

Tyler Burge

*Truth, Thought, Reason: Essays on Frege*

Oxford: Clarendon Press; New York: Oxford University Press, 2005

xii + 419 pp. ISBN 0199278539

## REVIEW

JOHN P. BURGESS

The volume under review reprints Tyler Burge's well-known historical papers on Frege with a good deal of supplementary material in the form of lesser papers, postscripts, and a long introduction. To keep this review of manageable length, I will focus on just a handful of items that give a good indication of the diversity of the issues Burge addresses.

"Frege on Truth" is a practical demonstration of the maxim that the best way to understand a philosopher is often to see how one might defend his least attractive views. Burge undertakes to defend three unpopular claims of Frege's: (1) that the denotations of sentences are their truth-values; (2) that these truth-values are objects; (3) that each of these objects is identical to its unit class.

An unsympathetic view of Frege's case for (1) would go as follows. Frege begs half the question by assuming sentences *have* denotations, then offers the pitifully weak argument for the claim that these denotations are truth-values, that we are interested in the existence of a denotation for a singular term in a sentence only when we are interested in the truth-value of the sentence: If we are interested in the *Iliad* only as literary art, we may be content with the sense of "Odysseus;" only if we are interested in its historical truth must we seek after a denotation for "Odysseus." This is an interesting choice of example, when one considers that Frege is writing a year or two after the death of Schliemann, whose claims had kept the German academic world in ferment for decades; but how is (1) supposed to follow?

Burge makes two effective points in defense of Frege. The first point is that the word translated as "denotation" is an ordinary German word that is unmistakably being used with an extraordinary Fregean meaning. One should not, therefore, think that there is some intuitive,

pre-theoretic notion of “denotation” for sentences against which Frege’s thesis can be judged. Rather, the “denotation” of sentence will be whatever completes the proportion

$$\frac{\text{_____}}{\text{the sense of a sentence}} : \frac{\text{the denotation of a singular term}}{\text{the sense of a singular term}} ::$$

Completing the proportion means that analogues of the compositionality principles relating the senses of parts of a sentence to that of the whole sentence must hold between the denotations of parts of a sentence and whatever goes into the blank.

The second point is that Frege’s diction shows that the remark about Homer is only intended as an heuristic motivation for considering (1) as a *conjecture*, that Frege’s real argument for (1) consists in the verification that the compositionality principles demanded by the above proportion do hold, and that even after this verification Frege only claims the thesis to be probable, so that throughout he maintains an entirely reasonable experimental attitude. Taken together, these points amount to a quite successful defense of (1), the only item Burge sincerely wishes to defend. His defense of (2) and (3) is in the nature of devil’s advocacy, since he does not in the end accept them himself, but there are some particular criticisms of (2) and (3) he does think it important to refute.

An unsympathetic view of Frege’s case for (2) might go as follows. Frege’s great contribution to logic was to challenge the logical grammar of traditional syllogistic logic. *Begriffsschrift* has suggested to later logicians an alternative logical grammar, with a corresponding ontology. There are two fundamental categories or types,  $N$  for “names” or singular terms, denoting objects, and  $S$  for sentences, denoting truth-values, and then a range of other categories or types for expressions with gaps, such as type  $N^k \rightarrow N$  for an expression with  $k$  gaps that filled in by  $N$ s produces a more complex  $N$ , a  $k$ -place function-expression, denoting a function, or  $N^k \rightarrow S$  for an expression with  $k$  gaps that filled in by  $N$ s produces an  $S$ , a  $k$ -place predicate, denoting a concept. Concepts are evidently analogous to functions, and there are other function analogues such as “the number of,” a functional of type  $(N \rightarrow S) \rightarrow S$  converting concepts to objects, and “the ancestral of,” an operator of type  $(N^2 \rightarrow S) \rightarrow (N^2 \rightarrow S)$  that converts a relation into another relation.

It is a beautiful picture, but then Frege goes and spoils it all by claiming, with wanton perversity, that predicates and concepts are not just *analogous* to  $k$ -place function-expressions and functions, but literally *are* function-expressions and functions. This requires him to maintain

that sentences and truth-values are “names” and objects, and thus he arrives at the grotesque mode of expression “naming the True” for being true.

A lot of what Burge has to say in defense of Frege consists simply in stressing the analogies, and this is by itself insufficient, since what needs to be motivated is the conversion of an *analogue* of a function into a literal *function*. (And note that the in the case of number and ancestral, one is going to *have* to leave it at the level of analogy.) Heck has suggested that there are motivating considerations that, if too weak to motivate anyone else to follow him, are at least strong enough to show that Frege’s step is not wanton perversity. The relevant considerations, however, are technical, pertaining to the reduction of many-place to one-place, and minimization of the number of primitives, and Burge does not cite them. Burge thus seems to have no real answer to the long-standing complaints of Michael Dummett and others that Frege’s step is unnecessary, unnatural, unattractive, and so on.

But then Burge’s real, or at least his primary, aim is not to defend Frege against any and all such complaints, but rather against Dummett’s most extreme formulations, which seem to say that Frege’s step actually involves the loss of Frege’s own most important insights, the primacy of sentences over names. Burge has no trouble showing—and I wonder if Dummett could ever really have doubted it—that even after his misstep Frege does still recognize sentences as at least a very special and especially important subspecies of singular terms.

An unsympathetic view of Frege’s case for (3) might go as follows. If “names of truth values” are a very special kind of names, one might expect that the truth values, the objects they are names of, would be a very special kind of objects. Surely the natural course would be to take them to be objects *sui generis*, not identical to any objects presented as anything other than truth-values. But Frege will not allow this, and insists in the notorious §10 of the *Grundgesetze* that they must be courses-of-values of some kind. With this radical step he threatens to destroy the priority and independence of the notion of “concept” with respect to that of “course-of-values,” by making the distinction between concepts and other functions depend on the notion of course-of-values (in that the concepts are the functions whose values for any arguments are always the one or the other of two particular courses-of-values). He then makes the specification that the truth values are to be their own unit classes, which is simply bizarre.

Burge tries to defend the particular choices Frege makes. He suggests, for instance, that it is more reasonable to identify a truth-value with a course-of-values than, say, to identify the True with the sun and

the False with the moon. This is correct as far as it goes, but it doesn't take us very far. But again I think Burge's primary aim is to refute interpreters who would turn Frege into something like a nineteenth century Paul Benacerraf, a structuralist who when it comes to identifying numbers, takes any old  $\omega$ -sequence to be as good as any other. To achieve this aim all that is really needed is to show that Frege's specifications are not *completely* arbitrary. Still, it sometimes looks as if Burge would really like to go further, and perhaps as far as denying that there is *any* arbitrariness in what Frege identifies with what. If so, he faces an obstacle neither he nor many other commentators seem to have recognized.

There is a dizzying self-referentiality or circularity in Frege's specification as to what the True and the False are to be. To bring out the circularity I need some notation, so for any pair of objects  $a$  and  $b$  let  $c_{ab}$  be the course-of-values of the function whose value for argument  $a$  is  $a$  and whose value for any other argument is  $b$ , and let  $d_{ab}$  be the course-of-values of the function whose value for argument  $b$  is  $a$  and whose value for any other argument is  $b$ . Now the key point is that since concepts are just functions whose values are truth-values, *until it is pinned down which objects are truth-values, it is up in the air which functions are concepts, and which courses-of-values are extensions*. Once this is appreciated, it will be seen that Frege's supposed specification of what objects the True and the False are to be is in fact is no *specification* at all, but merely a pair of *constraints*, according to which they should be objects  $a$  and  $b$  satisfying  $a = c_{ab}$  and  $b = d_{ab}$ . It is far from obvious that there is a unique solution to this pair of equations. Despite such residual reservations, insofar as Burge's real aim was not a total vindication of Frege, but rather an improved understanding, he has succeeded quite well.

"Frege and the Hierarchy" is a much shorter paper, and the least historical of the ones I have chosen for special comment. The starting point is a glitch in Frege's verification of the proportion discussed in connection with (1) above. Frege wants to maintain that substituting one "name" for another with the same denotation does not change the truth-value of a sentence. And yet it seems it sometimes does, in belief and related contexts. For example, in the list of Beethoven's works, Opus 123 is the Missa Solemnis, and Opus 132 is String Quartet 15, and yet, until I looked it up for purposes of this example, "J. B. believes that Opus 132 was composed after Opus 123" was true, in that I believe that opus numbers are assigned in a usually successful attempt to maintain chronological order, while "J. B. believes that

String Quartet 15 was composed after the Missa Solemnis” was false, since I had only the vague idea that both were late works.

As a way out, Frege proposes (a) that pairs of expressions that have the same denotation in ordinary contexts may have different denotations in belief contexts. This obliges him to maintain (b) that an expression has a different denotation in belief contexts from its ordinary denotation. Owing us a specific indication of what this alternate denotation is, Frege proposes (c) that the denotation of an expression in a belief context is the same as the sense of that expression in ordinary contexts. Since sense determines denotation, this obliges Frege to hold (d) that the sense of an expression in a belief context is different from the sense of that expression in ordinary contexts: Ordinarily, “Opus 132” is a certain mode of presentation of a musical composition, but in belief contexts it is a certain mode of presentation *of* that mode of presentation, the *indirect* sense of “Opus 132.”

Now what of belief contexts inside belief contexts, as in “Yehudi believes that Yitzhak believes that Yo-Yo believes that Opus 132 is a masterpiece”? Here Burge defends the view of Alonzo Church that we must shift each time we embed, from direct or primary sense to indirect or secondary sense to doubly indirect or tertiary sense to triply indirect or quaternary sense, and so on. Burge attempts to enforce this view with an abstract argument to the effect that to stop with one or two levels of sense will require abandoning the Fregean principle that a sense determines a *unique* denotation. Abstract arguments in philosophy seldom succeed in convincing (even when perhaps they ought to), since any argument involves some auxiliary assumptions and formal apparatus about which critics can quibble; and so it may be no surprise that the paper proper is followed by a postscript replying to critics that is three or four times the length of the original. The informal discussion in parts of the postscript may in practice be more effectively convincing than the formal argument of the paper proper.

According to Burge, a Fregean sense or mode of presentation of an object is a way of thinking about an object, of representing it in thought. A *full* report of a thought of Yehudi’s should include an indication of the way in which Yehudi was representing in thought that about which Yehudi was thinking; if that about which Yehudi was thinking was a thought of Yitzhak’s, a *really* full report should include an indication of the way in which Yehudi was representing in thought the way in which Yitzhak was representing in thought that about which Yitzhak was thinking; if that about which Yitzhak was thinking was a thought of Yo-Yo’s, a *really, really* full report should include an indication of the way in which Yehudi was representing in thought the way in

which Yitzhak was representing in thought the way in which Yo-Yo was representing in thought that about which Yo-Yo was thinking. And so we are led to modes of presentation of modes of presentation of modes of presentation of objects.

But how, critics ask, can a language in which each expression has an infinite hierarchy of modes or presentation or senses ever be learned? Such learnability objections are almost always mistaken for the same reason: that one doesn't have to learn the whole infinite sequence, but only the first term and the method by which each term generates the next. So Burge replies, "There is no more difficulty in learning a language committed to an infinite hierarchy of senses than there is in learning a language that iterates quotation marks." Needless to say, the key question is: What *is* the method that leads from the primary sense to the secondary sense to the tertiary sense and beyond? In Burge's terminology, objects in general admit different modes of presentation, but while for ordinary objects none among these various possible modes of presentations stands out as privileged, it is otherwise with modes of presentation of *senses*: The secondary sense is the *canonical* mode of presentation of the primary sense, the tertiary sense the *canonical* mode of presentation of the secondary sense, and so on. So the real issue is whether Burge's notion of canonicity is intelligible. Burge's answer is that anyone who understands how the embedding of that-clauses works does (implicitly) understand it.

The common objection against those of us who sympathize with the Fregean approach is that most of the time in reporting beliefs we aren't very interested in reporting how the believer thinks of the thing the belief is about. This objection gets only a few words from Burge, though he certainly acknowledges the point. Quine famously distinguished cases where the mode of presentation matters and cases where it doesn't, in his contrast between the *de re* "Dr Smith believes of the patient in room 100 that she is going to die," and the *de dicto* "Dr Smith believes 'The patient in room 100 is going to die.'" Though Quine's regimented way of marking the distinction may not be attractive, it seems as if Fregeans are going to have to mark it somehow. Or perhaps they may need to mark more than just a binary distinction, since as Heck says (in private correspondence)—and I believe this is Burge's view as well—"Ordinarily, we tolerate a good deal of variation in how beliefs are reported, and how much variation we tolerate seems to vary from case to case." (It occurs to me that there even seem to be cases where one might want to distinguish between reports that are identical in sense, differing only in tone.) The systematic representation of all the exponentially growing number of possibilities as we go

on to deeper and deeper embeddings would require an apparatus much more elaborate than the already complicated formal apparatus Burge presents (but which I have left out of my account). I am not suggesting Burge suggests otherwise.

There also is a residual worry about, say, “It is common knowledge among the members of the Budapest String Quartet that Opus 132 is in A minor,” where “common knowledge” means that each knows it, each knows that each knows it, each knows that each knows that each knows it, and so on *ad infinitum*. One might fear we will turn out to need  $\omega$ th-level senses, and  $(\omega + 1)$ st-level senses to cover, say, “Burge knows that it is common knowledge among the members of the Budapest String Quartet that Opus 132 is in A minor.”

“Sinning Against Frege” and its postscript mainly aim to defend Frege against the “anti-Fregean” arguments of Saul Kripke in *Naming and Necessity* (though this is Burgess’s and not Burge’s way of characterizing what the paper is about). As to naming, there are two sets of “anti-Fregean” arguments, Kripke’s rigidity arguments, and epistemic arguments from error and ignorance, including older and weaker arguments of John Searle and Peter Strawson for “non-Fregean” descriptivism, and later and stronger arguments of Kripke and Keith Donnellan for outright anti-descriptivism. Not to mention work of Burge’s own in his non-historical papers, there is work closely related to that cited so far by Kripke and Hilary Putnam on the topic of natural-kind terms, and work less closely related by David Kaplan on the topic of demonstratives (and indexicals). Burge gives much less attention to the former than to the latter, and tends to view proper as less similar to natural-kind terms, than to indexicals (and demonstratives), grouping names and these other expressions together under the rubric “context dependence.”

He also attributes such a view to Frege. In the main passage from Frege that Burge quotes, a well-known footnote to “Über Sinn und Bedeutung,” Frege speaks of idiolect, or variation in the sense of a proper name from speaker to speaker, but Burge thinks what Frege says should be taken to apply also to context-dependence, or variation on the part of one and the same speaker from occasion to occasion. The significance of this point will become clearer later.

Russell flatly states that real, ordinary proper names (as opposed to his mythical “logically proper names”) are mere “truncated” descriptions. Burge is much concerned to argue that Frege, by contrast, is *not* committed to the view that the meaning of a name is the same as that of some description. This is for a double reason. First, Frege is not



committed to the view that the *sense* of a name is the same as that of a description. Second, a Fregean sense in general is not the same thing as a “conventional linguistic meaning.” On both points the correctness of Burge’s claim can hardly be questioned, though its precise significance can be.

On the first point, the key text is the footnote alluded to earlier, where after stating it as a norm that in an ideal scientific language every proper name should be associated by every user of the language with a unique sense, Frege points out that in fact, in real natural languages, this norm is not met. In giving concrete examples of the senses of proper names, Frege uses descriptions, but he does not formulate any abstract thesis to the effect that the sense of a proper name is always the same as that of some description. Moreover, Burge points out, the descriptions Frege gives as examples *themselves* involve proper names, so that Frege is even further from committing himself to a view that proper names have *purely* descriptive senses. What raises a question about the significance of this fact is that it would be idle to pretend, and Burge does not pretend, that Frege had any well-thought-through view according to which the sense of a proper name is usually or ever something *different* from the sense of any description.

On the second point, the easy proof is as follows. The conventional linguistic meaning of “you,” for instance, surely does not vary from context to context (unless one counts variation between singular and plural). But the denotation certainly does vary with the person or persons being addressed. Since sense is supposed to determine denotation, sense must correspondingly vary even as meaning remains constant. Thus sense is not meaning. Q.E.D. Burge takes the point about the variation of sense with context here illustrated in the case of an indexical to apply to “context dependent” expressions more generally, in which group he includes, as already noted, proper names. In any case, regardless of what one thinks about proper names, indexicals and demonstratives require consideration anyhow, for their own sakes.

Now when I say that Burge “defends” Frege, I do not mean that he defends him to the end. In fact, Burge shows much less interest in developing broadly Fregean theories of context-dependent expressions than does Heck, for one. *In the end* Burge concedes that epistemic considerations do raise a serious problem for Frege’s theory of sense, which he describes as follows. On the one hand, a Fregean sense was supposed to be a “cognitive value.” (This is Burge’s favorite characterization, and it recurs literally dozens of times in the volume, with the crucial addition “expressed in language” sometimes made explicit and sometimes left implicit.) On the other hand, a Fregean sense is



supposed to determine denotation or reference. The problem is that there is often not enough cognitive value to determine reference. Thus is one of Kripke's conclusions expressed in Burgean language.

Returning now to the topic I set aside at the outset, that of rigidity, at the time Burge was writing it was widely taken to provide a devastating objection to "Fregean" theories, and it is still thought to do so by many today. Burge accordingly discusses it at considerable length. He does not, however, himself seem to think rigidity considerations raise problems as serious as the problem of insufficient cognitive value just noted. He has what I consider a convincing response, and even an obvious one. (Obvious in the sense that *once he has pointed it out* it may seem obvious *by hindsight*.)

One of the most important lessons of *Naming and Necessity* is that we must distinguish the notion of the "metaphysically" necessary, what could not have been otherwise, from the epistemological notions of the analytic and the *a priori*. Rigidity intuitions are *specific* to the "metaphysical" modal context "it could not have failed to be the case that," and without analogue for contexts such as "So-and-so knows/believes/says that." Frege is committed to a very definite view about the latter kind of context, but to no view at all about the former kind, since he simply has nothing to say about "metaphysical" necessity. Frege is not committed to any particular line on the behavior of proper names in the context of such a kind of modality, and therefore rigidity objections cannot touch him.

If there is a problem for Frege over metaphysical modality, it lies elsewhere, in the difficulty of making any room for this notion in his overall system. The problem is one that has been noted in special cases off and on in published and unpublished discussions over the years. Before trying to state it in its most general form, let me stop for some background.

Frege and Russell are so far ahead of even the best of nineteenth century "traditional" logicians that it is easy to overlook the fact that the logic of Frege and Russell is in a couple of features further from our own than is, say, Lewis Carroll's. For one thing, with Carroll, as the preposterousness of many of his examples shows, the interest is in whether a conclusion is *implied* by certain premises, regardless of whether the premises are true; whereas the *Grundgesetze* and *Principia* alike are concerned with *inference* from axioms presumed true. For another thing, Carroll begins the treatment of every example by specifying a "universe of discourse;" whereas for Frege and Russell first-order variables always range over absolutely all objects or individuals whatsoever. Under the influence of the great logicians, deduction from

hypotheses, on the one hand, and variable domains, on the other, did not receive adequate attention until the work of Gentzen and Tarski, respectively.

The features of Frege's logic just complained of are not accidental, but closely connected with his positive achievements. The insufficient attention to hypothetical deduction as opposed to categorical demonstration results from Frege's distinction of content and force, and his failure to consider closely any force but that of categorical assertion. (To this day the question of how the notion of force applies in the context of hypothetical reasoning remains troublesome.) A reason for a single all-encompassing category of objects is that it is needed for Frege's derivation of arithmetic from logic, and specifically for his "bootstraps" proof of the existence of successors.

These background points should not be very controversial; most are made by Burge himself in one place or another in the volume under review. But note a consequence of the last feature mentioned: Frege's notion of function per force differs greatly from that of his mathematical contemporaries. For mathematicians, a function always has a domain inside which it is defined and outside which it is undefined. The study of these domains was crucial to the analysis of complex-valued functions, not least at Göttingen, where Frege studied. Frege, however, notoriously abandons this feature of functions entirely, and demands that every function should give a value for absolutely every object whatsoever as argument.

But now what happens if we admit the Kripkean notion of necessity? Suppose there existed just one more concrete object than actually exists, say an electron  $e_0$  somewhere in the vastness of intergalactic space. Then to each actual function there would correspond countless functions differing from the actual function by assigning *some* value to  $e_0$ , and from each other by assigning *different* values to  $e_0$ . Inversely, if some actually existing object, say another and equally remote electron  $e_1$ , didn't exist, to every function that did there would correspond countless actual functions, differing from that function by assigning some value to  $e_1$  and from each other by assigning different values to  $e_1$ .

It seems hard to resist the conclusion that if there had been more or fewer or different objects from exactly the objects there actually are, then *none* of the functions there actually are would have been identical to any of the functions there would have been, which is to say that none of the functions there actually are would have existed. But since Frege insists (doubtless in some cases unnecessarily) on making almost every abstract object of interest into the course-of-values of some function, it

follows that no truth-values, no classes, no numbers, no directions, no linguistic expressions (as types rather than tokens) that actually exist would exist if the population of the universe were in any way different from what it actually is, though *other* truth-values, classes, numbers, directions, and linguistic expressions would exist in their places. Frege could avoid some of these embarrassing consequences by embracing some metaphysical fantasy such as Linsky's and Zalta's contingent concreteness or David Lewis's polycosmology, but as Heck says (again in private correspondence), "What a conclusion *that* would be!"

The foregoing *prima facie* difficulty will seem the more serious the more seriously one takes Frege's assertion that coextensiveness is the "analogue of identity" for concepts (and concurrence, or returning the same value for the same arguments, the analogue of identity for functions). This "extensionalist" reading of Frege is a common one, but Burge himself hints in more than one place at the possibility of a more "intensionalist" reading. He does not, however, in any of the works in this collection actually spell out how such a reading would deal with the difficulty noted. It would be interesting to see this attempted.

"Frege on Sense and Linguistic Meaning" is ultimately concerned to distinguish sense and meaning, but in a way unrelated to the distinction based on the behavior of context-dependent expressions that was so prominent in the paper just discussed. (The discussion of Frege's notion of sense turns on various things said by Frege in various places about concepts and thoughts, so it should be noted at the outset that while thoughts are senses and not denotations, concepts are denotations and not senses—or rather, as Heck reminds me, Frege's early usage of "concept" is ambiguous, but by "On Sense and Reference" at the latest, concepts are not senses.) Burge's proximate goal is to resolve two puzzles about sets of apparently incompatible views about concepts and/or thoughts endorsed by Frege. Since the solution is essentially the same in both cases, let me consider only the simpler of the two. Roughly speaking, the puzzle is this: Frege often seems to say that grasping concepts is easy and people do it every day, and yet often seems to complain that every other mathematician apart from himself lacks a sharp grasp of the most basic concepts of mathematics. As my rather artless formulation has perhaps given away, the solution Burge arrives at after carefully considering evidence from a number of different sources is that Frege crucially distinguishes mere grasp of a concept from *sharp* grasp of that concept.

For Frege, it is never correct to speak of the development of a concept, say the development of the concept of continuity from Newton

to Cauchy to Weierstraß, since a concept is a timeless abstract entity, not subject to change or development. Should we then speak of a series of *different* concepts being introduced (into discussion, not into existence, of course) at different times by different mathematicians? Or should we speak of the development of an increasingly sharp grasp of one and the same eternal and immutable concept? Burge rather disarms criticism by acknowledging that Frege speaks sometimes one way, sometimes the other; but his interest is in the latter way of speaking, and the aforementioned distinction among degrees of sharpness of grasp. Specifically, Burge is interested in this distinction as it bears on the relationship between Fregean sense and conventional linguistic meaning, though again he rather disarms criticism by acknowledging that Frege himself says directly very little about this matter.

Burge seems to favor the view that sense differs from meaning because in many cases where there is increasingly sharp grasp over time of one and the same sense, what we have on the side of meaning is a series of different meanings being introduced at different times. When Frege says that Weierstraß, one of the greatest mathematicians of his day, grasps the concept of natural number, but not sharply, so that his definitions are inadequate, Burge almost seems to want to take this to mean that the definitions in question are adequate as accounts of conventional linguistic meaning, but not as accounts of Fregean sense.

The question is complicated, however, by the fact (of which Burge is well aware, though he does not say much about it in the present context) that conventional linguistic meaning may go well beyond what even the most expert speakers can articulate after a little reflection. If we suppose grasp of conventional linguistic meaning involves internalization of certain rules of use, why should these semantic rules be any more directly available to consciousness on reflection, even on the part of the most eloquent speakers and writers, than syntactic or phonological rules? One may be able to discover what the general rules are only by considering first what *is* directly available to consciousness, the results of their application to a variety of cases, issuing in judgments of the type “That’s good English” or “That’s bad English,” and then looking for a pattern and conjecturing what the unconscious rules must be, and then testing the conjectures against further examples, and then revising the conjectures when exceptional cases are found to have been overlooked, and so on in a continuing dialectic until stability is achieved.

This is in fact how one proceeds in lexicography, skill at which is something distinct from skill at speaking or writing. (Shakespeare was an immeasurably greater writer than Doctor Johnson, but it does *not*

follow that if he had undertaken the task he would have produced a better dictionary.) Indeed, the methodology just sketched seems to be that not only of dictionary-makers but also of analytic philosophers. (The picture I have been drawing is simply a version of the standard solution to the “paradox of analysis.”) Thus it seems that we may distinguish mere grasp of conventional linguistic meaning (unconscious internalization of the conventional rules) from *sharp* grasp of conventional linguistic meaning (conscious ability to articulate a good definition). So though Weierstraß certainly had a grasp of the conventional linguistic meaning of “number,” we may still ask: was it a sharp one?

If we accept the view Burge attributes to Frege, we will have to distinguish *two* complex processes of gradually increasing, improving, and sharpening understanding: one for conventional linguistic meanings, the other for senses. Many (myself included) will be reluctant to embrace the idea that one can draw any real distinction among analytic philosophers between a Frege analyzing “senses” and, say, a Grice analyzing “meanings.” (Where would we class Moore, for instance?) Nonetheless, Burge brings forward quite an array of texts in attempting to make out such a distinction, and the things he has to say about them should be found informative by all who are interested in Frege studies, regardless of their theoretical orientations on such questions.

“Frege on Extensions of Concepts, From 1884 to 1903” is perhaps the most purely historical of the papers I have selected for special attention, the one the most concerned with close reading of particular texts. But if what one finds in the foreground is a particular textual puzzle, what is lurking in the background is a larger issue, already touched on in connection with (3) above, about how much room, if any, there is for arbitrary choice in definitions of the basic concepts of arithmetic, beginning with that of number. This latter question is one over which there can be disagreement even between writers quite opposed to any “structuralist” reading of Frege. The main puzzle text is the well-known footnote at the end of §68 of the *Grundlagen*, attached to the definition of number given there, in which Frege writes “I believe that for ‘extension of the concept’ we could simply write ‘concept.’” Can Frege really be suggesting in this passage that there is another, equally good, analysis as an alternative to the one he has given?

He cannot. This is generally agreed even if there are interesting differences in the *reasons* given. For Burge the main reason is simply that if Frege had held there were two or more equally good analyses, he would have trumpeted this finding and not hidden it away in an obscure footnote. For those of us who come at the subject from the

technical side, the main reason is that taking a number to be a concept rather than an extension of a concept and therefore an object would totally invalidate Frege's main proof. Either way, the puzzle is not over whether Frege meant to say there are two different, equally good, analyses, but over what he *did* mean, since he cannot possibly have meant *that*.

Anyone who has a taste for detective stories will enjoy and admire the way in which Burge hunts down clues in obscure sources as he searches for a solution to the mystery. Reviewers of detective stories generally make a point of not giving away the solution, so as not to spoil the reader's pleasure, and I almost regret having to give away the interpretation at which Burge eventually arrives, and for which he makes a convincing case. His conclusion is that Frege is saying, not that there are two equally good analyses, but that there are two equally good ways of wording one and the same analysis.

The Fregean doctrine being alluded to is that the denotation of "the concept *Horse*" is no more a concept than it is a horse, because "the concept *Horse*" is a noun phrase, and noun phrases that denote anything denote objects and not concepts. This is supplemented with the thought that the particular object "the concept *Horse*" denotes is *the extension of the concept Horse*. Of course, the extension of the concept *Horse* is also what is denoted by "the extension of the concept *Horse*," and *this*, according to Burge, is why Frege thought that for "extension of a concept" in his definition one could simply write "concept."

Burge then makes a good case for the claim that for a long time after the *Grundlagen* Frege actually thought he could motivate the claim that his notorious basic law V is an analytic or logical truth by appeal to the view that "the concept *F*" and "the extension of the concept *F*" denote the same thing. The opinion that they do was one of Frege's strangest, but the hope that he could motivate basic law V by appeal to this strange opinion is stranger still. And, of course, as Burge relates, Frege did eventually abandon this hope, and abandon the view that "the concept *Horse*" denotes an extension for the view that it is syncategorematic and denotes nothing.

After Burge's narrative reaches this point, he enters into controversial exegetical territory—notably containing a question, on which he differs from Heck, over whether Frege's personal doubts about basic law V were doubts about its truth or about its logical character—where I am hardly qualified to speak. Burge sketches an interesting view on which one may be sure that something is a law of logic *if it is true*, while being unsure that it is true. Bierce defines "self-evident" in the *Devil's Dictionary* as "evident to myself." According to Burge, this is



not what Frege means by self-evidence, and for Frege self-evidence is an intrinsic property of a thought that may not be evident to those who grasp it—if they do not grasp it sharply enough. Heck, by contrast, emphasizes Frege’s statement in the appendix to the *Grundgesetze*: that he has never concealed from himself Law V’s lack of the self-evidence that must properly be demanded of a law of logic.

Returning to the issue of how far, if at all, there is room for choice dictated merely by convenience in Frege’s series of definitions, and how far he takes his definitional choices to be forced upon him, if he is to remain faithful to a pre-existing eternal, immutable sense, there are other puzzles besides the one about the footnote that Burge discusses. One is the switchover from the definition in the *Grundlagen* (the number of a concept  $F$  is the extension of the concept “concept equinumerous with  $F$ ”) and that in the *Grundgesetze* (the number of a concept  $F$  is the extension of the concept “extension of a concept equinumerous with  $F$ ,” making the number two, in Russell’s words, “the class of couples”). I used to be confident that, as others have held, this is a case where Frege thought he had a choice, and realized as he increasingly familiarized himself with the powers of basic law V over the years between the two works, that the later version is more convenient. The two definitions do, however, only differ by the insertion of “the extension of,” so Burge’s discussion of the footnote does raise doubts.

Burge himself seems to favor the view that Frege saw himself as being so constrained by eternal and immutable senses as to have little if any room for free choice. I wonder how far he would take this view. If we consider the constant  $e$  in mathematical analysis, it may be characterized in two ways:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

Some analysts define  $e$  as the limit of the sequence on the left, and deduce from this definition the theorem that  $e$  is equal to the sum of the series on the right, while for others the series representation is the definition and the limit-of-a-sequence representation the theorem. Did Frege suppose, or does Burge suppose Frege supposed, that even in such a case a pre-existing “sense” of  $e$ , if not any “conventional linguistic meaning,” made one of these two choices objectively right, and the other objectively wrong? Go too far in the direction of an affirmative answer, and you will arrive at the zany view that of all the dozens and scores of known proofs of the Pythagorean theorem, there is one that is “the” proof. But where to stop?



Well, I myself will stop here. The reader should be aware that there is a great deal more food for thought in the volume than what I have discussed, and moreover that my discussion has been in a way slanted. I devoted most space to areas of disagreements with Burge, where I rightly or wrongly thought I had something to add, and usually passed quickly over areas where I could only say, like too many of Socrates' interlocutors in too many of the dialogues, "How true!" There are a lot of such areas in the volume.

One last disclaimer. I have mentioned Richard Heck so often that the reader may wonder why I haven't listed him as a co-author. So I should emphasize that he is *not* a co-author in the sense of bearing any responsibility for any errors in the above discussion. Those are all mine.

DEPARTMENT OF PHILOSOPHY, PRINCETON UNIVERSITY, PRINCETON, NJ  
08544-1006

*E-mail address:* [jburgess@princeton.edu](mailto:jburgess@princeton.edu)