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APPENDIX No. 6.

A NEW SYSTEM OF BINARY ARITHMETIC, BY BENJAMIN PEIRCE, CONSULTING GEOMETER, UNITED STATES COAST SURVEY.

CAMBRIDGE, *February* 25, 1876.

DEAR SIR: In sending you the inclosed paper upon a new system of binary arithmetic, I have no such extravagant thought as that of a substitute for our decimal system. I presume, however, that it is not unsafe to follow in the footsteps of Leibnitz, even in his excursions of pleasure. It seems to me, also, that it may be interesting to compute some of the fundamental numbers of science by a new arithmetic, for the purpose of comparison and verification.

Yours, very truly,

BENJAMIN PEIRCE.

CARLILE P. PATTERSON,
Superintendent United States Coast Survey.

1. Leibnitz proposed a system of binary arithmetic which he thought to be peculiarly fitted to exhibit the symmetry of certain arithmetical operations. Misled by erroneous reports, he believed that a similar system was originally used by the Chinese, as long as two thousand years before the Christian era.

2. The system here proposed retains the advantages of that of Leibnitz, while it is more economical of space, more so even than our ordinary decimal arithmetic. It admits of ready transformation into any other system of which the base is some power of two.

3. In the new system, as in that of Leibnitz, there are only two elementary characters, a vertical straight line and a circle, but their mode of use is interchanged. Leibnitz adopted the ordinary mode of ciphering, in which the circle, called the cipher, occupies each vacant space, while the vertical line is the only significant digit, and represents unity. In the new

system, on the contrary, the straight line denotes zero, and occupies each vacant space, while the circle is the significant digit, and stands for unity.

4. The places in the arithmetic of Leibnitz proceed continuously from right to left, as in ordinary arithmetic. But in the proposed system each odd place has, written above it, the next higher even place. Two such successive places constitute a *pair*, and the pairs succeed each other regularly from right to left.

5. Leibnitz required only two forms of type, corresponding to his two characters. But the new system involves three different forms of types, conforming to three different forms of the pairs. The circles, which represented unity, must be small enough to be written over each other, so that they are naturally reduced to a size such that the width of space occupied by a pair will be only half that of the ordinary figure. The three different form of pair are | ! :, and there is also ¡, which is the inversion of the second form.

6. Leibnitz proposes no nomenclature for his arithmetic, without which it is practically useless. In the new system, it is proposed to call each combination of two successive pairs, of which the right hand is an odd pair, a *quadrate*. There are, then, sixteen (or in the new system *onety*) distinct quadrates, of which the names and numerical representatives are as follows:

zero = || = |, four = !|, eight = ¡|, douze = :|
one = |! = !, five = !!, nine = ¡!, treize = !:
two = |¡ = ¡, six = !¡, ten = ¡¡, quorze = :¡
three = |: = :, seven = ¡:, onze = ¡:, quinze = ::

The successive places of the quadrates, counting from the right, are called *units*, *ties*, *tries*, *quads*, *quints*, *sies*, *septs*, and *octs*, which eight places constitute an *octad*. Each successive octad is divided into two parts, each of which consists of four quadrates. The right division of the octad is distinguished by the termination *illion*, and the left moiety by the termination illiad. The right-hand divisions of the octads are thus names, successively, *illions*, *billions*, trillions, and so on, while the left-hand divisions are named illiads, billiads, *trilliads*, and so on. In the case of very high numbers, however, and especially when the lower portion does not require to be designated, the Greek letter η may be placed after the numbers of the octade, with an exponent m, which shall express the place of the octade, and which shall be designated in enunciation, whether second, third, &c. This latter system corresponds to that adopted by Archimedes in his Arenarius.

7. The number

$$2^m - 2^{m-1} = 2^{m-1} = \text{¡}^m - \text{¡}^{m-1} = \text{¡}^{m-1}$$

is written with only one significant digit in the m^{th} place.

8. The number

$$2^m - 2^n = ¡^m - ¡^n$$

in which

$$m > n$$

consists of $m - n$ significant digits placed in continuous order, of which the highest digit occupies the m^{th} place, and the lowest digit the $n+!^{\text{st}}$ place.

9. Hence it follows that every number can be expressed in the form

$$¡^m - ¡^n + ¡^{m'} - ¡^{n'} + ¡^{m''} - ¡^{n''} + \&c.,$$

in which

$$m > n > m' > n' > m'' > n'' >, \&c.,$$

and where m, n, m', m'', &c., correspond to the highest places of the successive groups of continuous digits, and $n,'$ n', n'', &c., to the highest places of the successive groups of continuous zeros.

10. In multiplication, we have

$$\begin{aligned}(¡^m - ¡^n)(¡^{m'} - ¡^{n'}) &= !^{m+m'} - ¡^{m+n'} - ¡^{m'+n} + ¡^{n+n'} \\ &= ¡^{m+m'} - ¡^{m+n'+!} + ¡^{m+n'} - ¡^{m'+n} + ¡^{n+n'},\end{aligned}$$

in which we may assume

$$m > n \qquad m' > n' \qquad m - n > m' - n'$$

so that the product is a number of which the highest unit occupies the $m + m'^{\text{th}}$ place, and is the first of $m' - n' - !$ continuous units, at the rate of which is a single vacancy, followed by $(m - n)$ - $(m' - n')$ successive units, and afterward by $m' - n' - !$ vacancies, and lastly a unit in the $m' + n' - !^{\text{st}}$ place. This proposition supplies the place of a large multiplication-table. A skillful arithmetician will readily apply the principle upon which it is founded to facilitate multiplication and division.

11. Whenever a given number is divided into sets of m places, beginning at the right, it is evident that the sum of these sets divided by the number

$$¡^n - !$$

gives the same remainder as the number itself when divided by this divisor; and if the remainder is zero, the given number is divisible by the proposed divisor. The principle of this proposition is that which in ordinary arithmetic justifies the casting out of the nines. This divisor consists wholly of units.

12. Whenever the difference of the sume of the first, third, &c., sets of the preceding article — *i. e.* of the uneven sets and of the sum of the even sets — vanishes or is divisible by

$$¡^{n} + !,$$

the given number is also divisible by this divisor. The principle of this proposition is identical with that of the criterion in ordinary arithmetic for divisibility by eleven. The divisor of this article consists of two units separated by $n - !$ zeros.

13. The well-known expression of a perfect number becomes in this arithmetic $¡^{¡n+!} - ¡^{n}$ in which $¡^{n+!} - !$ must be a prime number.

This requires that $n+!$ should be a prime number, although this condition is not generally sufficient.

14. Were this arithmetic to be much used, which is quite improbable, the forms of the figures would undoubtedly become more flowing. They might come to resemble the present 6, 9, and 8, or they might assume the character of the Greek φ, or the circles might simply degenerate into crooks, more or less sharp, according to the peculiarity of the writer. But these variations would readily be understood, and would not embarrass the reader.