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Philippe De Rouilhan,
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REVIEW

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The only article in the proceedings of the ASL Summer Colloquium 2000 of direct historical pertinence is the contribution of Philippe De Rouilhan on "Russell's Logic". Of potential interest to philosophers of logic and to many historians of logic and mathematics is William Ewald's, "Hilbert's Wide Program" (pp. 228–251). In this review I shall restrict my attention to De Rouilhan's article.

The title which de Rouilhan chose for his article is somewhat misleading, and suggests a far broader scope than is actually presented. De Rouilhan does not present, as his title suggests, a survey of Russell's contributions to logic. Rather, the chief, indeed the sole focus, of the article is one aspect of Russell's efforts to deal with the paradoxes, and to do so within the broader context of Russell's various approaches to treating the paradoxes. To provide the historical background, De Rouilhan sets forth three versions of Russell's work on the paradoxes. He begins with the canonical—or as he calls it, "popular"—account, namely the discovery of the Russell paradox as described in the appendices of the *Principles of Mathematics*, and the theory of types, as first provided in "Appendix B" of the *Principles*, and as elaborated in his and Whitehead's *Principia*, as the means to avoid the paradox. Next, we are given the "scholarly" account, which is a more detailed and complex story, and examines the various adumbrations and development of the theory of types. Lastly, De Rouilhan offers his rational reconstruction, to explain the development of Russell's thought in working through to the final presentation of the theory of types as given in *Principia*. What is unique in De Rouilhan's presentation is the notion, not fully appreciated by Russell himself, according to de Rouilhan, that Russell hit upon the hyperintensional paradox, which was dealt with by the tested, but discarded, substitutional theory of propositions.

De Rouilhan argues, both in the present article and in the more detailed *Russell et le cercle des paradoxes* [De Rouilhan 1996] that in late 1902 Russell discovered the paradox of the hyperintensional notion of *proposition*, according to which we deal with the meaning of possible sentences. It is of the same genre as the better known Russell Paradox, which depends upon the notion of *set*, or, in Russell's terminology, *class*, involving the familiar Russell set. Both the hyperintensional paradox and the Russell Paradox are far more intractable than the semantic paradoxes, such as the Liar. But, having once discovered the hyperintensional paradox, Russell, says De Rouilhan, thereafter ignored it; or, at the very least failed to realize its significance. It was likewise ignored by the rest of the logical community, until it was rediscovered by John R. Myhill (1923-1987), working on the formalization of intensional logic [Myhill 1958].

Russell sought to devise a tool that would be sufficiently powerful to abolish both the logical (or set- theoretical) paradoxes *and* the semantic paradoxes. It was to this end that he arrived at the ramified theory of types. But, De Rouilhan suggests (p. 336), Russell's efforts to deal with the set-theoretic paradoxes alone would be adequately handled with the simple theory of types. It was Russell's desire to handle at once both the set-theoretic and semantic paradoxes that called forth the ramified theory of types. Unfortunately, ramified types was of itself too weak to permit construction of classical mathematics without the adjunction to the system of *Principia* of non-logical and controversial axioms, namely, the Axioms of Reducibility, Infinity, and Choice (see, *e.g.* [Myhill 1974]).

Within a short time of the appearance of the second edition of the *Principia*, and before Myhill rediscovered the hyperintensional paradox (which De Rouilhan proposes naming the Russell-Myhill Paradox), a number of logicians noted problems of the kind that de Rouilhan mentions. Thus, for example, Benjamin Abram Bernstein (1881-1964) in "Relation of Whitehead and Russell's Theory of Deduction to the Boolean Logic of Propositions" [Bernstein 1932a] explored the relationship between what he called the Boolean logic of propositions, which is the same as the *Aussagenkalkül* of Schröder, to the propositional logic presented in the *Principia Mathematica*. He found that the theory of deduction of *Principia* did not allow derivation of propositions of Boolean logic, whereas the theory of deduction of the *Principia* is derivable from the Boolean logic of propositions, and he concluded therefrom that the theory of deduction of *Principia* is inadequate for formulating the propositional calculus. He also provided a proof [Bernstein 1932b] of the consistency and independence of the postulate system of Jean Georges Pierre Nicod (1893-1924) that Russell's primitives $f(x)$, $\varphi(f(x))$, and *1.11 and *1.72 of *Principia* are redundant, and that Nicod's postulates are derivable from the primitives of the *Principia* (see [Nicod 1916]). A few years later, Susanne Katarina Knauth Langer (1895-1985) complained about the confusion of symbols and confusion of logical types in an article of that title [Langer 1926]. And in [Smart 1950], philosopher John Jamieson Carswell Smart (b. 1920) argued that it is false that, just because there is a clear meaning to compound propositional functions such as $f[\varphi(\psi(x))]$ in algebra, there must also be a clear meaning to the same expressions where $f(x)$, $\varphi(x)$, $\psi(x)$ are propositional functions. From Gödel's perspective, Whitehead and Russell's ramified theory of types neglected syntax, expressed by Gödel [1944, 126], as omitting a "precise statement of the syntax of the formalism," and pointing out that there were several varieties of the Vicious Circle Principle operative behind and within the ramified theory of types. Russell presumably came to agree, declaring in *A History of Western Philosophy* that certain expressions, such as "Scott exists?" are, as he expressed it: "Bad grammar, or rather bad syntax."

That Russell was clearly burdened by the paradoxes is evident from the manifold attempts and methods he undertook in dealing with them. Besides the ramified theory of types and the extensional simple theory of types, we are also familiar with his "no-class" theory, and his "zigzag" theory, and, thanks to Myhill, and now De Rouilhan, with the hyperintensional theory of types, along with the quickly discarded substitution theory which remained largely in manuscript form but which is Russell's version of the hyperintensional theory later elaborated by Myhill. Indeed, Christine Ladd-Franklin (1847-1930) became so frustrated with the succession of solutions proposed by Russell that she argued in "Symbolic Logic and Bertrand Russell" [Ladd-Franklin 1918] that there was little point in attempting to evaluate Russell and his logical work until he finally and once for all settled upon a system.

The simplified theory of types proposed by Frank Plumpton Ramsey (1903-1930) [Ramsey 1931] in 1925 required only the adjunction of the Axiom of Choice (although, ultimately ZFC was preferred over the Ramsified theory of types). Ramsey's point was that logic need worry only about the logical paradoxes, keeping its own house in order without worrying about the semantic neighbors. He therefore distinguished between "two groups—those expressed in symbols and those expressed in words. Those expressed in words are nearly all nonsense by the Theory of Types and should be replaced by symbolic conventions" [Ramsey 1931, 174].

Meanwhile, of course, a vast secondary literature has grown around both the theory of types in its various incarnations as well as an equally vast literatre devoted to the evolution of Russell's various efforts at dealing with the paradoxes.

The hyperintensional theory of types is rooted in the idea of the *incomplete symbol* (p. 337). It had already been pointed out (as we already noted), long before Myhill or De Rouilhan, for example by Langer, and later by Smart, that the confusion of incomplete symbols by Russell was a source of difficulty. Moreover, each of the hyperintensional theories of types, simple, ramified, and Ramsified, give us an inconsistent system (p. 346). Moreover, they might well be considered pointless, insofar, at least according to De Rouilhan (p. 341), as Frege's system was typed. Further, a typed logical system was incompatible with Russell's (and Frege's) requirement that one has a fixed *Universum* or universal domain, outside of which one cannot go (see DeRouilhan, p. 341; *cf.* [van Heijenoort 1987]).[1] The solution was to do away with the hyperintensional, and to develop an extensional theory of types.

[1]This universality is such that one cannot distinguish between the system and the metasystem, so that one is unable to ask extrasystematic questions about the system (say its completeness, categoricity, consistency) One must, perforce, do so from within the system; in effect, there is no metasystem. This was, in an important sense, a point of Gödel's incompleteness theorems; it is also the source of the difficulty in following the technical details of Gödel's original proofs, as he weaves back and forth between system and metasystem, to demonstrate (1) that the there is in $\mathbb{Z}$ an undecidable proposition, and then to prove (2) that the consistency of $\mathbb{Z}$ is not provable in $\mathbb{Z}$.

For De Rouilhan, (p. 340), Russell's universalism in logic meant very strictly that it is "impossible to speak about anything at all and about logic in particular, from *outside* logic." If De Rouilhan is correct in telling us, in the next sentence, that Frege was also a universalist, but of a less strict sort, insofar as he admitted the possibility of a kind of "absolute metalanguage" as a language of exposition, heuristically useful but theoretically "illegitimate", then we can readily appreciate Gödel's choice, in proving the incompleteness of "*Principia*-type systems," and van Heijenoort's specific attention [van Heijenoort 1987] to Russell on system and metasystem.

Now the hyperintensional paradox can be dealt with by the substitutional theory, which Russell briefly considered (p. 342). Gregory Landini argues in Russell's *Hidden Substitutional Theory* [Landini 1998] that Russell developed the substitutional theory of propositions as a consequence of the theory of descriptions, as a way to give syntactic meaning to the hyperintensional propositions, or incomplete symbols.[2] The theory of types replaced the proposed substitutional theory as a means of providing a semantic limitation to predicate variables. Landini [1998, 260] argues that "since the μ of $A\mu$ is typically ambiguous, it does not follow that $(\mu)(A\mu \vee B\mu)$ is a wff simply because $A\mu$ and $B\mu$ are wffs. The μ of $A\mu$ may have a different order/type index than the μ of $B\mu$." It is for this reason that Whitehead and Russell require the principle of "sameness of type" (*9.131). Thus, Landini ([1998, 260]; quoting letters #210.057451 and #710.057442 in the Russell Archives) notes that Whitehead detected a confusion arising from the kind of ambiguity resultant from a different order/type index while the printing of volume 1 of *Principia* was already in progress, writing to Russell in May 1910 that: "It appears to me as if two ideas are muddled up together—namely a true logical premise and a test which supplements the incompleteness of our symbolism," and explaining that:

> A true logical premise must [be] such as would still be required, if our symbolism were complete and adequate. Now in such a case the type is always in evidence. E.g. let every letter representing an individual have I as a subscript, then we have $\vdash .\phi x_i$ and $\vdash .\psi x_i$, we do not need any axiom to to assume that x_i and y_i are of the same type, and that any possible value of x_i is a possible value of y_i and vice versa.

What Whitehead seems to be suggesting to Russell is the indexing of variables (and quantifiers) that was already found in Schr"oder?s quantification theory (see, *e.g.* [Church 1976]) and was initially developed by Charles Peirce [1883] and his student Oscar Howard Mitchell [1883] in 1883. In effect, Whitehead is proposing a syntactical means of embedding a many-sorted (typed) quantificational logic into a generalized one-sorted quantificational logic and of concomitantly preserving

[2]In the present article, De Rouilhan mentions (n. 15, p. 342), but does not elaborate upon, the differences between his views on the substitutional theory and Landini's. In a review of Landini's treatment (see [Bozon 2001]), De Rouilhan refers the readers instead to the comparison between Landini's views and his own, as he elaborated it in his [De Rouilhan 1996].

the semantic distinction between the type-levels and their underlying ontologies.

Perhaps as good an example of the hyperintensional paradox as one could hope for is that which, presumably inadvertantly, occurs in De Rouilhan's list of references (p. 348), and which I here reproduce exactly as it appears:

> [2] Alonzo Church, Schröder's anticipation of the simple theory of types, **Erkenntnis**, vol. X (1976), pp. 407–411, "This paper was presented at the Fifth International Congress for the Unity of Science in Cambridge, Massachusetts. Preprints of the paper were distributed to the members of the Congress and the paper was to have been published in **The Journal of Unified Science (Erkenntnis)**, Volume IX, pp. 149-152. But this volume never appeared and the paper has not otherwise had publication", p. 411.

(To this, I can only add that Church was apparently unaware of the publication of this paper in *Erkenntnis* in 1976, until I contacted him about it, requesting an offprint.)

De Rouilhan culminates his treatment by considering that the standard historical account of Russell's work on the paradoxes includes two errors of fact (p. 347). One is that Russell was not, contrary to all appearances, seriously interested in the semantic paradoxes, because these involved the notion of truth relative to sentences. He was interested rather in the epistemological versions of these paradoxes, because these involved the logical notion of truth relative to propositions and the epistemological concept of assertion. The "scholarly" account corrects this error. The other is that the ramified theory of types was justified by the epistemological paradoxes. Rather, the logical paradoxes of hyperintensionality, that is, the logical paradoxes of propositions, especially those bearing incomplete symbols, required the ramified theory. The "rational" account, as presented by de Rouilhan, corrects this error.

He ends his account on an apologetic note (p. 348):

> The great Russellian logical enterprise was subject to powerful constraints quite exogenous to Russell's thought and mind. Russell may not have been aware of them, but their effects were in no way diminished by his ignorance, and these same forces apply, as is better realized today, to any possible effort to construct a hyperintensional logic.

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