Penelope Maddy, *Realism in mathematics*, Oxford, Clarendon Press, 1990.

Reviewed by

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This book, devoted to the defense of a kind of mathematical realism called "set theoretic realism," is to be highly recommended. Realism concerning a given discipline is, as Professor Maddy defines it, "the contention that its subject matter exists objectively, that various efforts to reinterpret its claims should be resisted, and that most of its well-supported hypotheses are at least approximately true" (p. 177).

After the publication of Benacerraf's two well-known papers, "What Numbers Could Not Be" (*1965*) and "Mathematical Truth" (*1973*), every mathematical realist was forced to respond to his objections to the view that numbers are independent objects. In "Mathematical Truth," Benacerraf highlights the epistemological problem that every realist has to face, to wit, the connection between knowers and objects known, by which we may determine what objects exist, and what their properties and relationships are. Traditional realists, from Plato to Gödel, have postulated a peculiar epistemic faculty, sometimes called "intuition," which provides the desired contact. Benacerraf's argument in "What numbers could not be" poses the problem of how to decide, among the different possibilities, what objects numbers really are. He focuses on the set theoretical proposals of von Neumann and Zermelo: if numbers were to be sets, what reasons might we have to decide between, for example, Zermelo's numbers and von Neumann's ordinals? Benacerraf answers that numbers are not objects at all but suggests that mathematics is about some kind of structure. Professor Maddy is aware that this argument, if it works – and she thinks it does, can be extended and used against the thesis that reals are sets. If both Cantor and Dedekind offer suitable accounts of real numbers, how can we choose between them?

Professor Maddy looks at Benacerraf's criticisms in the face and so Chapter Two of her book deals with Benacerraf's epistemological challenge; and in Chapter Three she addresses his arguments against the identification of numbers and any kind of entity. In answer to the first problem, Professor Maddy stresses her disagreement with Gödel's platonism. She does not accept abstract entities as non-spatio-temporal objrcts as Gödel does although she does agree with him in distinguishing the two kinds of intuitive justification he introduces: the immediate evidence of some very basic principles and the theoretical evidence that justify some axioms through the fruitfulness of their consequences. Maddy's realism also incorporates Putnam and Quine's indispensability arguments for abstract entities: we should accept them because they are required by our best theories of the world. Maddy's position is at one with the general project to which Putnam and Quine are committed, epistemology naturalized. Her sort of realism nestles between Gödel's and Putnam's branches. Her scheme is to bring mathematical entities into the physical world and so naturalize mathematical epistemology. Nevertheless, contrary to Putnam's and Quine's purposes, she does not attempt to reduce the richness of mathematics to a mere tool for empirical science. Maddy's is a kind of set theoretic realism because in a way she takes sets as basic but does not identify numbers with them. Her position is thus immune to Benacerraf's criticism. For her, numbers are neither sets nor any other kind of object but properties of sets, and sets themselves are perceptible spatio-temporal entities (at least those at the lowest level of the iterative hierarchy) whose essential feature is to have a number. She explains the relationship between sets and their numbers as being of the same kind as that which exists between an object and its length. If we are happy with an impure set theory, Maddy's position will require us to identify physical objects with their unit sets – as Quine does, by the way – and if we prefer to deal only with pure set theories we should interpret different proposa1s, for example Zermelo's and von Neumann's series, as diiierent yardsticks, just as we do with different measurement systems. By conceiving numbers as properties of sets and some sets as spatio-temporal, Maddy breaks down thls epistemological barrier because in her theory numbers have a causal relation with the physical world. Numbers are properties of sets and at least some sets have a spatio-temporal location. In this sense, accepting numbers stands at the same level as admitting many other sorts of theoretical properties we encounter in physical theories.

Chapters Four and Five are devoted to the still open theoretical problems in set theory and to discussng whether set theoretic realism finds itself in a somewhat handicapped position to take account of them compared to its competing philosophies, the most widespread of them being nominalism and structuralism. A sound and complete philosophy of mathematics should, one might think, help us to decide if, for instance, the Continuum Hypothesis or its negation is true or, in terms of the contention between set theories, if we should add to ZF an axiom of constructibility or one affirming the existence of a supercompact cardinal. Both theories, ZF + Constructibility and ZF + SC, have their strong points but they both cannot be true at the same time. Are the problems of choosing between highly theoretical proposals particularly damaging for set theoretic realism? Maddy's answer is that they are not. Sooner or later, every philosophy or mathematics must offer criteria for faclng up to these theoretical decisions and neither nominalism nor structuralism can do anything to avoid them. Neither Nominalism or structuralism are in a better position, so this difficulty of deciding between alternative theories cannot be used as an argument against set theoretic realism.

On the last page of her book, Professor Maddy concludes:

> Theories or mathematical knowledge tend either to trivialize it as conventional or purely formal or even false, or to glamorize it as perfect, a priori, and certain, but set theoretic realism aims to treat it as no more nor less than the science it is, and to be fair, all at once, to the mathematician who produces the knowledge, the scientist who uses it, and the cognitive scientist who must explain it.

If Maddy's project is successful, it will provide an account or mathematics which will turn it into a science just like any other. In my opinion her project deserves the strongest support and, although it is not completely worked out as yet, the arguments she offers cannot easily be dismissed. I do not think that the controversy between nominalism, structuralism and set theoretic realism can ever be settled on the basis of rational and theoretic arguments. I am rather inclined to think that sympathizers or any of these philosophies live in different paradigms that, strictly speaking, cannot be refuted. But defenders of mathematical realism should warmly welcome Maddy's book for it offers a new store of arguments that will be very useful, if not for overcoming nominalism and structuralism, at least for rendering realism a clearer, more compact, better developed, and more coherent position in the philosophy or mathematics.

Besides its scientific value and Professor Maddy's enchanting sexist language, the book is a pleasure to read. Both mathematicians with interest in philosophy and philosophers of mathematics and science will enjoy and profit from it.

REFERENCES

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