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SOME TOPOLOGICAL PROPERTIES OF A NONCONVEX INTEGRAL INCLUSION

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Dedicated to the memory of Juliusz P. Schauder

Abstract. We consider a conconvex parametrized integral inclusion and we prove that the solution set is a retract of Banach space.

1. Introduction

This paper is concerned with the following integral inclusion system

$$x(t) = \int_0^t f(t,\tau,u(\tau))\,d\tau, \tag{1.1}$$

$$u(t) \in F(t,V(x)(t),s) \text{ a.e. } I := [0,T], \quad \text{for all } s \in S, \tag{1.2}$$

where $F : I \times X \times S \to \mathcal{P}(X)$, $f : I \times I \times X \to X$, $V : C(I,X) \to C(I,X)$ are given mappings.

The aim of this paper is to prove that the solution set of the problem (1.1)–(1.2) is a retract of a Banach space. At the same time this result provides the existence of continuous selections of the solution set multifunction. Moreover, we prove that any two continuous selections from the solution map are homotopic.

Several results concerning problem (1.1)–(1.2) as a relaxation theorem and the continuous dependence of the set of relaxed solutions on various parameters may be found in [7], where additional references may also be found.

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In our approach we use a result of Bressan, Cellina and Fryszkowski ([5]) concerning the existence of a retraction of a Banach space on the set of the fixed points of a contractive set-valued map, at the same way as this result was used in [3], [4] by Blasi, Pianigiani and Staicu to obtain similar topological properties for hyperbolic differential inclusions and semilinear differential inclusions, respectively.

The paper is organized as follows: in Section 2 we present the notations, definitions to be used in the sequel, and in Section 3 we prove the main results.

2. Notations, definitions and preliminary results

Let $T > 0$, $I := [0,T]$ and denote by $\mathcal{L}(I)$ the σ-algebra of all Lebesgue measurable subsets of I. Let X be a real separable Banach space with the norm $|\cdot|$ and let (S,d) be a separable metric space. Denote by $\mathcal{P}(X)$ the family of all nonempty subsets of X, by $\mathcal{B}(X)$ the family of all Borel subsets of X and by $L(X,X)$ the space of bounded linear operators from X to X. If $A \subset I$ then $\chi_A(\,\cdot\,) : I \to \{0,1\}$ denotes the characteristic function of A. For any subset $A \subset X$ we denote by $\mathrm{cl}(A)$ the closure of A.

In what follows, as usual, we denote by $C(I,X)$ the Banach space of all continuous functions $x(\,\cdot\,) : I \to X$ endowed with the norm $|x(\,\cdot\,)|_{C(I,X)} = \sup_{t\in I}|x(t)|$ and by $L^1(I,X)$ the Banach space of all (Bochner) integrable functions $x(\,\cdot\,) : I \to X$ endowed with the norm $|x(\,\cdot\,)|_{L^1(I,X)} = \int_0^T |x(t)|\,dt$.

We recall first several preliminary results we shall use in this section.

Definition 2.1. A subset $D \subset L^1(I,X)$ is said to be *decomposable* if for any $u(\,\cdot\,), v(\,\cdot\,) \in D$ and any subset $A \in \mathcal{L}(I)$ one has $u\chi_A + v\chi_B \in D$, where $B = I \setminus A$.

We denote by $\mathcal{D}(I,X)$ the family of all decomposable nonempty closed subsets of $L^1(I,X)$ and by $\mathcal{D}_1(I,X)$ the family of all decomposable nonempty closed bounded subsets of $L^1(I,X)$.

Definition 2.2. Let Y be a Hausdorff topological space. A subspace X of Y is called retract of Y if there is a continuous map $h : Y \to X$ such that $h(x) = x$, for all $x \in X$.

Let M, N be metric spaces with distances d_M, resp. d_N. We denote by $\mathcal{K}(M)$ the space of all nonempty closed bounded subsets of M endowed with the Hausdorff metric d_M, given by

$$d_M(x,y) = \max\{\sup_{y\in Y} d_M(y,X), \sup_{x\in X} d_M(x,Y)\}, \quad x,y \in \mathcal{K}(M).$$

By $B_M(x,r)$ we denote the open ball in M centred at x with radius $r > 0$. A multifunction $F : N \to \mathcal{K}(M)$ is called *Hausdorff lower* (resp. *upper*) *semicontinuous* if for all $x_0 \in N$ and $\varepsilon > 0$ there exists $\delta > 0$ such that $F(x_0) \subset$

$\{y \in M \mid d_M(y, F(x)) < \varepsilon\}$ (resp. $F(x) \subset \{y \in M \mid d_M(y, F(x_0)) < \varepsilon\}$) for every $x \in B_N(x_0, \delta)$. F is called Hausdorff continuous if it is Hausdorff lower and upper semicontinuous.

In what follows we shall use the following result.

Theorem 2.3 ([5]). *Let E be a measure space with a finite, positive, nonatomic measure μ and let $L^1 := L^1(E, X)$ be the Banach space of all Bochner integrable functions $u : E \to X$ with the norm*

$$\|u\|_1 = \int_E |u(t)|\, d\mu(t).$$

We assume that L^1 is separable. Let $a(\,\cdot\,,\,\cdot\,) : S \times L^1 \to \mathcal{D}_1(E, X)$ be a Hausdorff continuous multifunction, that is contractive with respect to u. Consider the set of fixed points

$$\mathcal{F}_s := \{u \mid u \in a(s, u)\}.$$

Then there exists a continuous mapping $g : S \times L^1 \to L^1$ such that:

$$g(s, u) \in \mathcal{F}_s \quad \text{for all } u \in L^1, \quad g(s, u) = u \quad \text{for all } u \in \mathcal{F}_s.$$

We note first that the apparently more general problem:

$$x(t) = \int_0^t f(t, \tau, u(\tau))\, d\tau + Q(t), \tag{2.1}$$

$$u(t) \in F(t, V(x)(t), s) \text{ a.e. } I \quad \text{for all } s \in S, \tag{2.2}$$

defined by the mappings $f : I \times I \times X \to X$, $Q : I \to X$, $F : I \times X \times S \to \mathcal{P}(X)$, may be reduced to the one in (1.1)–(1.2).

As it is easy to see, if $(x(\,\cdot\,), u(\,\cdot\,))$ is a solution pair of (2.1)–(2.2) then $(x_1(\,\cdot\,), u(\,\cdot\,)), x_1(\,\cdot\,) := x(\,\cdot\,) - Q(\,\cdot\,)$ is a solution of (1.1)–(1.2) defined by $V(x_1) := V(x_1 + Q)$ and by the same mappings f and F; therefore any result for the problem (1.1)–(1.2) may be translated into a corresponding result for (2.1)–(2.2).

System (2.1)–(2.2) encompasses a large variety of differential inclusions and control systems and, in particular, those defined by partial differential equations.

Example 2.4. Set $f(t, \tau, u) = G(t - \tau)u$, $V(x) = x$, $Q(t) = G(t)x_0$ where $\{G(t)\}_{t \geq 0}$ is a C^0-semigroup with an infinitesimal generator A. Then a solution of system (2.1)–(2.2) represents a mild solution of

$$x'(t) \in Ax(t) + F(t, x(t), s), \quad x(0) = x_0. \tag{2.3}$$

In particular, this problem includes control systems governed by parabolic partial differential equations as a special case. When $A = 0$, relation (2.3) reduces to classical differential inclusions.

To simplify the notations, we set

$$\phi(u)(t) = \int_0^t f(t,\tau,u(\tau))\,d\tau, \quad t \in I. \tag{2.4}$$

Then the integral inclusion system (1.1)–(1.2) becames

$$x(t) = \phi(u)(t), \quad u(t) \in F(t, V(x)(t), s) \quad \text{a.e. } (I) \tag{2.5}$$

which may be written in the more "compact" form

$$u(t) \in F(t, V(\phi(u))(t), s) \quad \text{a.e. } (I)$$

but the integral operator $\phi(\,\cdot\,)$ in (2.4) plays a certain role in the proofs of our main results.

Definition 2.5. A pair of functions (x,u) is called a *solution pair* of (2.5), if $x(\,\cdot\,) \in C(I,X)$, $u(\,\cdot\,) \in L^1(I,X)$ and relation (2.5) holds.

In what follows we assume the following:

Hypothesis 2.6.

(i) *$F(\,\cdot\,,\,\cdot\,) : D \subset R \times X \times S \to \mathcal{P}(X)$ has nonempty closed values and is $\mathcal{L}(I) \otimes \mathcal{B}(X \times S)$ measurable.*

(ii) *The set-valued map $s \to F(t,x,s)$ is lower semicontinuous for all $(t,x) \in I \times X$.*

(iii) *There exists $L(\,\cdot\,) \in L^1(I,R_+)$ such that, for almost all $t \in I$ and for any $s \in S$, $F(t,\,\cdot\,,s)$ is $L(t)$-Lipschitz in the sense that*

$$d(F(t,x,s),F(t,y,s)) \le L(t)|x-y| \quad \textit{for all } x,y \in X.$$

(iv) *The mapping $f : I \times I \times X \to X$ is continuous and there exist the constants $M_1, M_2 > 0$ such that*

$$\begin{aligned} |f(t,s,u_1) - f(t,s,u_2)| &\le M_1|u_1 - u_2| && \textit{for all } u_1,u_2 \in X, \\ |V(x_1)(t) - V(x_2)(t)| &\le M_2|x_1(t) - x_2(t)| && \textit{for all } t \in I. \end{aligned}$$

3. The main results

In order to prove some topological properties of the solution set of the integral inclusion (2.5), we need some additional assumptions.

In what follows S is a separable metric space and the following hypothesis is satisfied.

Hypothesis 3.1. *Hypothesis* (2.5) *is satisfied. Moreover, one has:*

(i) *For each* $(t,x,s) \in I \times X \times S$, $F(t,x,s) \in \mathcal{K}(X)$ *and for each* $(t,x) \in I \times X$ *the set-valued map* $s \to F(t,x,s)$ *is Hausdorff continuous on* S.

(ii) $d(\{0\}, F(t,x,s)) \le L(t)$ *for all* $(t,x,s) \in I \times X \times S$.

(iii) $f(t,s,u) = K(t,s)u$, *for all* $(t,s) \in I \times I, u \in X$, *where* $K(\,\cdot\,,\,\cdot\,) : I \times I \to L(X,X)$ *is a continuous mapping such that for any* $t \in I$ $K(t,t)$ *is nonsingular and for any* $s \in I$ $K(\,\cdot\,, s)$ *is* C^1.

Denote $M_1 := \max_{(t,s)\in I\times I} |K(t,s)|$. On $L^1 := L^1(I,X)$ we consider the following norm

$$|u|_1 = \int_0^T \exp(-2Mm(t))|u(t)|\,dt, \quad M := M_1M_2, m(t) = \int_0^t L(\tau)\,d\tau \tag{3.1}$$

which is clearly equivalent with the usual one. We put:

$$A(s,v) := \{g \in L^1 \mid g(t) \in F(t, V(\phi(v))(t), s), \text{ a.e. } (I)\}, \tag{3.2}$$

$$B(s) := \{g \in L^1 \mid g \in A(s,g)\}, \tag{3.3}$$

$$E := \{\phi(v) \mid v \in L^1\} = \phi(L^1).$$

Proposition 3.2. *Let Hypothesis* 3.1 *be satisfied. Then the operator* $\phi(\,\cdot\,) : L^1 \to E$ *is a one-to-one mapping.*

Proof. If there exist $v_1, v_2 \in L^1$ such that $\phi(v_1) = \phi(v_2)$, then

$$\int_0^t K(t,s)[v_1(s) - v_2(s)]\,ds = 0, \quad t \in I.$$

Put $v(t) = v_1(t) - v_2(t)$, $t \in I$. For any Lebesque point of the map $t \to K(t,t)v(t)$, according to Hypothesis 3.1(iii) one has

$$K(t,t)v(t) + \int_0^t D_1K(t,s)v(s)\,ds = 0,$$

and hence

$$v(t) = \int_0^t (K(t,t))^{-1} D_1K(t,s)v(s)\,ds, \quad \text{a.e. } (I),$$

thus there exists $M_3 > 0$ such that

$$|v(t)| \le M_3 \int_0^t |v(s)|\,ds \quad \text{a.e. } (I).$$

We define

$$g(t) := \exp(-M_3t) \int_0^t |v(s)|\,ds$$

for any $t \in I$. Obviously, $g(\,\cdot\,)$ is an absolutely continuous mapping and $g'(t) \le 0$ a.e. (I). We infer that $g(t) \le g(0) = 0$, hence $v(t) = 0$ a.e. (I).

For arbitrary $v \in L^1$ we set

$$|\phi(v)|_E = |\phi(v)|_{C(I,X)} + |v|_1 \tag{3.4}$$

where $|v|_1$ is given by (3.1). Denote by $\mathcal{T}(s)$ the solution set of (2.5). Using Proposition 3.2 it is easy to check that (3.4) defines a norm on E. and $\mathcal{T}(s) \subset E$, for all $s \in S$. □

Proposition 3.3. *Let Hypothesis* 3.1 *be satisfied. Then* $E = \phi(L^1)$ *with the norm given by* (3.4) *is a Banach space.*

Proof. Let the sequence $\{\phi(v_n)\}$ be Cauchy in E. For $n, m \in \mathbb{N}$ we have

$$|\phi(v_n) - \phi(v_m)|_E = |\phi(v_n) - \phi(v_m)|_{C(I,X)} + |v_n - v_m|_1.$$

It follows that $\{v_n\}$ converges to some v in L^1. The sequence $\{\phi(v_n)\}$ converges to $\phi(v)$ in $C(I, X)$ since, for each $t \in I$ one has:

$$\begin{aligned}|\phi(v_n)(t) - \phi(v)(t)| &= \left| \int_0^t K(t,s)(v_n(s) - v(s))\, ds \right| \\ &\le M_1 \int_0^t |v_n(s) - v(s)|\, ds \le M_1 \exp(2Mm(T))|v_n - v|_1.\end{aligned}$$

Hence $\{\phi(v_n)\}$ converges to $\{\phi(v)\}$ in E and so E is complete. □

Theorem 3.4. *Let Hypothesis* 3.1 *be satisfied. Then there exists a continuous mapping* $\psi : S \times E \to E$ *satisfying the following properties:*

$$\psi(s, x) \in \mathcal{T}(s) \quad \text{for all } x \in E,\ s \in S, \tag{3.5}$$

$$\psi(s, x) = x \qquad \text{for all } x \in \mathcal{T}(s),\ s \in S. \tag{3.6}$$

Proof. It is easy to verify that for any $(s, v) \in S \times L^1$, $A(s, v)$ defined in (3.2) is a nonempty closed bounded subset of L^1, thus (3.2) defines a multifunction $A(\cdot\,,\cdot) : S \times L^1 \to \mathcal{D}_1(L^1)$.

We prove first that $A : S \times L^1 \to \mathcal{D}_1(L^1)$ defined by (3.2) is Hausdorff continuous.

Let us suposse that A is not Hausdorff lower semicontinuous. Then there exist an $\varepsilon > 0$, a sequence $(s_n, v_n) \in S \times L^1$, converging to (s_0, v_0) in $S \times L^1$, and a sequence $g_n \in L^1, g_n \in A(s_0, v_0)$, for all $n \in \mathbb{N}$, such that

$$d_1(g_n, A(s_n, v_n)) \ge \varepsilon, \quad \text{for all } n \in \mathbb{N}. \tag{3.7}$$

For $n \in \mathbb{N}$ define $G_n : I \to \mathcal{P}(X)$

$$G_n(t) := F(t, V(x_{v_n})(t), s_n) \cap B(g_n(t), d(g_n(t), F(t, V(x_{v_n})(t), s_n))).$$

As G_n is measurable, there exists a measurable selection $\overline{g}_n \in A(s_n, v_n)$, such that

$$|g_n(t) - \overline{g}_n(t)| = d(g_n(t), F(t, V(\phi(v_n))(t), s_n) \quad \text{a.e. } (I).$$

Since $g_n(t) \in F(t, V(\phi(v_0))(t), s_0)$, one has

$$\begin{aligned}
\int_0^T & \exp(\,-2Mm(t))|g_n(t) - \overline{g}_n(t)|\,dt \\
&\leq \int_0^T d(g_n(t), F(t, V(\phi(v_n))(t), s_n) \\
&\leq \int_0^T d(F(t, V(\phi(v_0))(t), s_0), F(t, V(\phi(v_n))(t), s_n)\,dt \\
&\leq \int_0^T d(F(t, V(\phi(v_0))(t), s_n), F(t, V(\phi(v_0))(t), s_0)\,dt \\
&\quad + \int_0^T d(F(t, V(\phi(v_n))(t), s_n), F(t, V(\phi(v_0))(t), s_n))\,dt.
\end{aligned}$$

Therefore, one has

$$\begin{aligned}
|g_n - \overline{g}_n|_1 \leq &\int_0^t d(F(t, V(\phi(v_0))(t), s_n), F(t, V(\phi(v_0))(t), s_0)\,dt \\
&+ \int_0^T M_2 L(t)|\phi(v_n)(t) - \phi(v_0)(t)|\,dt.
\end{aligned}$$

Let $n \to \infty$. The second integral vanishes for $\phi(v_n)$ converges to $\phi(v_0)$ in $C(I, X)$. The first integral also vanishes because of the Lebesque dominated convergence theorem and of Hypothesis 3.1(ii). Therefore, there exists $n_0 \in \mathbb{N}$ such that $|g_n - \overline{g}_n| < 1/2\varepsilon$ for all $n \geq n_0$. In particular,

$$d_1(g_n, A(s_n, v_n)) < \varepsilon/2 \quad \text{for all } n \geq n_0,$$

which contradicts (3.7). Hence $A(\,\cdot\,,\,\cdot\,)$ is Hausdorff lower semicontinuous. The proof that $A(\,\cdot\,,\,\cdot\,)$ is Hausdorff upper semicontinuous is similar. Consequently, $A(\,\cdot\,,\,\cdot\,)$ is Hausdorff continuous.

We prove next that, for every $s \in S$, $A(s,\,\cdot\,)$ is a contraction on L^1. If $v_i \in L^1$, $i = 1, 2$ one has

$$|\phi(v_1)(t) - \phi(v_2)(t)| \leq \int_0^t M_1 |v_1(t) - v_2(t)|\,dt. \tag{3.8}$$

Let $g_1 \in A(s, v_1)$ be arbitrary. Take $g_2 \in A(s, v_2)$ such that

$$|g_1(t) - g_2(t)| = d(g_1(t), F(t, V(\phi(v_2))(t), s) \quad \text{a.e. } (I).$$

Using (3.8), one has

$$\begin{aligned}
|g_1 - g_2|_1 &\leq \int_0^T \exp(-2Mm(t)) d(F(t, V(\phi(v_1))(t), s), F(t, V(\phi(v_2))(t), s))\,dt \\
&\leq \int_4^T \exp(-2Mm(t)) M_2 L(t) |\phi(v_1)(t) - \phi(v_2)(t)|\,dt
\end{aligned}$$

$$
\begin{aligned}
&\leq \int_0^T \exp(-2Mm(t))M_1M_2L(t)\bigg(\int_0^t |v_1(\tau)-v_2(\tau)|d\tau\bigg)\,dt\\
&= \int_0^T |v_1(\tau)-v_2(\tau)|\bigg(\int_\tau^T \exp(-2Mm(t))ML(t)\,dt\bigg)\,d\tau < \frac{1}{2}|v_1-v_2|_1.
\end{aligned}
$$

It follows $d_1(g_1, A(s,v_1)) < |v_1-v_2|_1/2$ and thus, since $g_1 \in A(s,v_1)$ is arbitrary

$$\sup_{g_1\in A(s,v_1)} < \frac{1}{2}|v_1-v_2|_1.$$

From this and from the analogus inequality obtained by interchanging the roles of v_1 and v_2, we infer that $A(s,\cdot\,)$ is a contraction.

By Theorem 2.3 there exists a continuous map $\theta : S\times L^1 \to L^1$ satisfying for each $s\in S$ the following properties

$$\theta(s,v)\in B(s) \quad \text{for all } v\in L^1, \tag{3.9}$$

$$\theta(s,v)=v \quad \text{for all } v\in B(s). \tag{3.10}$$

Let $(s,x)\in S\times E$ be arbitrary. Since $x\in E$, for some $v\in L^1$ we have $x=\phi(v)$. Hence $(s,x)=(s,\phi(v))$. Let $\psi(s,\phi(v)) : I\to X$ be given by

$$\psi(s,\phi(v))(t) := \phi(\theta(s,v))(t). \tag{3.11}$$

Since $\psi(s,\phi(v))=\phi(\theta(s,v))$, this equality defines a map $\psi : S\times E\to E$.

Finally, we prove the continuity of ψ, (3.5) and (3.6).
Let $(s_1,\phi(v_1)),(s_2,\phi(v_2))\in S\times E$ be arbitrary. One has

$$
\begin{aligned}
|\psi(s_1,\phi(v_1))-\psi(s_2,\phi(v_2))|_C &= |\phi(\theta(s_1,v_1))-\phi(\theta(s_2,v_2))|_{C(I,X)}\\
&\quad + |\theta(s_1,v_1)-\theta(s_2,v_2)|_1\\
&\leq (M_1\exp(2Mm(T)+1)|\theta(s_1,v_1)-\theta(s_2,v_2)|_1.
\end{aligned}
$$

Since θ is continuous, from the last inequality it follows that ψ is continuous.

Let $s\in S$, $x\in E$ be arbitrary, thus $x=\phi(v)$ for some $v\in L^1$. By (3.9) $\theta(s,v)\in B(s)$ and hence $\phi(\theta(s,v))\in \mathcal{T}(s)$. Since $\psi(s,\phi(v))=\phi(\theta(s,v))$, it follows that $\psi(s,\phi(v))\in\mathcal{T}(s)$, proving (3.5).

Let $s\in S$, $x\in\mathcal{T}(s)$, i.e. $x=\phi(v)$ for some $v\in A(s,v)$ and so, by (3.10), $\theta(s,v)=v$. From (3.10) it follows that

$$\psi(s,x)=\psi(s,\phi(v))=\phi(\theta(s,v))=\phi(v)=x$$

and the proof is complete. □

Remark 3.5. Theorem 3.4 may be considered as an extension to the more general problem (1.1)–(1.2) of the results in [4], namely Theorem 3.1, that are obtained in the particular case of the differential inclusion (2.3).

Corollary 3.6. If the assumptions in Theorem 3.4 are satisfied and if we define $\alpha : S \to E,\ \alpha(s) = \psi(s, \phi(0))$, with ψ defined in Theorem 3.4, then $\alpha(\,\cdot\,)$ is a continuous selection of the set-valued map $\mathcal{T}(\,\cdot\,)$.

Moreover, if $\alpha_i : S \to E,\ i = 1, 2$ are two continuous selections of $\mathcal{T}(\,\cdot\,)$, then α_1 and α_2 can be joined by a homotopy with values in $\mathcal{T}(s)$. Indeed, if $\beta : S \times I \to E,\ \beta(s,t) = \psi(s, (1-t)\alpha_1(s) + t\alpha_2(s))$, then $\beta(s,0) = \alpha_1(s),\ \beta(s,1) = \alpha_2(s)$ and $\beta(s,t) \in \mathcal{T}(s)$ for all $(s,t) \in I \times I$.

Corollary 3.7. For any $s \in S$, the set $\mathcal{T}(s)$ of solutions of (2.5) is a retract of the Banach space E.

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