

Alfred Tarski, *Collected Papers*, Volume 1, 1921–1934, xvi+659 pp., Volume 2, 1935–1944, xvi+699 pp., Volume 3, 1945–1957, xi+682 pp., Volume 4, 1958–1979, xvi+757 pp., plus Acknowledgements, insert, 3 pp. Edited by Steven R. Givant and Ralph N. McKenzie. (Contemporary Mathematicians, Gian-Carlo Rota, Editor). Birkhäuser, Basel, Boston, and Stuttgart, 1986.

Reviewed by

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The work under review is a reprint of two others:

The collected work of Alfred Tarski, Volume 1, 1921–1934, vii+666 pp., Volume 2, 1935–1944, vi+700 pp., Volume 3, 1945–1957, vi+682 pp., Volume 4, 1958–1979, xiii+737 pp., S.R. Givant, R.N. McKenzie, editors, University of California, Berkeley, California, 1981,

and

Bibliography of Alfred Tarski, Steven Givant, *The Journal of Symbolic Logic* 51 (1986), 913–941.

In this review the reference codes from Givant's bibliography are used to designate Tarski's works. The two digits in each reference code are the last two digits of the year of publication. For example, [21] was published in 1921. (In a few cases two pairs of digits appear, reflecting the numbering system of the relevant journal.) Additional publications from the same year are distinguished by additional letters. For example, the five papers published in 1924 are [24], [24a], [24b], [24c], and [24d]. A superscript "a" on the digits, as in [24^aa], indicates that the item is an abstract, a superscript "m" indicates a monograph (e.g., [47^m]), a superscript "c" indicates a contribution to a discussion (e.g., [28^ca]) a superscript "p" indicates a problem (e.g., [24^p]), and a superscript "r" indicates a review (e.g., [37^r]). (Here are some corrections to Givant's bibliography. Line 1, page 744, volume 4: "relations" should be replaced with "relation" in the title of [49^a]; line 23, page 753, volume 4: "[36^c]" should be replaced with "[36^cc]"; line 32, page 730, volume 4: the second line of information for [24^a] should read: "*Mathematicae*, vol. 5 (1924), pp. 147–154.")

Tarski's *Collected Papers* is a treasure trove of twentieth century logic and mathematics. It is described on the flyleaf by the following text:

Alfred Tarski was one of the two giants of the twentieth century development of logic along with Kurt Gödel. The four volumes of this collection contain all of Tarski's published papers and abstracts (but not the books), and include a comprehensive bibliography. Here will be found many of the works, spanning the period 1921 through 1979, which are the bedrock of contemporary areas of logic — whether in mathematics or in philosophy. These areas include the theory of truth in formalized languages, decision methods and undecidable theories, foundations of geometry, set theory, and model theory, algebraic logic and universal algebra.

Facing the title page of each volume is a photograph of Tarski. (It was his favorite, taken on a train in Russia around 1965; see [G1]). Each volume begins with the table of contents for that volume, followed immediately by the tables of contents for the other three volumes. Then comes a single page with the following information.

This collection of the published papers of Alfred Tarski was conceived as a gift to the author, in connection with the celebration of his eightieth birthday. Its original publication, in a very limited edition, was made possible by the generous contributions of Tarski's friends, students, and colleagues from around the world. The organizing committee consisted of John Addison, Steven Givant, Leon Henkin, Ralph McKenzie, Judith Ng, Julia Robinson, and Robert Vaught. The editors of the volumes were Givant and McKenzie.

According to the tables of contents, the four volumes of the *Collected Papers* include 110 papers and monographs, 96 abstracts, 18 items classified as problems, reviews, or contributions to discussions, and the bibliography by Givant. Each paper begins with a page giving the title and bibliographic information. Subsequent translations, reprints, and revised versions are listed on the next page. The papers and monographs are all photographically reproduced from the originals. The abstracts, problems, contributions to discussions, and reviews, however, are not reproductions of the original publications. Tarski maintained a personal file of typewritten copies of these items, each of which included bibliographic information as part of the heading. It is these versions which are reproduced in the collected papers. The reader thus gets a glimpse into Tarski's papers that cannot be obtained by looking up the original publications. Some of the abstracts had errors in them, for which corrections were published later. For example, abstracts [49^{aa}], [49^{ab}], and [49^{ac}] appear on pp. 63–64 in volume 55 of the *Bulletin of the American Mathematical Society*, and all three have corrections on page 1192 of the same volume. Such errors have

been removed from the versions reproduced in the *Collected Papers*. In other cases, new errors have been introduced. For example, problem [30Pe] is Exercise 136 in the first volume of the journal *Parametr*, but the number was entered incorrectly in Tarski's personal version and therefore is incorrect in the *Collected Papers*. Thus "NR. 135" in line 4 on page 690 of volume 4 should be replaced by "NR. 136". A more serious case is that problem [25P] (problem number 38 on page 381 of volume 7 (1925) of *Fundamenta Mathematicae*) does not appear in the *Collected Papers*. Someone mistakenly copied problem 39 instead of problem number 38. Consequently the last three lines on page 685 of volume 4 of the *Collected Papers* contain the text of problem number 39, which is actually due to O. Nykodym, and should be replaced with the following text.

NR. 38. Un carré et un cercle dont les aires sont égales peuvent-ils être décomposés en un nombre fini de sous-ensembles disjoints respectivement congruents?

Many of Tarski's publications were reprinted, translated, or revised. (Almost all of them are reprinted in [81^{ma}] *The collected works of Alfred Tarski*, and the *Collected Papers* is itself a reprint of [81^{ma}], supplemented with Givant's bibliography.) Reprints, translations, and revisions are indicated in the reference codes by an additional number in parentheses, outside the brackets. For example, [29](1) is the revised English translation of [29]. (It appeared in the monograph [56^m]. The second, revised edition of this monograph is [56^m](1).) Each such item appears in the bibliography directly under the original publication. A very few exceptions were made for those items whose translations were more often cited than the originals. In particular, [35c], [36f], and [36g] are the German translations of the original Polish publications [34], [36], and [36a], respectively.

The *Collected Papers* usually includes only original publications, but there are several exceptions that should be noted. First, the German translations [35c], [36f], and [36g] (and not the Polish originals [34], [36], and [36a]) were included in the *Collected Papers*. Tarski's doctoral dissertation [23] (in Polish) is not included in the *Collected Papers*. However, Tarski published two papers in French which are revised portions of his dissertation, namely [23a] and [24], and they are both included in the *Collected Papers*. (They also appear together in [56^m], revised and translated to English, as [23](1) *On the primitive term of logistic*. When [23a] was incorporated into [56^m], the lemmas were rearranged into a different order, and some notational changes were made in the proofs.) Both of the Polish papers [31b] and [31/32a] appear in the *Collected Papers*, and each is followed by a 1950 English translation by I. Wirszup, namely [31b](1) and [31/32a](1). Tarski's article [69] for *Scientific American* is not included, presumably because the pages are too large. The revised version [69](1) was included instead. Finally, [87?] (*What are the logical notions*, edited by J. Corcoran, *History and Philosophy of Logic* 7 (1986), pp. 143–154) was not yet in print when Givant's bibliography was written. It is not in [81^{ma}], and consequently does not appear in the *Collected Papers*.

There are 20 monographs in the bibliography. Three of them are included in the *Collected Papers*, namely [47^m], [48^m](1) (the second, revised, 1951 edition of [48^m]),

and [67^{ma}]. The monograph [67^m] is represented in the *Collected Papers* by its revised version [69a]. Furthermore, [35b] in the *Collected Papers* is the German translation (with an added appendix) of the monograph [33^m] and it was also printed as a seperatum. One excluded monograph, *Undecidable theories* [53^m], might reasonably have been included on the ground that it actually consists of three separate papers, namely, *A general method in proofs of undecidability* by Tarski, *Undecidability and essential undecidability in arithmetic* by A. Mostowski, R. Robinson, and Tarski, and *Undecidability of the elementary theory of groups* by Tarski.

Among the 96 abstracts in the *Collected Papers* are all but one of the 94 abstracts listed in the bibliography. The missing abstract is [58^{af}] *What is elementary geometry*, 7 pp., a mimeographed summary of [59]. Three other items were included among the abstracts because they contain announcements of results by Tarski, namely, [28^{ca}] *A contribution to the discussion of an article by C. Czeżowski*, Knaster [27] *Un Théorème sur les fonctions d'ensembles*, and Editors of *Fundamenta Mathematicae* [34] *Editorial remark after an article by Th. Skolem*.

The *Collected Papers* includes all 23 problems listed in the bibliography, all 21 contributions to discussions, all four reviews, and three of the four items listed in the appendix to the bibliography. The missing item from the appendix is Łukasiewicz [28/29] *O definicyach w teoryi dedukcyi* <*On definitions in the theory of deduction*>. Also not included are three books for which Tarski was one of the editors, seven project reports, and one published letter.

The Collected Papers is a reprint of *The collected works of Alfred Tarski*, which was issued in a very limited edition as a gift to Tarski. Unfortunately, the reprint, unlike the gift to Tarski, does not retain the page numbers of the original papers. It is therefore difficult to look up references to the page numbers of Tarski's publications. Further examples of the hardships imposed on the reader by this shortsighted omission are easy to find. For example, the 124-page paper [30f] has an index of symbols, which, of course, uses the missing original page numbers. However, since the papers were reproduced photographically from the originals, there is a one-to-one correspondence between pages in the *Collected Papers* and the original pages. The bibliographic information at the beginning of each paper includes the range of the original page numbers, so the enterprising reader can reconstruct the original pagination. In particular, for each paper it is possible to compute a number which yields the correct page number of the *Collected Papers* when it is added to the original page number. For example, the range of pages for [30f] is 181–304 in volume 16 of *Fundamenta Mathematicae*, and the first page of [30f] in the *Collected Papers* is 393. The key number for this paper is therefore 212 (= 393 – 181). The index of [30f] asserts that *S* is defined on page 233. Adding 212 to 233 yields 445, so *S* is defined on page 445 of the *Collected Papers*.

The contents of the four volumes are listed at the end of the review. Each item has been supplemented with page numbers and with a reference code from the bibliography. The

items are written in four languages: Polish, French, German, and English. Translations of the Polish titles are included in both the tables of contents and in the bibliography. Languages are explicitly indicated in the list below for those items whose language cannot be determined from the title in the tables of contents. A very few items were not actually written by Tarski, and therefore have their authors listed before the reference code.

It would be impossible for a single individual to fully assess the impact that Alfred Tarski has had on the development of the foundations of mathematics (and geometry in particular), linguistics, logic, mathematics in general, metamathematics, philosophy, and semantics. Fortunately, there is a series of articles in *The Journal of Symbolic Logic*, written by experts in many different fields, devoted to a survey of Tarski's work. See Blok and Pigozzi [BP], Doner and Hodges [DH], Etchemendy [E], Jónsson [J], Levy [L], McNulty [Mc], Monk [Mo], Szczurba [Sz], Suppes [Su], van den Dries [vD], and Vaught [V] in the list of references at the end of this review. For a more personal glimpse at Tarski, see the fascinating article by Givant [G1].

Tarski is famous for his many results, which include, to name but a few examples, the Banach-Tarski Paradox (found independently by Banach and Tarski), the general form of the Löwenheim-Skolem theorem, the characterization of universal theories, the equivalence of the Axiom of Choice with the statement that $\mathfrak{m}^2 = \mathfrak{m}$ holds for all infinite cardinals \mathfrak{m} , the undefinability of truth, the completeness and decidability of the elementary theory of real closed fields, which followed from Tarski's characterization of definable sets of real numbers, the definition, completeness, and decidability of elementary geometry, the ultrafilter theorem (also called the prime ideal theorem), Tarski's Lemma (on existence of ultrafilters preserving a countable set of meets), Tarski's Fixed Point Theorem, the undecidability of many elementary theories, including those of groups and rings, and the decidability of many others, such as the elementary theory of Boolean algebras.

Tarski is also well known for his definitions, theories, problems, intuition, talks, writings, and students. For example, Tarski defined algebras of formulas (sometimes, unfortunately, called "Lindenbaum algebras"). He is perhaps best known for the definition of satisfaction and truth for formalized languages. Tarski was the principle architect of model theory and universal algebra, and he invented several kinds of algebras, including relation algebras, cylindric algebras, ordinal algebras, and cardinal algebras. Tarski's problem of determining whether any two free groups with two or more generators are elementarily equivalent has generated much work and remains unsolved. Tarski's writings have been reprinted and translated extensively. For example, his paper [44a] *The semantic conception of truth and the foundation of semantics* has appeared 14 times in 6 languages, and his textbook on logic ([37^m] and [41^m]) has appeared 27 times in 14 languages. Several of Tarski's students have achieved great prominence, such as C. C. Chang, S. Feferman, H. Gaifman, W. Hanf, B. Jónsson, H. J. Keisler, A. Levy, J. D. Monk, R. Montague, A. Mostowski, J. Robinson, W. Szmielew, and R. Vaught.

There is a saying, of unknown origin, which was circulating in Berkeley in the 1970's, that the four greatest logicians in the history of the world are Aristotle, Frege, Gödel, and Tarski. Perhaps this list was intended to be in the order of greatness, but in any case it is clearly alphabetical, and it is also very nearly chronological. (Tarski was born in 1901, a few years before Gödel, but died a few years later 1983.) On the other hand, Tarski expressed his opinion in a conversation with the reviewer around 1978 that just two people exhibited genius in logic in this century, namely Gödel and Paul J. Cohen. When asked, "What about you?", Tarski replied, "I came close". Perhaps Tarski was thinking literally, for he came close to Gödel's incompleteness theorem (see [G1]). Perhaps he was just being uncharacteristically modest, for Tarski is truly a twentieth century Aristotle, a "genius of logic", whose influence was so great that large parts of the subject are justifiably called his creation, and whose spirit still presides over its destiny. The *Collected Papers* is a fitting monument to a man who had a profound impact on the shape of the intellectual world.

Contents of volume 1:

- [21], pp. 1–12. Przyczynek do aksjomatyki zbioru dobrze uporządkowanego (A contribution to the axiomatic of well-ordered sets).
- [23a], pp. 13–19. Sur le terme primitif de la logistiqu.
- [24] pp. 21–38. Sur les truth-fonctions au sens de MM. Russell et Whitehead.
- [24a], pp. 39–48. Sur quelques théorèmes qui équivalent à l'axiome du choix.
- [24b], pp. 49–64. O równoważności wielokątów (On the equivalence of polygons).
- [24c], pp. 65–117. Sur les ensembles finis.
- [24d], pp. 119–154. Sur la décomposition des ensembles de points en parties respectivement congruentes (with S. Banach).
- [25], pp. 155–170. Quelques théorèmes sur les alephs.
- [26], pp. 171–204. Communication sur les recherches de la Théorie des Ensembles (with A. Lindenbaum).
- [28], pp. 205–224. Sur la décomposition des ensembles en sous-ensembles presque disjoints.
- [29], pp. 225–231. Les fondements de la géometrie des corps.
- [29a], pp. 233–241. Geschichtliche Entwicklung und gegenwartiger Zustand der Gleichmächtigkeitsstheorie und der Kardinalzahlarithmetik.
- [29b], pp. 243–248. Sur les fonctions additives dans les classes abstraites et leur application au problème de la mesure.
- [29e], pp. 249–252. Zjazd matematyków (The meeting of mathematicians).
- [29c], pp. 253–265. Sur la décomposition des ensembles en sous-ensembles presque disjoints (supplément à la note sous le même titre).
- [29d], pp. 267–273. Na marginesie «Rozporządzenia Prezydenta Rzeczypospolitej o ubezpieczeniu pracowników umysłowych z dnia 24 listopada 1927 r.» (On the margin

of «Decree of the President of the Republic concerning the insurance of non-manual workers of November 24, 1927»).

[30], pp. 275–285. Une contribution à la théorie de la mesure.

[30a], pp. 287–297. Sur une propriété caractéristique des nombres inaccessibles (with W. Sierpiński).

[30b], pp. 299–310. Über Äquivalenz der Mengen in Bezug auf eine beliebige Klasse von Abbildungen.

[30c], pp. 311–320. Über einige fundamentale Begriffe der Metamathematik.

[30d], pp. 321–343. Untersuchungen über den Aussagenkalkül (with J. Łukasiewicz).

[30e], pp. 345–390. Fundamentale Begriffe der Methodologie der deduktiven Wissenschaften, I.

[30f], pp. 391–516. Sur les classes d'ensembles closes par rapport à certaines opérations élémentaires.

[31], pp. 517–548. Sur les ensembles définissables de nombres réels, I.

[31a], pp. 549–559. Les opérations logiques et les ensembles projectifs (with C. Kuratowski).

[31b], pp. 561–570. O stopniu równoważności wielokątów (On the degree of equivalence of polygons).

[31b](1), pp. 571–580. The degree of equivalence of polygons (English translation by Isaak Wirszup of the preceding article).

[31/32], pp. 581–593. Teorja długości okręgu w szkole średniej (The theory of the measure of the circumference of a circle for high school teaching).

[31/32a], pp. 595–601. Uwagi o stopniu równoważności wielokątów (Remarks on the degree of equivalence of polygons).

[31/32a](1), pp. 603–611. Further remarks about the degree of equivalence of polygons (English translation by Izaak Wirszup of the preceding article).

[32], pp. 613–617. Der Wahrheitsbegriff in den Sprachen der deduktiven Disziplinen.

[33], pp. 619–636. Einige Betrachtungen über die Begriffe der ω -Widerspruchsfreiheit und der ω -Vollständigkeit.

[35c], pp. 637–659. Einige methodologische Untersuchungen über die Definierbarkeit der Begriffe.

Contents of volume 2:

[35], pp. 1–24. Zur Grundlegung der Boole'schen Algebra, I.

[35a], pp. 25–50. Grundzüge der Systemenkalküls, Erster Teil.

[35b], pp. 51–198. Der Wahrheitsbegriff in den formalisierten Sprachen.

[35d], pp. 199–202. Wahrscheinlichkeitslehre und mehrwertige Logik.

[36b], pp. 203–212. Über die Beschränktheit der Ausdrucksmittel deduktiver Theorien (with A. Lindenbaum).

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- [36c], pp. 213–221. Über die Erweiterungen der unvollständigen Systeme des Aussagenkalküls.
- [36d], pp. 223–243. Grundzüge der Systemenkalküls, Zweiter Teil.
- [36e], pp. 245–258. Sur les classes d'ensembles closes par rapport aux opérations de Hausdorff.
- [36f], pp. 259–268. Grundlegung der wissenschaftlichen Semantik.
- [36g], pp. 269–281. Über den Begriff der logische Folgerung.
- [36h], pp. 283–288. Ideale in den Mengenkörpern.
- [37], pp. 289–321. Über additive und multiplikative mengenkörper und Mengenfunktionen.
- [37a], pp. 323–333. Sur le méthode déductive.
- [37b], pp. 335–350. Appendix E to Woodger, "The axiomatic method in biology".
- [38], pp. 351–355. Einige Bemerkungen zur Axiomatik der Boole'schen Algebra.
- [38a], pp. 357–380. Über unerreichbare Kardinalzahlen.
- [38b], pp. 381–406. Drei Überdeckungssätze der allgemeinen Mengenlehre.
- [38c], pp. 407–416. Ein Überdeckungssatz für endliche Mengen.
- [38d], pp. 417–423. Eine äquivalente Formulierung des Auswahlaxioms.
- [38e], pp. 425–443. Über das absolute Mass linearer Punktmengen.
- [38f], pp. 445–450. Ein Beitrag zur Axiomatik der Abelschen Gruppen.
- [38g], pp. 451–472. Algebraische Fassung des Massproblems.
- [38h], pp. 473–506. Der Aussagenkalkül und die Topologie.
- [39], pp. 507–527. Ideale in vollständigen Mengenkörpern, I.
- [39a], pp. 529–548. Boolesche Ringe mit geordneter Basis (with A. Mostowski).
- [39b], pp. 549–558. On well-ordered subsets of any set.
- [39c], pp. 559–568. On undecidable statements in enlarged systems of logic and the concept of truth.
- [41], pp. 569–587. On the calculus of relations.
- [43], pp. 589–605. On families of mutually exclusive sets (with P. Erdős).
- [44], pp. 607–659. The algebra of topology (with J.C.C. McKinsey).
- [44a], pp. 661–699. The semantic conception of truth and the foundations of semantics.

Contents of volume 3:

- [45], pp. 1–17. Ideale in vollständigen Mengenkörpern, II.
- [46], pp. 19–61. On closed elements in closure algebras (with J.C.C. McKinsey).
- [46a], pp. 63–67. A remark on functionally free algebras.
- [47^m], pp. 69–143. Direct decompositions of finite algebraic systems (with B. Jónsson).
- [48], pp. 145–161. Some theorems about the sentential calculi of Lewis and Heyting (with J.C.C. McKinsey).

- [48a], pp. 163–169. A problem concerning the notion of definability.
- [48b], pp. 171–198. Axiomatic and algebraic aspects of two theorems on sums of cardinals.
- [48c], pp. 199–231. Measures in Boolean algebras (with A. Horn).
- [49], pp. 233–250. Cancellation laws in the arithmetic of cardinals.
- [51], pp. 251–296. Distributive and modular laws in the arithmetic of relation algebras (with L. H. Chin).
- [48^m], pp. 297–367. A decision method for elementary algebra and geometry (prepared for publication with the assistance of J.C.C. McKinsey).
- [51a], pp. 369–419. Boolean algebras with operators, I (with B. Jónsson).
- [52], pp. 421–458. Boolean algebras with operators, II (with B. Jónsson).
- [52a], pp. 459–476. Some notions and methods on the borderline of algebra and metamathematics.
- [52b], pp. 477–479. Mutual interpretability of some essentially undecidable theories (with W. Szmielew).
- [52c], pp. 481–504. On algebras whose factor algebras are Boolean (with J.M.G. Fell).
- [54], pp. 505–513. Theorems on the existence of successors of cardinals, and the axiom of choice.
- [54a], pp. 515–526. Contributions to the theory of models, I.
- [54b], pp. 527–535. Contributions to the theory of models, II.
- [55], pp. 537–547. Contributions to the theory of models, III.
- [55a], pp. 549–575. A lattice-theoretical fixpoint theorem and its applications.
- [56], pp. 577–591. The fundamental ideas of pansomatism (English translation of the article «Zasadnicze myśli pansomatyizmu» by T. Kotarbiński; translated by A. Tarski and D. Rynin).
- [56a], pp. 593–602. Equationally complete rings and relation algebras.
- [56b], pp. 603–610. Equilaterality as the only primitive notion of Euclidean geometry (with E.W. Beth).
- [56c], pp. 611–619. A general theorem concerning primitive notions of Euclidean geometry.
- [57], pp. 621–650. Higher degrees of distributivity and completeness in Boolean algebras (with E.C. Smith, Jr.).
- [57a], pp. 651–674. Arithmetical extensions of relational systems (with R.L. Vaught).
- [57b], pp. 675–682. Remarks on direct products of commutative semigroups.

Contents of volume 4:

- [58], pp. 1–8. The sentential calculus with infinitely long expressions (with D. Scott).
- [58a], pp. 9–16. Remarks on predicate logic with infinitely long expressions.

- [59], pp. 17–32. What is elementary geometry?
- [61], pp. 33–41. On two properties of free algebras (with B. Jónsson).
- [61a], pp. 43–75. Cylindric algebras (with L. Henkin).
- [61b], pp. 77–111. On some problems involving inaccessible cardinals (with P. Erdős).
- [62], pp. 113–125. Some problems and results relevant to the foundation of set theory.
- [64], [64](1), pp. 127–213. From accessible to inaccessible cardinals. Results holding for all accessible cardinals and the problem of their extension to inaccessible ones (with H.J. Keisler).
- [64a], pp. 215–249. Refinement properties for relational structures (with C.C. Chang and B. Jónsson).
- [65], pp. 251–271. A simplified formalization of predicate logic with identity.
- [65a], pp. 273–287. Metamathematical properties of some affine geometries (with L.W. Szczerba).
- [67^{ma}], pp. 289–346. The completeness of elementary algebra and geometry.
- [68], pp. 347–362. Equational logic and equational theories of algebras.
- [69a], pp. 363–397. An extended arithmetic of ordinal numbers (with J. Doner).
- [69](1), pp. 399–423. Truth and proof.
- [75], pp. 425–434. An interpolation theorem for irredundant bases of closure structures.
- [78], pp. 435–490. The elementary theory of well-ordering – A metamathematical study (with J. Doner and A. Mostowski).
- [79], pp. 491–530. Metamathematical discussion of some affine geometries (with L. W. Szczerba).

Abstracts (pp. 531–681):

- [24^a], pp. 533–534. Sur les principes de l'arithmétique des nombres ordinaux (transfinis).
- [24^{aa}], pp. 535–536. Une remarque concernant les principes d'arithmétique théorique.
- [26^a], p. 537. Remarque concernant l'arithmétique des nombres cardinaux.
- [26^{aa}], pp. 538–540. Sur l'indépendance des notions primitives dans les systèmes mathématiques (with A. Lindenbaum).
- [27^a] pp. 541–543. Sur quelques propriétés caractéristiques des images d'ensembles.
- [27^{aa}], pp. 544–545. Quelques théorèmes généraux sur les images d'ensembles.
- Knaster [27], pp. 546–547. Un Théorème sur les fonctions d'ensembles (by B. Knaster).
- [28^a], p. 548. Remarque de M. Tarski.
- [28^{ca}], p. 549. A contribution to the discussion of an article by C. Czeżowski.
- [28^{aa}], pp. 550–551. Sur les groupes d'Abel ordonnés.
- [28^{ab}], pp. 552–554. Remarques sur les notions fondamentales de la méthodologie des mathématiques.
- [30/31^a], pp. 555–559. O pojęciu prawdy w odniesieniu do sformalizowanych nauk dedukcyjnych (On the concept of truth in reference to formalized deductive sciences).

- [30^a], pp. 560–561. Über definierbare Zahlenmengen.
- [31^a], pp. 562–563. Neue Resultate und unentschiedene Probleme der Kardinalzahlarithmetik.
- [31^aa], pp. 564–566. Untersuchungen über den Aussagenkalkül.
- [32^a], p. 567. Sur les propriétés géométriques de la mesure de Banach.
- Editors of *Fundamenta Mathematicae* [34], p. 568. Editorial remark after an article by Th. Skolem.
- [37^a], pp. 569–570. Über das Repräsentationsproblem.
- [39^a], p. 571. New investigations on the completeness of deductive theories.
- [45^a], p. 572. On direct products of algebras (with B. Jónsson).
- [45^aa], p. 573. A generalization of Wedderburn's theorem (with B. Jónsson).
- [48^a], p. 574. Measures in Boolean algebras (with A. Horn).
- [48^aa], p. 575. Boolean algebras with operators (with B. Jónsson).
- [48^ab], p. 576. Representation problems for relation algebras (with B. Jónsson).
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