Gila Sher, *The Bounds of Logic: A Generalized Viewpoint.* MIT Press, Cambridge, Massachusetts, 1991, 178 + xv pages.

Reviewed by

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Gila Sher opens her book *The Bounds of Logic* by quoting from the introductory chapter to Barwise and Feferman's 900-page survey of a quarter century's experimentation with those bounds: "Whatever the fate of the particulars, one thing is certain. There is no going back to the view that logic is [standard] first-order logic" [Barwise and Feferman *1985*, 23]. The research effort devoted to model-theoretic and alternative logics since the 1960s has been immense. Most of this effort has gone into the study of their properties, focusing especially on completeness, compactness, and Löwenheim-Skolem theorems. By contrast, little has been done to clarify the philosophical implications of the very existence of these logics for the status of standard first-order logic. What *is* logic if it is no longer standard first-order logic? What exactly differentiates the logical from the non-logical? Could there be explicit criteria for logicality which would force even the advocate of the "first-order thesis" to concede that logic is more than what we continue to call *standard* first-order logic?

Determining what is or is not logic depends on deciding which expressions in the language under consideration should count as logical. Therefore a question underlying all of the above questions is that concerned with distinguishing between logical and nonlogical constants. In a recent paper addressing this question and including a brief survey of other attempts to do so, Kosta Došen notes that in the history of modern logic the dominant attitude even among "philosophically inclined" logicians has been "a certain skepticism as to whether the distinction between logical and nonlogical expressions can be clearly drawn. Most logicians, like so many followers of Protagoras, are content with just listing what they take as logical constants" [Došen *1989*, 363]. Alfred Tarski, for example, was being either skeptical or open-minded when he made the following comment in his famous paper, "On the Concept of Logical Consequence":

> Underlying our whole construction is the division of all terms of the language discussed into logical and extra-logical. This division is certainly not quite arbitrary.... On the other hand, no objective grounds are known to me which permit us to draw a sharp boundary between the two groups of terms. It seems to be possible to include among logical terms some which are usually regarded by logicians as extra-logical without running into consequences which stand in sharp contrast to ordinary usage. [Tarski *1936a,* 418–419]

Nevertheless, a few philosophically-inclined logicians (and logically-inclined philosophers) have tried to define that boundary precisely. For the most part, they have chosen either a proof-theoretic or a model-theoretic approach.

Proof-theoretic approaches to the problem of characterizing the logical constants naturally tend to emphasize those aspects of logic having to do with the study of deduction, and therefore seem to be based, more often than not, on Gentzen's sequent calculus. An early attempt along such lines was made by Karl Popper in the 1940s, although his purpose may have been more to show how the meanings of logical constants could be defined syntactically, than to distinguish the logical from the nonlogical (see [Schroeder-Heister *1984*]). W. Kneale does something similar in "The Province of Logic" [Kneale *1956*] In a more recent paper, Ian Hacking suggests that a logical constant is one that can be defined using certain Gentzen-like rules of inference which preserve a very small number of basic facts about deducibility. In this way, he extends standard logic up to the ramified theory of types but no further towards second-order logic, noting that "this is just what the authors of *Principia* expected; for in their opinion the ramified theory is logic, but the simplified theory obtained by adding the axiom of reducibility is not logic" [Hacking *1979*, 287]. Finally, Kosta Došen has introduced an interesting Gentzen-derived criterion based on the idea that logical constants "serve as 'punctuation marks' [in the object language] for some structural features of deductions" [Došen *1989*, 362]. As far as this reviewer knows, Došen's paper is the latest to address the question from a proof-theoretic angle.

Among model-theoretic results should probably be included Per Lindström's well-known theorem that no proper extension of standard first-order logic which is either complete or compact can also have the Löwenheim-Skolem property (the property that every formula having an infinite model has a countable model). In "Which Logic Is the Right Logic?" [Tharp *1975*], Leslie Tharp uses Lindström's theorem as a starting point for examining the appropriateness of taking one or more of completeness, compactness, or the Löwenheim-Skolem property as necessary characteristics of a genuine logic, and seems to opt for completeness. With regard to logical constants, he suggests some criteria based on the notion of continuity of quantifiers which help to explain why the quantifiers of standard logic are in some sense primary, but which seem inapplicable to anything more complex than monadic logic. Other proposals for characterizing logical constants which have a model-theoretic flavor are Christopher Peacocke's [*1976*] and Timothy McCarthy's revision of Peacocke's criterion [McCarthy *1981*].

Gila Sher's book appears to be the most recent contribution, on the model-theoretic side, to answering the question, What is a logical constant? and hence, What is logic? Her interest in such questions, and in identifying what she calls the "philosophical force" of "core" logic — "Fregean Russellian mathematical logic with Tarskian semantics" — motivated her work as a graduate student under Charles Parsons at Columbia University, and *The Bounds of Logic is* a revised version of her Ph.D. thesis. Parts of the book have also appeared previously in two of her published papers [Sher *1989a* and *1990*].

Perhaps because defining new quantifiers has proved to be the most venerable method over the past three decades for testing the boundaries of "core" logic, Sher was soon led to a study of generalized quantifiers as a concrete means of approach to the philosophical understanding she was looking for. In the preface to her book, Sher justifies this approach and indicates how the various philosophical questions become interrelated through pursuing it:

> The generalization of quantifiers gives rise to the question, What is logic? in a new, sharp form. In fact, it raises two questions, mutually stimulating, mutually dependent. More narrowly, these questions concern quantifiers, but a broader outlook shifts the emphasis: What is it for a *term* to be logical? What are *all* the terms of logic? Sometimes in the course of applying a principle, we acquire our deepest understanding of it, and in the attempt to extend a theory, we discover what drives it. In this vein I thought that to determine the full scope of logical terms, we have to understand the idea of logicality. But the actual expansion of quantifiers gives us hands-on experience that is, in turn, valuable in tackling "logicality." (pp. ix–x)

The emphasis shifts several times in the course of Sher's investigation — from one question to another, related one, and back again. Rather than making a detour around tentative formulations of the problem and its solution in order to go more directly to her conclusions, Sher describes in detail the road her investigation actually followed — as she puts it, "leading the reader from the questions and gropings of the early chapters to the answers in the middle and from there to the formal developments and the philosophical ending" (p. x). This has the advantage of illuminating more than one side of the problem before proposing the solution, and prepares the reader to appreciate the full force of the solution when it finally comes. Sher's writing is a model of clarity, preventing any impatience one might have with this plan, as well as making her book accessible to readers with a low tolerance for technical details.

Most of this review will be devoted to tracing Sher's itinerary, showing how she develops her new conception of logic from ideas inspired by the work of Andrzej Mostowski and Alfred Tarski especially, and highlighting some of the interesting historical aspects revealed along the way. Sher's point of view is not explicitly historical. For example, concerning her use of Tarski's early papers she writes at one point: "...we are interested in the legacy of Tarski, not this or that historical stage in the development of his thought. For the intuitive ideas we go to the early writings, where they are most explicit, while the formal constructions are those that appear in his mature work" (p. 41). Nevertheless, *The Bounds of Logic* appears to be an important contribution not only to the philosophy of logic but also to a clearer understanding of some of the main currents in its history.

In her introduction, Sher observes that "...few in philosophy have suggested that the very principles underlying the 'core' first-order logic might not be exhausted by the 'standard' version" (p. 8). The first logician to make a suggestion of this kind may well have been Andrzej Mostowski, who, by a simple extrapolation from the standard version, invented a new family of generalized quantifiers expressing cardinality properties such as "for exactly five *x*...", "for more *x* than not...", and so on. In his pioneering 1957 paper, he ventured the following philosophical remark:

> ...we believe that some at least of the generalized quantifiers deserve a closer study and some deserve even to be included into systematic expositions of symbolic logic. This belief is based on the conviction that the construction of formal calculi is not the unique and even not the most important goal of symbolic logic. [Mostowski *1957*, 12]

The idea that Mostowski's generalized quantifiers might be genuine logical terms has not been immune from criticism, at least of an indirect sort. This has ranged from W. V. Quine's assurance that we need not take logic to be anything more than standard first-order logic, to observations that any logic containing Mostowskian quantifiers must be incomplete, to those proposals for a criterion for logical term which turn out to be too restrictive to include very many such quantifiers. Among the latter are the criteria of Christopher Peacocke and Timothy McCarthy.

Although Sher provides a rebuttal to McCarthy's views later in her book, here she prefers to consider at length certain criticisms of Mostowskian quantifiers due to logicians working in the field of natural language. Mostowski's quantifiers can be used to analyze many English sentences for which the standard quantifiers are clearly inadequate — sentences such as those of the form, "most things in the universe have property *P*" — but they fail in the analysis of sentences such as, "most things which are *P* are *Q*." Sher concentrates on the influential [*1981*] paper by Jon Barwise and Robin Cooper, "Generalized Quantifiers and Natural Language," in which the authors pointed this out and tried to avoid the limitations of Mostowskian quantifiers by developing a theory of nonlogical quantification in which quantifiers are identified with noun phrases.

In general, Sher has a number of reasons for devoting as much space to linguistic applications as she does throughout her book. For one thing, as her detailed discussion of Barwise and Cooper's theory shows, the use of quantifiers to analyze the logical structure of natural language raises new questions about the nature of quantification. Elsewhere, Sher is able to cite in support of her own conclusions the research of linguists such as James Higginbotham and Robert May who lean toward the view that quantifiers in natural language are logical. Finally, there is Sher's evident interest in linguistics for its own sake. Nevertheless, linguistic considerations remain secondary: a source of ideas, but not the primary motivation for her work, whatever ramifications her work may have for linguistics — and judging from the favorable reviews her book has received from some linguists, these could prove to be substantial.

After concluding that Barwise and Cooper's theory only succeeds in explaining certain interesting "linguistic regularities," Sher opts for the view that Mostowski simply did not take his generalization far enough. He considered only *one*-place quantifiers modeled syntactically on the familiar quantifiers of standard first-order logic. But allowing n–place quantifiers for $n > 1$ eliminates many of the objections of Barwise and Cooper. For example, "most" in the sentence, "most things which are P are Q," is a 2-place quantifier. In any given model $\mathfrak{M}$ with universe A, this quantifier is defined by the set of all pairs $\langle B, C\rangle$ of subsets of A such that $|B \cap C| > |B - C|$. Syntactically, the sentence quoted above would be rendered $(Mx)(Px, Qx)$, where (Mx) is the 2-place quantifier "most" and where Px and Qx are formulas with the variable x free.

Further generalization would yield logical predicates in addition to the familiar predicate of equality, logical quantifiers over relations, logical functors and logical quantifier functors. An example of a quantifier over relations might be the 1-place "well-ordering" quantifier (Wxy) over 2-place relations, where $(Wxy)Pxy$ is true in a model if P is a well-ordering of some subset of the universe. And whereas a 1-place logical quantifier is defined in a given model by a *set* of subsets of the universe (for example, the existential quantifier by the set of all the non-empty subsets), a 1-place logical quantifier *functor* would be defined by a *function* mapping subsets to subsets; one of Sher's examples is a "complement quantifier functor," defined in a given model by a function mapping each subset of the universe to its complement (p. 58).

We have sketched what is essentially a "maximal" extension of Mostowski's system due to Per Lindström (whose 1966 paper, unlike Mostowski's, contained no comments of a philosophical nature). At this point, Sher evidently wants to adopt Lindström's system as a basis for defining the notion of logical quantifier, but there is a problem. Mostowski had considered it fundamental that a logical quantifier "not allow us to distinguish between different elements" of the universe [Mostowski *1957*, 13]. He was able to make this requirement precise for his own quantifiers, and so far Sher has taken his formulation for granted. But now it is not so clear how Mostowski's precise formulation applies to the more complex quantifiers of Lindström's system, nor even why his requirement should be considered necessary in the first place. In order to resolve these issues, Sher decides she must take her investigation up another level of abstraction and consider the question of what it means for a general *term* to be logical.

Sher soon became convinced that the one practical way to answer this question was to first "identify a central role of logic" and only then, "relative to that role, ask what expressions can function as logical terms" (p. 36) — an approach for which she gives Leslie Tharp the credit (see [Tharp *1975*, 4–5]). It is in Alfred Tarski's papers from the 1930s that Sher finds "the most suggestive discussion" she has yet encountered of the role of logic: according to her reading of Tarski, it is "to develop and study deductive systems" (p. 38). Specifically, the primary subject-matter of logic is the concept of *logical consequence*. And in fact, it is in Tarski's foundational work on logical consequence that Sher will also find the very key to her answer to the question, What are the logical terms?

In his [*1936a*] paper, "On the Concept of Logical Consequence," Tarski proposed two essential, intuitive conditions on the relation of logical consequence: that it be *necessary* — a sentence *X* follows logically from a class of sentences *K* only if it is not possible for *X* to be false when every sentence in *K* is true — and that it be *formal*, in the sense that the relation of logical consequence is "uniquely determined by the form of the sentences between which it holds" and therefore "cannot be affected by replacing the designations of the objects referred to in these sentences by the designations of any other objects" [Tarski *1936a*, 414–415]. Sher thinks that the conditions of necessity and formality on the concept of logical consequence "delineate the scope as well as the limit of Tarski's enterprise: the development of a conceptual system in which the concept of logical consequence ranges over *all* formally necessary consequences and nothing else" (p. 44). But we cannot know that we have all formally necessary consequences unless we know that we have all logical terms, since on Tarskian principles the former depend on the latter via the logical form of sentences. So, what could be more natural than to search for "the widest notion of logical term" that fits Tarski's concept of necessary and formal consequence?

Yet Tarski himself was not at all sure at this time how to characterize the logical terms. He even suggested the possibility that "in the extreme case we could regard all terms of the language as logical. The concept of *formal* consequence would then coincide with that of *material* consequence" [Tarski *1936a*, 419]. Whatever Tarski might have meant by "regarding all terms of the language as logical," Sher is skeptical about this possibility. In fact: "The distinction between logical and extralogical terms is founded on our pre-theoretical intuition that logical consequences are distinguished from material consequences in being necessary and formal. To reject this intuition is to drop the foundation of Tarski's logic. To accept it is to provide a ground for the division of terms into logical and extralogical" (p. 51). In other words, Tarski's concept of logical consequence in some essential way *implies* the distinction being sought. But in what way?

Tarski's major insight was that the definition of logical consequence had to depend on "certain connexions between the expressions of a language and the objects and states of affairs referred to by these expressions" [Tarski *1936b*, 401] — that is, on semantics. Only by means of semantics, via models, could the definition capture the intuitive notion of logical consequence.

From an elementary standpoint, Tarskian semantics does, of course, provide a distinction between logical and extralogical terms. In Sher's words, "extralogical terms have no independent meaning," but "are interpreted only *within* models" (p. 47). Or, to put it another way, an extralogical term has every possible meaning, in the sense that it has as many meanings (denotations) as there are models for the language. On the other hand, a *logical* term derives its unique meaning from a rule in the "metatheory" outside of all models, a rule uniformly applicable in every model for the language because it is *defined over all models*.

This last idea provides Sher with the key to characterizing the logical terms. In order to discover all the logical terms of a logic with Tarskian semantics, we look for all the purely formal, structural patterns of elements, sets and relations occurring in all models for that logic. These formal "structures" — that is, the formal patterns that can be defined by a rule uniformly applicable in all models — are to be identified with the logical constants.

More precisely, Sher's definition is this: A *logical constant* for a Tarskian first-order logic is, syntactically, an n-place predicate or functor ($n > 0$) whose arguments are individual terms and/or predicates; semantically, it is a single "extensional function" defined over all models for the logic, mapping each model to a structure in that model in such a way that the function is invariant under isomorphic structures. A *logical term is* then either a sentential connective or a term which satisfies the above definition of logical constant. The definition ensures that logical constants represent "necessary" features of models by requiring that the extensional function be defined over *all* models, and ensures that logical constants represent "formal" features by requiring this function to be invariant under isomorphic structures.

A few examples should make it clear how the definition works. In a given model $\mathfrak{M}$ with universe A, the formal structures corresponding to 1-place quantifiers over 1-place predicates are sets of subsets of A. Hence, the extensional function defining the universal quantifier maps each model $\mathfrak{M}$ to the structure consisting of the singleton $\{A\}$. For the existential quantifier, the corresponding structure is the set of all nonempty subsets of A, while for the quantifier "for more x than not..." it is the set of all subsets of A having cardinality greater than the cardinality of their complement. Clearly all of Mostowski's quantifiers qualify as logical terms under Sher's definition; Mostowski's requirement that a logical quantifier "not allow us to distinguish between different elements" corresponds to the invariance of the extensional function under isomorphic structures. But the non-Mostowskian quantifiers mentioned earlier also satisfy the definition. The 2–place quantifier *Mx* is defined by the extensional function mapping each model $\mathfrak{M}$ to the set of all pairs $\langle B, C\rangle$ of subsets of A such that $|B \cap C| > |B - C|$. For the well-ordering quantifier *Wxy*, the extensional function maps $\mathfrak{M}$ to the structure consisting of the set of all relations $R \subseteq A \times A$ such that *R* is a strict linear ordering and every nonempty subset of the union of the domain and range of *R* has a least element. In short, all of Lindström's quantifiers are logical terms according to Sher's definition.

The "general framework" for logics provided by this definition is what Sher calls "Unrestricted Logic" (UL). A first-order logic with Tarskian semantics belongs to UL if it has the usual sentential connectives plus a nonempty set of logical constants satisfying Sher's definition. UL captures genuine logic because in UL "every formal and necessary consequence is identified by some logic, and only necessary and formal consequences pass the test of logicality" (p. xiii).

The language of standard logic does not appear to be rich enough for Tarskian semantics: in general, there are "too many" (non-isomorphic) models for a given set of sentences. But UL can distinguish between non isomorphic models, since it is non-isomorphic "structures" within the models that make them non-isomorphic. In other words, UL has the power to express all of the *mathematical* properties of models. "Any higher-order mathematical predicate or relation can function as a logical term, provided it is introduced in the right way into the syntactic-semantic apparatus of first-order logic" (pp. xii–xiii). Clearly Tarskian semantics can serve no richer language than the language of UL, for it is essentially the language of mathematics itself.

Sher presents a formal semantics for UL which (unlike Lindström's) is constructive, in the sense that it shows how the logical constants for a given Tarskian logic can be built up from concrete mathematical functions defining the formal structures in an arbitrary model for the logic. In her opinion this provides an answer to the question, What are all the logical constants? analogous to the well-known answer, in terms of Boolean functions, to the question, What are all the sentential connectives? Of course, it may be awhile before Sher's answer to the first question is generally held to be as incontestable as the classical answer to the second is now. But Sher predicts optimistically that the vast amount of research on alternative logics has already laid the foundation for a shift in the paradigm as to what should count as logical, so she feels that it is only a matter of time before the standard will be revised (p. 131).

One of the chief merits of Sher's book is that her consideration of the notion of logical term in light of Tarski's early work comprises an excellent summary of that work which places some of his major contributions in sharp, philosophical focus. It may even inspire a few readers to go back to the early papers just as she has, and her book is as much a perceptive and original appreciation of what Tarski did as it is an important extension of his results.

But how far was Tarski from formulating that extension himself? Sher cites Tarski's 1966 lecture "What Are Logical Notions?", in which he suggests "that we call a notion 'logical' if it is invariant under all possible one-one transformations of the world onto itself" [Tarski *1986*, 149]. This sounds almost like an anticipation of Sher, but, as she points out, invariance under "one-one transformations" is only one part of her definition: a logical term is not just a higher-order mathematical term, but one that has been incorporated into logic "in the right way" — namely, in a way that preserves the notion of logical consequence which Tarski had developed thirty years before his lecture (pp. 63–64). Anyone satisfied with Sher's definition might be tempted to think that there is some irony in this.

A chapter in *The Bounds of Logic* dealing with branching quantifiers — a type of syntactic structure allowing non-linear ordering of quantifier symbols — seems to resemble a long footnote to the entire work. Although branching quantifiers were first introduced in 1959 by Leon Henkin as a way of interpreting certain infinitely long formulas, most of the subsequent literature on them has been confined to their application within finitistic logic. The simplest example of a branching quantifier that cannot be rendered linearly in ordinary first-order logic is

$$\begin{matrix} (\forall x)(\exists y) & \\ & \Phi(x,y,z,w) \\ (\forall z)(\exists w) & \end{matrix}$$

which Henkin defined semantically in terms of Skolem functions, as follows:

$$(\exists f)(\exists g)(\forall x)(\forall z)\Phi(x, f(x), z, g(z))$$

It is natural to ask whether branching quantifiers are genuine logical terms. Sher quickly found that she was not in a position to answer this question, for there seems to be no agreement on how branching quantifiers should be defined semantically in the first place. Henkin's definition in terms of Skolem functions may suffice for branching structures built out of the standard quantifiers of first-order logic, but is of little use when the components are generalized quantifiers, as in some branching structures that have turned up in the study of natural language. Attempts at settling the issue at least in certain general cases have been made by Jon Barwise, Johan van Benthem, Dag Westerstahl, and others. Sher proposes a definition which may be the most comprehensive to appear in the literature so far. She introduces a classification scheme for partially ordered quantifier prefixes, based on "quantifier dependence" — ranging from totally independent