Martin Goldstern and Haim Judah, *The Incompleteness Phenomenon. A New Course in Mathematical Logic*, Wellesley, Massachusetts, A. K. Peters, 1995, xiii + 247 pp.

Reviewed by

ROMAN MURAWSKI

Wydział Matematyki i Informatyki
Uniwersytet im. Adama Mickiewicza
ul. Matejki 48/49
60–769 Poznań, Poland
E-mail: rmur@math.amu.edu.pl

The book under review is a course in mathematical logic. Divided into four chapters which, in the authors' opinion, can be taught in two semesters, it is based on the second author's lectures in logic given in Berkeley and Bar Ilan. The forward to the book was written by Saharon Shelah, who explains in it that logic is a branch of mathematics dealing with problems of exactness of mathematics itself, of what is a mathematical proof, of what mathematical theories are like, of what does it mean to be computable, of what are the powers of a mathematical theory, etc. He points out that logic is "one of the oldest intellectual disciplines yet also one which has developed enormously in this century" (p. ix).

The main focus of the book is the incompleteness phenomenon discovered by Gödel. It consists of the fact that "axiom systems cannot capture all semantical truths" (p. xi). This is the main result in basic mathematical logic.

The book is divided, as indicated above, into four chapters. The first two chapters provide a basic backgroud in the subject. All details are explained here so that a student not familiar with the abstract method used in mathematical logic can read about it. The chapter "The Framework of Logic" is devoted to induction, propositional calculus, predicate calculus and proof systems. The concept of induction is introduced in a very general way through the notion of an "inductive structure". This notion is essential for everything in the rest of the book. In the section devoted to proof systems a notion of a formalized proof and of a deductive system are introduced and their basic metamathematical properties are defined and proved. The notion of validity is also introduced in this chapter.

The second chapter is devoted to completeness and its aim is to show that the syntactical concept of "provability" coincides with the semantical concept of "validity". So one finds here the compactness theorem for propositional logic and first-order logic as well as the completeness theorem in the version stating that a theory is consistent if and only if it has a model. Henkin's proof of it is provided. The chapter closes with an application of the completeness theorem: it is shown that there are nonstandard models of the arithmetic of natural numbers.

The last two chapters of the book, on model theory (Chapter 3) and the incompleteness theorems (Chapter 4), are more sophisticated. Not all details are explained here, and it is assumed that the reader will be able to supply them.

In Chapter 3 several tools used in model theory are introduced and it is shown how they can be applied to classical problems of model theory such as finding the number of nonisomorphic countable models of a first-order theory. In particular, one finds here theorems on elementary substructures and chains, on joint consistency and interpolation, on ultraproducts and compactness, on types and saturated models, on universal, prime and atomic models as well as on categorical theories.

The main chapter of the book is Chapter 4 devoted to Gödel's theorems. It is shown that Peano arithmetic is incomplete and then a general version of this theorem is given. The method of arithmetization due to Gödel being the essential tool in incompleteness proofs is explained in detail. The proof of Gödel's theorem is based on the liar paradox. To simplify the proofs the authors decided to prove everything semantically rather than syntactically (working in the model of natural numbers rather than talking about derivations from Peano's axioms). The chapter ends with an introduction to recursion theory.

It should be stressed that at the end of each section several exercises have been included which are an intrinsic part of the book because the authors believe that "it is impossible to *understand* mathematics without actually *doing* mathematics" (p. xiii). The exercises help the reader to see better the meaning of the notions presented as well as the theorems proved in the book. Various new notions are also introduced in exercises. The text of the book itself also contains many examples which should help the reader to understand the new notions.

The book under review is a nice introduction to mathematical logic presenting its basic notions and theorems. I agree with the authors who say in the Introduction that "this book should be part of the basic background of every student in any discipline which employs deductive and formal reasoning as a part of its methodology" (p. xi). In fact, the understanding of the role and the necessity of logic is actually becoming much less clear. For example, courses in logic are reduced not only in faculties of humanities but even in faculties of science (at least this is the case at Polish universities) feeding the irrationalism and . . . postmodernism. We logicians should try to oppose these fatal tendencies. The book by Goldstern and Judah is a valuable contribution to this fight.