REVIEWS

Karel Lambert (introduction and editor), *Philosophical Applications of Free Logic*. Oxford: Oxford University Press, 1991. 309 pp.

Reviewed by

JEAN PAUL VAN BENDEGEM

Vrije Universiteit Brussel
Department of Philosophy
Pleinlaan 2, B-1050 Brussel, Belgium

There is a long philosophical tradition concerning the problem of non-existing entities. what status do they have? What do they refer to? Can statements involving non-existing entities have truth-values? Are all non-existing entities the same? If so, does this imply that "Pegasus = Sherlock Holmes" is true? If not, how do we distinguish between non-existing entities, if such is possible? In contemporary philosophy, the problem crops up when one is dealing with, e.g., possible worlds. If these are considered to be fictional entities, how many possible worlds are there? According to some philosophers of mathematics, the nominalists in particular, most mathematical concepts are fictions. If so, how do we deal with them?

Obviously, any answer(s) to this set of problems must first of all deal with the problem of distinguishing between existent and non-existent entities. To a certain extent — and this is the position defended by Karel Lambert in his introduction to this volume *The Nature of Free Logic* — the history of logic can be interpreted as a gradual explicitation of the underlying existence assumptions in formal reasoning. In Aristotelian syllogistic reasoning, the inference from "All *S* are *P*" to "Some *S* are *P*" was considered correct because it was implicitly assumed that a universal statement carried an existential commitment that there is at least one *S* that is *P*. Hence the correctness of the conclusion. In Fregean logic this is no longer the case. It must be explicitly added that there is at least one *S* that is *P* to derive, trivially, the conclusion. But modern logic allows the use of singular terms (or individual constants). If t is such a singular term then it is true that t = t. Apply the rule of existential generalization and one obtains $(\exists x)(x = \mathrm{t})$. Hence, whatever it is that t refers to, it must exist. Obviously, when talking about Pegasus or Sherlock Holmes, I do not wish to commit myself to their existence. A similar move as in the transition from Aristotelian to modern logic is required *vis-à-vis* singular terms. Free logic has set itself this task. Or, in Lambert's terms: "Free logics are logics devoid of existence assumptions with respect to their terms..." (p. 3). What must be clear right from the start, is that free logic is not meant as an alternative to modern classical logic. Rather, it is a search for a minimal system, i.e., as far as existence assumptions are concerned. This justifies Lambert's claim that "the usefulness of free logic as a *neutral* instrument in much ontological argument thus seems assured" (p. 13; my emphasis).

Probably, most, if not all, philosophers and logicians are familiar with Russell-Quine's standard solution. Eliminate all singular terms from the language of logic through the use of definite descriptions. Thus the phrase, "The present king of France is bald," seems to refer to a non-existent entity that is named, but in Russell's transcription this reads as $(\exists x)(Bx \ \& \ Kx \ \& \ (\forall y)(Ky \supset y = x))$, where B is the predicate "is bald" and K is the predicate "is the present king of France". This is summarized as $B((\iota x)Kx)$, where "ι" is the famous Russellian iota-operator. Elegant though this solution may be, it does imply that our natural language use is lacking in precision and that, therefore, it needs to be reconstructed. It belongs to the category: Don't say ..., but say ... (Most) free logicians are not interested in this move. They take singular terms seriously and hence they face the question how to deal with them.

The first proposal dates back to 1956, more particularly the paper of H. S. Leonard, "The Logic of Existence," (*Philosophical Studies* 7, 49–64), followed by Jaakko Hintikka, "Existential Presuppositions and Existential Commitments" (*The Journal of Philosophy* **56** (1959), 125–137), Hugues Leblanc and Theodore Hailperin, "Non-designating singular terms" (*Philosophical Review* **68** (1959), 129–136), Karel Lambert, "Existential Import Revisited" (*Notre Dame Journal of Formal Logic* **4** (1963), 133–144) and its sequels, Rolf Schock, "Contributions to Syntax, Semantics and the Philosophy of Science" (N*otre Dame Journal of Formal Logic* **5** (1964), 214–290), Bas C. van Fraassen, "The Completeness of Free Logic" (*Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik* **12** (1966), 219–234) and Nino B. Cocchiarella, "A Logic of Possible and Actual Objects" (*Journal of Symbolic Logic* 31 (1966), 688–689; abstract). Since then a quite extensive literature has been developed, both on the technical-logical level and on the philosophical level. The book being reviewed here focuses on the latter though one third (under the heading "Logic and Language") is reserved for more technical studies.

For those interested in the formal and technical details, there is the superb overview in *The Handbook of Philosophical Logic* by Ermanno Bencivenga, "Free Logics" [D. Gabbay and F. Guenthner (eds.), *Volume III: Alternatives in Classical Logic* (Reidel, Dordrecht, 1986), 373–426]. Nevertheless, let me briefly present the details of two particular systems, FS1 and FS2 to have some idea of what free logic looks like. Actually, the plural form is better, for although there is agreement on the general desiderata for a free logic, there are divergences in choice of specific axioms and/or rules, leading to a diversity of non-equivalent free logics (as will be illustrated in the sequel). This should not amaze us for, after all, e have to rely on our basic intuitions concerning non-existent entities and we all know how treacherous these intuitions are. For that matter, do we have any intuition at all when we talk about "the round square"?

The formation rules for FS1 and FS2 are basically the formation rules for first-order predicate logic and most of the axioms will look familiar too:

1. All tautologies.
2. $A \supset \forall xA$ where x does not occur free in A.
3. $\forall X(A \supset B) \supset (\forall xA \supset \forall xB)$.
4. $\forall y(\forall xA \supset A(y / x))$.
5. $\forall x \forall yA \supset \forall y \forall xA$.
6. $\forall xA(x / a)$ if A is an axiom where $A(x/a)$ is the result of replacing a by x in A and x does not occur in A.
7. $a = b \supset (A \supset A(b \,//\, a))$ where $A(b \,//\, a)$ is the result of replacing a at one or more places in A by b, if any at all.

The only rule of inference is *modus ponens*.

What is obviously lacking in this axiom system in comparison with classical logic, is the axiom of universal instantiation:

$$\forall xA \supset A(x / a).$$

This is indeed not derivable, but the following is:

$$\forall xA \supset (\exists x(x = a) \supset A(a / x)). \qquad (*)$$

The expression $\exists x(x = a)$ is often abbreviated to E!a so that the theorem reads:

$$\forall xA \supset (\text{E!}a \supset A(a / x)).$$

Thus, the existential conditions are indeed made explicit.

FS2 differs from FS1 in the following. E! is introduced as a primitive predicate, axiom 4 is replaced by (*) and axiom 5 by axiom 5′: $\forall x\text{E!}x$. In full, this says: $\forall x \exists y(y = x)$. This may appear strange at first sight, for does it not say that, given any x, there exists something identical with it. It would, if one could deduce from 5′ that $\exists y(y = a)$, but that is universal instantiation which is not allowed for.

My earlier remark on the origin of the multiplicity of free logics can be best illustrated by considering the semantics. A model $\mathfrak{M}$ for a free logic will of course consist of a domain D, an assignment function f and a valuation function v, $\mathfrak{M} = \langle D, f, v\rangle$. The first choice to be made concerns D. Will it consist of just one type of entities — those that exist — and will it consider singular terms that do not refer, to actually refer to nothing? If so, the function f will not be total, for some terms will not correspond to an element $d \in D$. This Lambert calls the Russellian world picture. An alternative is to consider D as the disjoint union of two sets D_0 and D_1 such that D_0 has all the existing entities and D_1 all the non-existing ones. In this case f is a total function for every term now refers to something. This is appropriately called the Meinongian world picture. There is, however, a second choice to be made. If $A(a)$ is a statement about a non-existing entity do we want that $A(a)$ be true, false or without truth-value? Negative free logic has all these statements false, positive free logic allows for some of them to be true, and neuter free logic opts for no truth-value. The two systems considered above, FS1 and FS2, can be interpreted as positive free logics based on a Meinongian world picture. It is however sufficient to add the axiom $A(a / x) \supset E!a$ (provided $A(a / x)$ is atomic), to obtain a negative free logic based on a Russellian world picture.

Perhaps one is inclined to accept the idea of a positive free logic, because, after all, would we not want "Pegasus = Pegasus" to be a true statement. Surely. But, equally well, we understand the neuter position. Given a statement like "Sherlock Holmes was in love with Irene Adler (the woman)," a neutral position seems most appropriate (given, of course, that the Doyle stories do not allow an indisputable deduction on this point). Incidentally, the search for a semantics that would express this position has led to one of the most important — both technically and philosophically — techniques available in logic today, namely the method of supervaluations, developed by van Fraassen. But negative free logic seems a bizarre position. Well, it is not. Take the Russell-Quine position. Singular terms are to be replaced by definite descriptions. Fine, but as one can easily see, the statement, "The present king of France is bald," or "$(\exists x)(Bx \;\&\; Kx \;\&\; (\forall y)(Ky \supset y = x))$" must be false, as the existential quantifier is not satisfied. But so must "The present king of France is not bald" if this sentence is interpreted as "$(\exists x)({\sim}Bx \;\&\; Kx \;\&\; (\forall y)(Ky \supset y = x))$" (i.e., what Russell calls the primary occurrence of the description). Hence, ${\sim}B((\iota x)Kx)$ is equally false. In fact, on this account, all statements about non-existent entities will turn out false. Hence the obvious need for a negative free logic.

As said, from the technical and logical point of view, free logic is a very well developed field. Any decent handbook that claims to give an overview of modern logic must include a chapter on free logic (as, for example, *The Handbook* does). It is therefore a very welcome addition to the field to have this volume that brings together material to illustrate the applications, both technical and philosophical, of free logic. The sixteen essays, with the exception of the papers by James W. Garson, Nino B. Cocchiarella, and Peter M. Simons, have been published elsewhere before, but have been brought together here for the first time. The value of the book as a source of reference is without discussion.

Some of the contributions explore extensions of free logic. The basic free logic is on the same level as classical first-order predicate logic. Therefore, it is natural to ask what happens to extensions of classical logic, if the underlying logic is replaced by a free logic. This is done for set theory (Dana Scott, "Existence and Description in Formal Logic"), for quantified modal logic (James W. Garson, "Applications of Free Logic to Quantified Intensional Logic"), and for tense logic (Nino B. Cocchiarella, "Quantification, Time, and Necessity"). As the purpose of free logic is to make explicit any implicit existential commitments, there is no reason to restrict the "free-ing" process to classical logic. That it is worthwhile to do so is illustrated by Carl J. Posy's contribution, "A Free IPC is a Natural Logic: Strong Completeness for Some Intuitionistic Free Logics." I will not go into the details as far as these papers are concerned, but rather focus on the more philosophical ones.

Although the contents of the book are grouped in three major categories — the already mentioned "Logic and Language", "Knowledge and Truth", and "Metaphysics" — I found these too broad and I will therefore propose a fourfold classification.

The two contributions by Karel Lambert himself, "A Theory of Definite Descriptions" and "Predication and Ontological Commitment", stand somewhat apart. The first paper reexamines Russell's theory of definite descriptions. What happens if these are added to a free logic? In other words, what happens if an iota-operator is added? Does it solve the following intriguing problem within Russell's approach? A statement of the form $B((\iota x)Kx)$, for any predicate B, will turn out to be false, but that implies that $K((\iota x)Kx)$ is false as well. This runs counter to our intuition for the last statement says "The present king of France has the property of being the present king of France". On the other hand, allowing $K((\iota x)Kx)$ to be true for any predicate leads to the description paradox: Simply take the predicate $(\lambda x)(Ax \mathbin{\&} {\sim}Ax)$. Then the following should be true,

$$A((\iota x)(Ax \mathbin{\&} {\sim}Ax)) \mathbin{\&} A((\iota x)(Ax \mathbin{\&} {\sim}Ax)),$$

which is nothing but a clear contradiction. This is, of course, nothing but the round square, being a round square. In a free logic, however, the appropriate scheme is this:

$$\mathrm{E!}(\iota x)Ax \supset A((\iota x)Ax),$$

and no problem arises. In addition, one can without fear of contradiction state this:

$$(\iota x)Ax = (\iota x)Ax,$$

an elegant solution indeed.

The second paper is a proposal to improve upon Quine. In his famous "On What There Is", Quine joined with Russell in the elimination of singular terms in favour of definite descriptions. According to Lambert, this was an unnecessary move. If instead of classical logic a free logic is used, singular terms can be allowed for, while retaining the properties that Quine finds essential to logic, namely, the principle of bivalence, the principle of extensionality (i.e., coextensive terms can be substituted *salva veritate*), and the principle of purely referential position to determine quantification in context (i.e., to quote the famous example, the transition from "Homer believed that Pegasus is a flying horse" to "There exists an x such that Homer believed that x is a flying horse" is not acceptable because Pegasus in the former sentence does not hold a purely referential position). This is unquestionably a considerable strengthening of Quine's position.

The second group of papers is the largest one. They all have to do with truth in one form or another. Ronald Scales', "A Russellian Approach to Truth", is closest to Lambert's work. In this paper, he solves the ambiguity that is inherent in Russell's approach concerning the present king of France. Are we supposed to read "The present king of France is not bald" as "$(\exists x)(\sim Bx \ \& \ Kx \ \& \ (\forall y)(Ky \supset y = x))$" (the above mentioned primary occurrence) in which case it is false, or should we read it as "$\sim(\exists x)(Bx \ \& \ Kx \ \& \ (\forall y)(Ky \supset y = x))$" in which case it is true (Russell's secondary occurrence). Relying on classes, Scales eliminates the ambiguity with the bonus that all such sentences have a truth-value.

The two papers by Bas C. van Fraassen, "Singular Terms, Truthvalue Gaps, and Free Logic" and "Presupposition, Implication, and Self-Reference," are both illustrations of his method of supervaluations. Consider once more Pegasus. "Pegasus is a flying horse" is a statement van Fraassen would like to give no truth value to. Equally so to the statement "Pegasus is not a flying horse." Nevertheless, he would like to see "Either Pegasus is a flying horse or Pegasus is not a flying horse" come out true, as this is simple instantiation of the law of excluded middle. The solution is this: Consider two classical models, i.e., models that satisfy all the requirements a classical logic imposes on a model. Then there will be a model $\mathfrak{M}_1$ wherein v(Pegasus is a flying horse) = T (and thus its negation F) and a model $\mathfrak{M}_2$ wherein v(Pegasus is a flying horse) = F (and thus its negation T). A supervaluation V is defined such that $V(A)$ = T in case $v(A)$ = T for all models; $V(A)$ = F in case $v(A)$ = F for all models, $V(A)$ is undefined otherwise. It then follows that V(Pegasus is a flying horse) and V(Pegasus is not a flying horse) are undefined, but as the law of excluded middle holds in all models, we have that V(Either Pegasus is a flying horse or Pegasus is not a flying horse) = T. This technique is applied in the second paper to the problem of presuppositions with a direct application to the liar-paradox. Ermanno Bencivenga in his "Free Semantics" follows a similar route. Via the notion of a completion of a given domain (of existent objects) — which comes down to the assignment of denotations to the non-denoting terms — coupled with supervaluations, he arrives at a semantics whereof "we can say that a sentence is true not only when it corresponds with reality (i.e., when it is *factually* true) but also when every "mental experiment" of a certain sort makes it true (which we can call its being *formally* true)" (p. 99). It must be remarked that, although the supervaluation approach originated within the framework of free logic, it has since become a quite general technique applicable, among other things, to the logic of questions and answers.

Tyler Burge's "Truth and Singular Terms" is an attempt to narrow down possible axioms for free logics on the basis of the following criterion. If a theory of truth — in Burge's case via the concept of "model a satisfies A"— is developed for a particular logic, then it should be the case that "our truth theory and its underlying logic help clarify how with respect to singular terms we can use the language we use and in the same language believe in the world we believe in" (p. 201). Richard E. Grandy's "A Definition of Truth for Theories with Intensional Definite Description Operators" has a similar purpose. The object of the paper is "to explore the possibilities of developing a system that is sufficiently weak that the principle of extensionality is not valid, but that is sufficiently strong that a definition of truth is forthcoming" (p. 172). The outcome of this detailed study is that at least four different systems satisfy his requirements to which he adds the philosophical moral: "thus, it appears that we must either look for more criteria of correctness for a definition of truth or admit that the matter is highly relative" (p. 186).

I have refrained from giving any detailed criticism of this group of papers. The reason is quite simply that this cannot be done without discussing other theories of truth and see how they deal with non-existent entities (assuming they have something to say about the issue). Suffice it here to say that the free logic approach is an alternative one has to deal with in any such discussion. It is formally very well developed and there are many considerations that speak in its favour as these authors show.

For the third group, I have brought together two papers that can be considered as direct opposites. On the one hand, Peter M. Simons in his "Free Part-Whole Theory" develops a free mereology. Mereological theory is usually associated with a nominalistic framework. The founding father of the logical approach to mereology is Leśniewski, who was interested, as Simons says, "to provide a nominalistically acceptable alternative to set theory" (p. 289). On the other hand, Storrs McCall in "Abstract Individuals" goes the other way. What he proposes is "changing our basic theory of quantification from a logic of predicates to a logic of abstract individuals" (p. 231).

Simons notes that all "classical" mereological theories (referring to the theories of Leśniewski, Tarski, Leonard and Goodman) involve at least one non-existent, namely, a null element. But there is more. Consider:

$\sigma x(Fx)$ (i.e., the sum of all x such that x is F) if nothing is F,

$x \cdot y$ (i.e., the overlap of x and y) if x and y do not overlap,

$\pi x(Fx)$ (i.e., the product of all x such that x is F) if the Fs do not have any common part,
$x \setminus y$ (i.e., the mereological difference) if $x < y$.

All of these terms can possibly be empty. Thus, Simons argues, a free logic, as underlying logic for a mereology, is far better suited to the task. Doing so, opens up an interesting perspective. Consider the following principle:

$$(\exists x(x = s) \ \& \ (s < t \ \vee \ t < s)) \supset \exists y(y = t),$$

i.e., if s exists, and either t is part of s or s is a part of t, then t exists. In a free mereology, this principle can be rejected and the author argues that it should be. Take, e.g., the Aristotelian idea of an actual continuum having potential parts. An actual line $\overline{AC}$ can be divided in half by introducing the mid-point B. Thus the line has two potential parts $\overline{AB}$ and $\overline{BC}$. If these parts are considered to be non-existent, then which exists has parts that do not exist. "So (oddly) some sums of nonexistents are existents" (p. 297). Here is an alternative, I believe, that nominalists (such as the author of this review) should consider quite seriously to solve some of their eternal shortcomings and difficulties.

But, as Lambert said, free logic is, ontologically speaking, neutral, so it can just as well be applied to abstract entities. McCall proposes to read phrases such as "The color of her eyes" as stating that the abstract individual "color" is dependent on the individual substance "her eyes" in such a way that "we would say that there was such a thing if and only if the possessor of the thing in question were characterized in a certain specific way" (p. 230). In short, there is such a thing as the color of her eyes iff her eyes are colored. This leads to free logic in a quite obvious way. If Cx stands for "x is colored", and $c(x)$ for "the color of x", then the classical predicate is related to the abstract individual as follows:

$$Cx \ \text{ iff } \ (\exists y)(y = c(x)).$$

But, in classical logic, $x = x$ is an axiom. By existential generalization, $(\exists y)(y = x)$, substitute $c(x)$ for x and it follows that Cx is a theorem. One might wonder why McCall is so interested in abstract individuals. He believes that arguments such as:

Women are beautiful.
What Considine seeks does not exist.
Therefore Considine does not seek the beauty of any woman.

are logically acceptable, and, hence, it is required that such individuals be dealt with. In his logic, this argument is neatly formalized, whereas a transcription in classical logic is indeed very clumsy (as this reviewer has tried). But that is not the end of the story. The following argument:

> Harry is honest.
> Honesty is a virtue.
> Therefore one of Harry's virtues is honesty,

asks for an extension to express the second premise. But that is easier said than done; as McCall explicitly states: "The plethora of different logical principles, which hold or fail in different situations, ought to discourage any lighthearted attempt to axiomatize the logic of abstract individuals" (p. 239). I agree.

The last two papers are the most outspoken philosophical contributions to this volume. William E. Mann's "Definite Descriptions and the Ontological Argument" is easily summarized. Let Gx stand for "Nothing greater than x can be conceived." Then the following argument is valid in a Lambert type free

(P1)	$\text{God} = (\iota x)Gx$	premise
(P2)	$\sim \exists y(y = (\iota x)\ Gx) \supset \sim (G(\iota x)Gx)$	premise
(P3)	$(G(\iota x)Gx)$	premise
(1)	$\exists y(y = (\iota x)\ Gx)$	*modus tollens*, (P2), (P3)
(2)	$\exists y(y = \text{God})$	substitution

In plain words: here is a formalization of Anselm's proof of the existence of God in a logic that is free of existence assumptions! I will resist a detailed analysis but limit myself to drawing the reader's attention to the following bizarre consequence of the above proof. First note that existence is considered a positive property, since a is greater than b, if, all other things being equal, a exists and b does not. Call a lesser then b if there is property, attributed to God, that b has and a does not, all other things being equal. Thus, if a exists and b does not, then b is lesser than a. Let Lx stand for "nothing lesser than x can be conceived." Then the following argument seems valid, if the one above is:

(P1)	$* = (\iota x)Lx$	premise
(P2)	$\sim \exists y(y = (\iota x)Lx) \supset \sim (L(\iota x)Lx)$	premise
(P3)	$(L(\iota x)Lx)$	premise
(1)	$\sim \exists y(y = (\iota x)Lx)$	*modus tollens*, (P2), (P3)
(2)	$\sim \exists y(y = *)$	substitution

What is an appropriate name for *? It has all properties God has not. Is therefore Evil not a good choice? But then the argument says that Evil does not exist. As I said, a bizarre consequence.

The last paper is Jaakko Hintikka's "*Cogito, Ergo Sum*: Inference or Performance." As the title indicates, it is a detailed, profound and balanced discussion of the meaning of Descartes' famous dictum. It must be said, though, that this essay is somewhat the odd one out in this book. It does not use explicitly, as all others do, one or another system of free logic and it is rather non-formal in its approach. It does, of course, share the emphasis with free logic on problems of existence. E.g., he argues quite convincingly that the "*ergo*" is not a sign of logical inference because implicit existence assumptions show up that are equivalent to the conclusion. A classical case of *petitio principii*. Rather it is related to performance. Thus, e.g., the sentence "Hintikka does not exist" can be uttered by me without any problem, but not so by Hintikka himself. If he utters the sentence, an immediate contradiction follows. Furthermore, it is only while uttering the sentence that the contradiction is realized. Thus "*cogito, ergo sum*" comes down (in rather unqualified terms) to "Each time, I, René Descartes, think, I realize, by doing this and while I am doing this, that I exist." Published in 1962, this paper has become one of the classics on the philosophy of René Descartes.

The reader may have the impression that this is quite a heterogenous collection. And, in a way, it is. Intentionally so. For the point of this book is precisely to show that free logic has many applications in a number of fields that are widely divergent. That point has been made. Let me add one final remark. Free logic is not an isolated branch of the logical tree. Lambert himself states it quite clearly in the first paragraph of "Predication and Ontological Commitment":

> Current theory about what there is is awash in new and resurrected kinds of objects. Perry and Barwise have discovered situations [*Situations and Attitudes*, MIT, Cambridge, Mass., 1983]; Fine has resurrected arbitrary objects [*Reasoning with Arbitrary Objects*, Blackwell, Oxford, 1985]; a few years ago Scott was championing virtual objects [Scott's paper in the book under review]; Simons, Mulligan, and Smith, ..., have sought lasting status for moments [references in Simons' paper]; and, finally, Routley [*Exploring Meinong's Jungle and Beyond*, RSSS, Canberra, 1979], Parsons [*Nonexistent Objects*, Yale University Press, New Haven, 1980], and many others, have rediscovered nonexistent objects! (p. 273; references have been added by me, as they are mostly lacking in the book)

This is definitely no exaggeration. Free logic has become part of a larger tradition that seeks to take the problem of non-existent entities seriously. As far as I can judge, any philosopher should.