ℵ Modern Logic ω

DISCUSSION

HISTORY AND GRAMMAR

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Abstract. A brief summary of Colin McLarty's treatment of the history of topos theory.

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In an article in the September, 1990, issue of *The British Journal for the Philosophy of Science*, 'The Uses and Abuses of the History of Topos Theory', Colin McLarty makes some telling points about the history of mathematics, controversial observations about the foundations of mathematics, and one grammatical point. Since I disagree with him over the grammar, let me get that out of the way first. In a footnote on page 357, he writes, "Notice as a point of orthography that 'topos' is a French word, formed from 'topologie,' and not a Greek word. In writing, Grothendieck always forms the plural according to the French rule for words ending in 's,' so it is invariant – 'les topos.' So the English plural ought to follow the English rule – 'toposes.'" Without taking issue with the relevance of Grothendieck's practice, I feel that we are stuck with 'topos' as a word of Greek origin. If someone were to introduce the word 'ager' for field as derived from the English word 'agriculture', the fact that 'ager' is identical in appearance with a Latin word would suggest a Latin formation for the plural. On top of the linguistic argument there is the point that 'topoi' sounds less ugly than 'toposes'. It is safe to say that both forms will continue in use regardless of attempts to legislate their appearance.

Plural-formation aside, McLarty's article is striking from the title onward. The reader might have expected 'The History of the Uses and Abuses of Topos Theory' and that subject does come up in McLarty's article, but the title describes his enterprise precisely. He is concerned with showing how the representation and misrepresentation of the evolution of the subject have been and can be called upon to characterize its role in mathematics. In particular, McLarty indicates some of the dangers of calling on history to perform tasks for which it is not suited. The lessons he draws cut across the history of logic (and, indeed, of mathematics) in general.

The basic target of McLarty's attack is the view (particularly as espoused by Robert Goldblatt in his book *Topoi*) that topos theory was introduced by those seeking to provide a category-theoretic version of set theory. Although Goldblatt is not the only culprit identified by McLarty, his shortcomings receive the most attention. In addition to the general issue of the interpretation of the intentions behind the early works in topos theory, McLarty finds flaws in specific claims by Goldblatt. In fact, one suspects that part of the reason for the lack of popularity of the form 'topoi' in the eyes of McLarty and other topos theorists is its appearance in the title of Goldblatt's book.

In order to counter the impression that topos theory arose as an alternative foundation for set theory, McLarty examines in detail the work of the 1960's based on the testimony of a number of those involved (MacLane, Lawvere, Freyd, and others). He indicates convincingly that the origins of topos theory cannot be separated from algebra and geometry, questions from which were uppermost in the mind of those looking at the properties of categories. As he notes, "One measure of the relative unimportance of set theory as a model for categorical foundations is that, even when categorical axioms for the category of sets were given, no one pursued them" (p. 359).

Where did the myth of the connection between set theory and category theory arise? Apart from the errors of misreading of which

McLarty indicts Goldblatt, there is a natural temptation (not in a category-theoretic sense) for those accustomed to think in terms of sets to try to fit categories into those terms. As a result, those expounding categories to others will try to explain categories as an evolutionary successor to sets. Those for whom sets remain fundamental will present categories as though they are trying to occupy the same niche as sets.

McLarty argues, however, that the emphasis on set theory as a foundation for mathematics made its appearance relatively recently and that an approach to categories that makes them secondary to sets misrepresents them. The wealth of the historical evidence accumulated by McLarty convicts this approach of inaccuracy. The picture of the evolution of topos theory is different and much more complicated.

Here, however, the shortcomings of history as a pedagogical tool cannot be ignored. The complications of the historical record can send the student off in a number of different directions, while the notion of topos was intended to play something of a unifying role. McLarty, in fact, suggests a different rational reconstruction of the origins of topos theory from Goldblatt's, but admits that it is also a misrepresentation. "The history sketched here [in the section entitled 'Falsifying History Enough'] suits categorical foundations the same way the common sense history suits the set theoretic ones – in both cases, at the cost of truth" (p. 370).

McLarty ends with the suggestion that foundations for mathematics must change as mathematics changes. Those who put their trust in sets will have difficulties making use of the advantages of category theory. Students of foundations should study that for which they are trying to lay a foundation. Category theorists would presumably be ready to admit that further technical developments in the subject will move the centre of foundational interest to some other domain.

This closing argument of McLarty (which echoes Lawvere) sounds convincing against the defenders of set theory as a foundation for mathematics, at least insofar as they take the basic role of set theory for granted. Recent work in the philosophy of mathematics has also looked for alternatives to set theory in the hopes of discovering something underlying the shifts in mathematical fashion. It is an old practice, going back at least to Plato, and pessimism about the solutions proposed so far endorses the relativist picture presented by McLarty.[1] While agreeing that set theory may not be the last word on foundations, the Platonist may not have given up on the hunt for the objects of mathematics.[2]

[1] McLarty elsewhere defends a realist view of categorical foundations, a view that I question in my 'Shifting Sands and Foundations' (forthcoming).

[2] It is safe to say that the categorical concern for foundations owes more to Lawvere than to anyone else. His enthusiasm for the truth of the matter is a spur to others.

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