

**"GOVERNMENT BY LOGIC". REVIEW OF: A MATHEMATICAL  
APPROACH TO PROPORTIONAL REPRESENTATION: DUNCAN  
BLACK ON LEWIS CARROLL**

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by

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The editors of this book are responsible for the preservation of Duncan Black's (1908 – 1991) papers now at the University of Glasgow. Black, an economist with a strong interest in political science best known for his classic book, *The Theory of Committees and Elections* (1958), left an immense store of documents, books and papers, including chapter plans and versions of chapters for a book on Lewis Carroll's theory of proportional representation (PR). The book under review is the completion of this project which occupied the final thirty years of Black's life.

The title of the book, however, is misleading. Much more than PR is covered; in particular, Black establishes the broad connections between logic and voting theory (apportionment and PR being one piece; majority rule or social choice the other), and evaluates Carroll's contributions to this embryonic school of politics.

Voting theory is part of collective rational decision-making involving the relationships between the preferences of people and the resolution of those preferences as a group choice. The basic question addressed: what procedures utilizing those preferences yield the fairest outcomes as collective actions? depends on the notion of a social choice function, a rule that uses

the individual preference orders to determine a single order of all the outcomes. The history of the subject goes back to the eighteenth century and in one sense culminates in this century with the work of Kenneth Arrow [Arrow 1951] who proved the extraordinary theorem that rational group decision-making, rationality being defined by four axioms considered reasonable, from a group ranking based on individual rankings, is impossible.

Faced with this negative result, there have been several attempts at modification of the conditions of the theorem to enable rational group decision-making in an altered form. One of these attempts, the weakening of the form of the group choice ranking, is part of current research in the logic of non-monotonic reasoning. (On this topic, see [Doyle and Wellman 1992].)

In Black's judgment, Carroll's early work on majority rule entitles him to a place second only to the Marquis de Condorcet (1743 – 1794), arguably the greatest voting theorist prior to Arrow. Black puts forth, for the first time, the thesis that in developing his theory of the committee Carroll was "putting into logical form, something which cannot be expressed in the ordinary extensional Logic, but requires another form of Logic, intensional Logic . . . . And Carroll's theory of the committee, I would suggest, provides this Logic of Intensity which is not to be found in the textbooks" (p. 41). (Intensionality, a variant of intentionality, the property of mental states by which they are directed toward an object, refers to the content of the concept categorizing the object.)

If Black's judgment is correct, that Carroll provided the beginnings of a theory of intensive logic, then both the history of intensionality, begun by the philosopher Franz Brentano (1838 – 1917) who greatly influenced Edmund Husserl (1859 – 1938), as well as the history of the theory of intrapersonal preference given by the economists Vilfredo Pareto (1848 – 1923) and Ragnar Frisch (1895 – 1973) must be revisited.

Black also discusses Carroll's more general work in logic, particularly his use of diagrams, in two different ways. The first use is in deducing logical conclusions from the premises of formal arguments in syllogistic form. Recently, Anthony J. Macula [Macula 1995] presented an application of Carroll's set diagrams, allowing the depiction of all the possible intersections for ten or more sets, to a problem in combinatorics. The second use is in Boolean algebra as applied to the design of electric circuits in the form of an attribute map first constructed by Maurice Karnaugh in 1953. Black shows this map to be a topologically deformed Carroll diagram (p. 38).

However, Black's overall assessment of Carroll's contributions to Logic, suffers from his not knowing of the work of William W. Bartley [Bartley 1977]. Black writes, "Carroll's contribution to Logic . . . lies either in the indirect contribution through the *Alice* books, or in his contribution through the logical formation of political theories" (p. 39). Bartley's discoveries have led to a new appreciation of Carroll's logic writings, especially using his puzzle problems in automated deductive systems for sorted logics. (On this topic, see [Lusk and Overbeek 1985; Cohn 1989; Frisch 1991].)

Black planned a book in four parts. The editors have followed Black's wishes, confining their own overview to a thirty page Introduction with a list of ninety-nine references. They write, "*The Principles [of Parliamentary Representation (PPR)]* is the earliest known work to discuss both the assignment of seats to each of a number of multi-member districts (the apportionment problem) and the assignment of seats within each district to the parties (the PR problem)" (p. xxv).

Part 1, "The Life and Logic of Lewis Carroll" includes the major subsection, "Government by Logic". Here Black revisits Carroll's work on majority rule theory in three pamphlets [Dodgson 1873, 1874, 1876], and gives his view of Carroll as a mathematician.

Part 2, "The Principles of Parliamentary Representation", is an edited version of Black's previously published work on Carroll's theory of PR, three articles that appeared between 1967 and 1970 that included his understanding of Carroll's arguments in an historical and psychological setting.

The third and longest part is Black's analysis of Carroll's theory of PR, his unpublished material providing the *raison d'être* for this book. Here we have Black's views on Carroll's three pamphlets on PR [Dodgson 1884, 1885], and Black's formulation and proof of Lewis Carroll's Theorem on quota which he shows to be equivalent to the quota for allocating seats in a legislative body to the political parties given by the Belgian mathematician, Victor d'Hondt (1841 – 1901).

The final section of this part, dealing with the allocation of parliamentary members to each district in proportion to its number of voters, Black left incomplete. The editors have continued with the exposition they believe Black intended, and added their own evaluation of the result.

Part 4 contains the reprints of Carroll's pamphlets on PR and the main sources Black thought influenced them, by James Garth Marshall (1802 – 1873) and Walter Bailey (1837 – 1917), together with comments by the

editors. These are rare pieces, difficult to locate, and we should be grateful to the editors for making them available.

Both Black and Carroll wrote about voting theory: Black, the professional, because it was his work; Carroll, the dilettante, because he was responding to external events that required the support of fundamental principles to guarantee fairness of application. Black experienced much difficulty getting his work published; Carroll's serious work went unrecognized. Black established Carroll's high reputation in the theory of majority ruler (social choice) in his 1958 book, and the editors lay out the reasons. By presenting the fruits of Black's continued quest to establish the importance of Carroll's contributions to PR, the editors have shown for both of them the originality of their thinking and the priority of their work.

Regrettably, Black did not keep up with modern interpretations of Carroll's life and work, and this mars much of his evaluation of Carroll's intellectual and psychological sides. Surprising, too, is that Black did not seem to know the work of Peter Fishburn who has written extensively on Carroll's theory of majority rule (See, for example, [Fishburn 1973].)

Black wrote an article on Carroll's theory of PR in *Jabberwocky* that the editors inexplicably have not mentioned. [Black 1970]. In their discussion of the maximin criterion (Nash equilibrium strategy), the editors have not accurately described the contribution to game theory by the mathematician, John F. Nash. They write, "As game theory had not been invented and Nash not born, it was not surprising that Carroll's pamphlet [*Principles of Parliamentary Representation*] had not been understood." (p. xix) Nash did not define the concept of an equilibrium point — that had been done early in the nineteenth century by A. A. Cournot, [Cournot 1971]. What Nash achieved was the proof that equilibrium points actually exist in a wide variety of non-cooperative games [Nash 1951].

There is a number of typographical errors, none important, the most egregious being the duplicated section on pp. 72–73. However, Kluwer, a well-respected publisher of academic books, could have expended greater effort in producing the index which lacks sufficient depth and omits pages that ought to be listed under existing headings.

This book will be appreciated by the informed reader as an addition to the *operi* of both Black and Carroll, and by the specialist in voting theory interested in aspects of its history. But it cannot be considered a completely reliable addition to our understanding of the intellectual and psychological dimensions of Lewis Carroll.

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