la convergence des séries de Dirichlet. C. R. Acad. Sci. Paris, v. 151, pp. 375—377. 1910
5. Über die Summabilitätsgrenzgerade der Dirichlet'schen Reihen. Sber. Akad. Wiss. Wien, v. 119, pp. 1391—1397. 1910
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7. Lösung des absoluten Konvergenzproblems einer allgemeinen Klasse Dirichletscher Reihen. Acta math., v. 36, pp. 197—240. 1912
8. Über die gleichmässige Konvergenz Dirichletscher Reihen. J. reine angew. Math., v. 143, pp. 203—211. 1913
9. Über die Bedeutung der Potenzreihen unendlich vieler Variabeln in der Theorie der Dirichletschen Reihen $\sum \frac{a_n}{n^s}$. Nachr. Ges. Wiss. Göttingen 1913, pp. 441—488. 1913
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[1] The Collected Papers of Harald Bohr will be published by the Danish Mathematical Society with the support of the Rask-Ørsted and Carlsberg Foundations. The doctoral dissertation and the autobiographical lecture (no.s 3 and 150 of the list) will be done into English for this edition. The other Danish papers will be summarized in English.

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