GEORGE S. BOOLOS

GEORGE S. BOOLOS, professor of philosophy, well known logician from the Massachusetts Institute of Technology, and well known figure in logic circles in the Boston/Cambridge area, died of pancreatic cancer at his home in Cambridge, Massachusetts on Monday 27 May 1996. He was in his fifty-fith year. He is survived by his mother, his wife Sally Sedgwick who is a professor at Dartmouth College, and his son Peter from a previous marriage.

Boolos, of Greek-Jewish descent, was born in New York City and educated at Princeton University, from which he obtained his undergraduate degree in mathematics in 1961. He thereafter attended Oxford University as a Fulbright Scholar and received the B.Phil. in 1963. He received his Ph.D. in philosophy from MIT in 1966, the first such degree to be offered there, and taught at Columbia University from 1966 to 1969 before returning to MIT where he remainded thereafter. He was a long-time member of the Association of Symbolic Logic, serving on its council and executive committee from 1982 to 1985, its Vice President from 1992 to 1995, and became its President in 1995. He also served as an Editor for Reviews for the *Journal of Symbolic Logic* until his illness.

The Boston Globe of Friday, May 31, 1996, wrote of him (p. 28):

> A logician and philosopher, Mr. Boolos was an originator of provability logic and an expert on the work of 19th century philosopher Gottlob Frege.
>
> [...]
>
> The recipient of a 1996 Guggenheim Fellowship to complete a book on Frege, he was the author of *The Logic of Provability* and co-author, with Richard C. Jeffrey, of *Computability and Logic.*
>
> [...]

One of his students, Norman D. Megill, commented (Article 18288 in sci.logic): "he was a wonderful teacher. One of his trademarks, I recall, was always to write with a fountain pen containing brown ink." (Irving Anellis verifies that a letter he received from Boolos dated 7 December 1983 was indeed written in brown ink.)

Boolos developed provability logic in the late 1970s and early

1980s as an application of modal logic to study formal provability. In the "Preface" to his book *The Logic of Provability* [*1993*, ix], Boolos explained what provability logic is by saying simply that: "When modal logic is applied to the study of provability, it becomes provability logic." The "Introduction" to *The Logic of Provability* [*1993*, xv–xxvi] displays a sensitivity and appreciation for the history of the subject and after briefly outlining the history in particular of modal logic, goes on to establish the connection between provability logic and the development by Hilbert and Gödel of *Beweistheorie.* For example, in 1983 he wrote, with Giovanni Sambin of the University of Siena, the paper "An Incomplete System of Modal Logic", which, as he described to Anellis, "deals with the strength of Löb's theorem" (private communication, Boolos to Anellis, 7 December 1983).

In 1989 he published a new proof of Gödel's first incompleteness theorem [*1989*]. It is a nonconstructive proof of the theorem, using Berry's paradox, in the form

There is no algorithm A whose output contains all true statements of arithmetic and no false ones.

Unlike most proofs of the Gödel incompleteness result, Boolos's is rather short and does not use diagonalization. Richard Vesley noted (see [Barwise *1989*]) that Boolos did not also show that (1) there is an algorithm, A_1 such that if A is a correct algorithm of a certain kind (for a certain formal arithmetic system), then A_1 applied to A yields a truth not given by A, and (2) that if A is a correct algorithm, then A_1 applied to A yields a truth not given by A, whereas Gödel did prove (1) and even sought to provide an algorithm A_1. Responding to comments on his proof, Boolos [*1989a*] remarked that what is especially interesting is not the brevity of his proof, but that it "provides a *different sort of reason* for the incompleteness of algorithms."

Boolos planned to submit "An Incomplete System of Modal Logic" as a substitute for the planned, but never published proceedings of the American Mathematical Society Special Session on Proof Theory organized by Irving Anellis and held in January 1983 in Denver, Colorado; it was proposed for the proceedings in lieu of the paper "Don't Eliminate Cut!" on which Boolos spoke, under the title "Keep Cut" [*1983*], at the special session. "Don't Eliminate Cut!" was ultimately published [*1984*] in the *Journal of Philosophical Logic.* "An Incomplete System of Modal Logic", ultimately published [*1985*] in the *Journal of*

Philosophical Logic.

A close associate of Richard Jeffrey and friend of the late Jean van Heijenoort, Boolos was interested in Smullyan trees and contributed in an important way to certain recent modifications in its presentation in Jeffrey's textbook [*1967*] *Formal Logic*. He saw Raymond Smullyan's [*1968*] book *First-Order Logic* as clarifying the tableau method, and is quoted by Ira Mothner [*1985*, 302] as saying that "What characterizes Ray's books and papers is how astoundingly clear they are. He is able to strip away what is inessential and give you the core of an idea, undiluted and unvarnished. He is a great simplifier." In the early- to mid-1980s, a number of logicians, among them George Boolos, for example in his unpublished papers "Don't Eliminate Cut!" [*1982*], and "Keep Cut" [*1983*] — originally written the America Mathematical Society's Special Session of Proof Theory organized by Irving Anellis held in Denver, Colorado in January 1983 — which were early versions of his published paper "Don't Eliminate Cut" [*1984*], and especially his paper "Trees and Finite Satisfiability: Proof of a Conjecture of Burgess" [*1984a*], have produced and published results on trees that were first dealt with by van Heijenoort in his unpublished papers. In particular, Boolos in [*1984a*] proved Burgess's conjecture of 1982 that the Smullyan tree method proves the finite satisfiability of any finitely satisfiable first-order formula to which it is applied; he thereby proved the "weak" soundness (and completeness) of the tree method. Boolos's result was extended by Miodrag Kapetanović and Aleksadar Krapež, of the Mathematics Institute in Belgrade, (Serbia) Yugoslavia, to languages with function symbols in their [*1987*] paper "More on Trees and Finite Satisfiability: The Taming of Terms". This work of Boolos was in line with van Heijenoort's rather cursory proofs of the principles of converging and diverging induction (upward and downward induction) for trees in his paper "Falsifiability Trees" (see, e.g. [van Heijenoort *1974*, 24], and N. L. Wilson's 1981 proof of upward and downward induction on trees in [*1983*] in "The Transitivity of Implication in Tree Logic". In the second edition of his textbook *Formal Logic* [*1981*, §2.5], Jeffrey declares that restricting propositional logic to the tree method gives an incomplete view of the subject. One of the principal difficulties is that tree proofs lack the transitivity of proofs in virtue of which proofs are distinguished from tests of validity. By transitivity, we mean the following condition:

Let T be a proof of P from $Q_1, \ldots, Q_m$ and let $T_1, \ldots, T_n$ be proofs respectively of from $X_1, \ldots, X_n$ (where X_i is a set of formulæ, $i = 1, \ldots, n$).

Then there must be a uniform method, i.e. a method independent of the content of the given proofs, for constructing from T_1 ,..., T_n a proof of P from X_1, ..., X_n.

Jeffrey satisfies this condition in §2.5 of the second edition of his textbook by introducing the rule XM of excluded middle into the set of rules for trees to obtain deductive trees. XM had already been present in Toledo's [*1975*, 158] system C_5^2 as the cut rule, and was the basis for Boolos's [*1982*, *1983*, *1984*], plea not to eliminate cut and of Wilson's introduction of XM (what he calls the "forking laws") into Jeffrey's earlier [*1967*] set of rules. Toledo must have learned the rule directly from Smullyan inasmuch as it is, as Perry Smith has said "behind the scenes" in his [Smullyan's] proof of the cut elimination theorem in his [*1968*] *First-Order Logic.*

For Boolos, "Don't Eliminate Cut!" [*1982*] was intended as a contribution to the discussion of the comparative pedagogical merits of natural deduction and the tree method. In particular, it was a contribution to the question whether beginning students should encounter free but quantified variables and whether flagging of variables is natural. The aims of "Don't Eliminate Cut!" [*1982*] were (1) to display an example of an inference which is valid but whose validity cannot, humanly, practically, be proved by the tree method but which can be shown, however tediously (the one given by Boolos in the [*1982*] manuscript is one single-spaced typed page long), to be valid using natural deduction; and (2) to describe a virtue of natural deduction which the tree method lacks. The partivular virtue to which Boolos was referring was that, in common with Hilbert-style axiomatizations of logic, it allows development and use within derivations of "subsidiary conclusions", i.e. of lemmas. The proposal that we were left with by Boolos was precisely the adjunction to the tree method of the XM rule.

Richard Heck of Harvard University informs us that a fund has been set up at MIT, the George S. Boolos Memorial Fund, intended to provide support to graduate students interested in logic, philosophy of mathematics, etc., at MIT. Any contributions to this fund would, obviously, be much appreciated, as it is hoped that it will be able to provide significant support to those working in the areas George Boolos loved. They can be sent to:

George S. Boolos Memorial Fund

Department of Linguistics and Philosophy, Room 20D-213
Massachusetts Institute of Technology
Cambridge, MA 02139

References

I. SOME PUBLICATIONS OF GEORGE BOOLOS:

1968. (with Hilary Putnam), *Degrees of unsolvability of constructible sets of sentences*, Journal of Symbolic Logic **33**, 497–513.

1974. (with Richard C. Jeffrey), *Computability and logic*, Cambridge, Cambridge University Press; 2nd ed., 1981; 3rd ed., 1989.

1976. *On deciding nthe truth of certain statements involving the notion of consistency*, Journal of Symbolic Logic **41**, 779–781.

1977. *On deciding the provability of certain fixed point statements*, Journal of Symbolic Logic **42**, 191–193.

1979. *The unprovability of consistency: An essay in modal logic*, Cambridge, Cambridge University Press.

1979. *Reflection principles and iterated consistency assertions*, Journal of Symbolic Logic **44**, 33–35.

1980. *Provability, truth and modal logic*, Journal of Philosophical Logic, 1–7.

1980a. *Omega-consistency and the diamond*, Studia Logica **39**, 237–243.

1980b. *Provability in arithmetic and a schema of Grzegorczyk*, Fundamenta Mathematicæ **96**, 41–45.

1980c. *On systems of modal logic with provability interpretations*, Theoria **46**, 7–18.

1982. *Don't eliminate cut!*; preprint, 5pp.

1982a. *Extremely undecidable sentences*, Journal of Symbolic Logic **47**, 191–196.

1982b. *On the nonexistence of certain normal forms in the logic of provability*, Journal of Symbolic Logic **47**, 638–640.

1983. *Keep cut*, Abstracts of Papers Presented to the American Mathematical Society **4**, #801-03-415, p. 9.

1984. *Don't eliminate cut*, Journal of Philosophical Logic **13**, 373–378.

1984a. *Trees and finite satisfiability: proof of a conjecture of Burgess*, Notre Dame Journal of Formal Logic **25**, 193–197.

1984b. *The logic of provability*, American Mathematical Monthly **91**, 470–480.

1985. (with Giovanni Sambin), *An incomplete system of modal logic*, Journal of Philosophical Logic **14**, 351–358.

1987. (with Vann McGee), *The degree of trhe set of sentences of predicate provability logic that are true under every interpretation*, Journal of Symbolic Logic **52**, 165–171.

1989. *New proof of the Gödel incompleteness theorem*, Notices of the American Mathematical Society **36**, 388–390.

1989a. *A letter from George Boolos*, Notices of the American Mathematical Society **36**, 676.

1990. Review of V. A. Uspensky, *Gödel's incompleteness theorem*, Journal of Symbolic Logic **55**, 889–891.

1991. (with Giovanni Sambin), *Provability: The emergence of a mathematical modality*, Studia Logica **50**, 1–23.

1993. *The Logic of Provability*, Cambridge, Cambridge University Prress.

1993. *The analytical completeness of Dzhaparidze's polymodal logics*, Annals of Pure and Applied Logic **61**, 95–111.

II. ADDITIONAL REFERENCES:

BARWISE, J. 1989. *The last word on Boolos' proof of Gödel's theorem*, otices of the American Mathematical Society **36**, 1352.

JEFFREY, R. C. 1967. *Formal logic: its scope and limits*, New York, McGraw-Hill.

—. 1981. *Formal logic: its scope and limits*. New York: McGraw-Hill, 2nd edition.

KAPETANOVIĆ, M. & **KRAPEŽ**, A. 1987. *More on trees and finite satisfiability: the taming of terms*, Notre Dame Journal of Formal Logic **28**, 392–394.

MOTHNER, I. 1985. *Raymond Smullyan*, in D. J. Albers and G. L. Alexanderson (editors), *Mathematical people: profiles and interviews* (Boston/Basel/Stuttgart, Birkhäuser), 299–307.

SMULLYAN, R. M. 1968. *First-order logic*. Berlin/Heidelberg/New York: Springer-Verlag.

TOLEDO, S. A. 1975. *Tableau systems for first-order number theory and certain higher order theories*, Lecture Notes in Mathematics **447**, Berlin/ Heidelberg/New York, Springer-Verlag.

VAN HEIJENOORT, J. 1974. *Falsifiability trees*; ts., 25pp.; 30 September. Copy in Anellis archives.

WILSON, N. L. 1983. *The transitivity of tree logic*, Notre Dame Journal of Formal Logic **24**, 106–114.

THE EDITOR