maticians, rather than read about it. It is certainly a recommendation that can be applied to the work of André Weil.

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Raymond M. Smullyan, *Gödel's Incompleteness Theorems*, New York, New York, Oxford University Press, Inc., 1992. xiii + 139 pp.

Reviewed by

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Certainly, Gödel's work on incompleteness enjoys one of the highest profiles in mathematical logic. Not only are his First and Second Incompleteness Theorems familiar to every logician, but through various popularizations this material has managed to impinge on the general consciousness. (And without even needing cute computer-generated pictures!)

Smullyan's *Gödel's Incompleteness Theorems* is an introduction, but — unlike several other of his books — not a popularization for the public at large. To quote from the Preface, the book is intended "for the general mathematician, philosopher, computer scientist and any other curious reader who has at least a nodding acquaintance with the symbolism of first-order logic...and who can recognize the logical validity of a few elementary formulas. A standard one-semester course in mathematical logic is more than enough [background]." On the other hand, again quoting from the Preface, "There is a good deal in [Chapter VIII] that should interest the expert as well as the general reader." Smullyan lives up to his aims. The book provides a highly accessible, user-friendly introduction to incompleteness. At the same time the treatment is rigorous and contains material that even a professional logician can find informative and interesting.

Smullyan goes right to the heart of the matter in Chapter I by stripping incompleteness to its essentials. What basic features does a language need for an incompleteness theorem? Using these features, how does one prove such a theorem via diagonalization? In a sense, much of the rest of the book consists of an elaboration of the first chapter, examining how the abstract incompleteness scenario plays itself out in progressively more sophisticated contexts. Smullyan discusses, in turn:

Tarski's Theorem on the nonarithmeticity of the set of (Gödel numbers of) true sentences of arithmetic;

the **incompleteness of Peano Arithmetic** (at first, PA augmented with exponentiation; then, plain PA), under the assumption of PA's correctness;

Gödel's proof of the incompleteness of PA, under the weaker assumption of ω-consistency;

Rosser's proof, under the still weaker assumption of consistency;

Shepherdson's proof of the representability of Σ_1 sets, and related results;

and Gödel's Second Incompletoness Theorem (the unprovability of consistency) and its generalization in **Löb's Theorem.**

Finally, the book comes full circle by placing the main results in another abstract setting.

In covering this material, Smullyan has taken great pains to proceed along simple, direct paths that get the job done while minimizing technical entanglements. For example, the Gödel numbering, adapted from Quine, is a concatenation-based system that obviates the need for the Chinese Remainder Theorem. Using an axiom system for first-order logic due to Montague and Kalish, the book avoids many technicalities associated with the arithmetization of substituting terms for free variables. Likewise, the treatment of recursive functions is smooth and neat. Those who teach logic courses and/or write logic textbooks should consider the option of taking more advantage of these simplifications.

Accompanying the simplicity of the approach to the book's content, there is an admirable lucidity of style. Smullyan's experience in writing popularizations puts him in good stead here. As mentioned above, he accomplishes this without sacrificing rigor. *Gödel's Incompleteness Theorems* is filled with lemmas, propositions, and theorems, and they are proved without hand-waving. Indeed, page 109 contains a delightful polemic against some of the abuses perpetrated by other renditions of incompleteness for the nonspecialist. But Smullyan never confuses rigor with dullness or obscurity. His writing is clear and lively. He takes care that the reader knows what is going on and will even toss in a logic puzzle when it serves to make the point more comprehensible.

I will enter a *caveat*, however. Not only, of course, are many of the results in the book similar, or at least related, to each other, but also topics are examined and re-examined from different viewpoints, and often proofs are anticipated in the exercises. To many readers, all this may provide reinforcement of the main ideas; undoubtedly, it is meant to do so. But I can imagine some of the intended audience ending up somewhat disoriented — "Didn't we already do that? Or did we? Is there any real difference between those theorems, or is this just tinkering?" The danger in a spiral construction for a book is that some readers may feel that it just goes around in circles.

Another possible source of disorientation is the nonuniform labeling of results. For example, pages 57–64 feature, in order: Theorem G, Theorem A, Theorem B, Theorem 1, Lemma 1, Corollary, Theorem 1°, Lemma ω, Theorem 2, Theorem 3, Theorem 3°, Theorem A′, Lemma 2, and Theorem A*. And there is no index of theorem names to help the reader keep track of this motley collection.

The book also contains several glitches that should have been caught at the copy-editing/proofreading stages. These range from the merely annoying (e.g., the occasional scrambled sentence) to the potentially more serious (e.g., the disjunctions in the statement of Theorem 3 on page 94 should be conjunctions.)

The pluses much outweigh the minuses, though, and *Gödel's Incompleteness Theorems* is well worth reading — and even obtaining for one's own (unlike many other of the publisher's monographs, this one is not unduly expensive.) Although it contains many exercises, the book seems to be written more for self-study than for use as a classroom text, and it serves that purpose well. In addition, besides providing an excellent introduction to incompleteness and giving old hands at logic some food for thought, Smullyan has another aim. Quoting from the Preface one last time, "this volume...was also intended as a preparation for our sequel, *Recursion Theory for Metamathematics*, in which we explore in depth the fascinating interrelations between incompleteness and recursive unsolvability." I am eagerly awaiting the sequel's appearance.

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Katalin G. Havas, *Thought, Language and Reality in Logic*, Budapest, Akadémiai Kiadó, 1992. Translation by J. Kovács & M. Gulyás, revised by B. Dajka, of *Gondolkodas, nyelv, valóság a logikában*, Budapest, Akadémiai Kiadó, 1983.

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