PEIRCE'S LOGIC TODAY

(A Report on the Logic Program of the Peirce Sesquicentennial Congress)

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Mathematicians and logicians who pay attention to historical references have probably noticed that Charles Peirce is being mentioned with increasing frequency. In part this is because we are becoming more historically-minded, perhaps because we are growing less dogmatic about our foundations and are thus more open to earlier approaches, or maybe we are finally realizing that the community we are part of still includes those scholars who are no longer with us. But interest in Peirce's work is not all backward looking, not just for the historian who wants to become clearer about our roots and how old ideas have evolved into modern mathematics and logic. Part of what Peirce had to say has relevance for the newest and most forward looking areas of investigation. The growing and wide ranging interest in Peirce's work in logic and the foundations of mathematics was evident at the meetings of the Peirce Sesquicentennial Congress recently held at Harvard University and co-hosted by Texas Tech University.

From 5-10 September 1989 over two hundred and fifty scholars from around the world met in Cambridge to deliver papers on Peirce's work in the many fields to which he contributed. Most of the meetings were held in Sever Hall, which stands almost directly on the spot where Peirce's boyhood home stood in Harvard Yard. His father, Benjamin Peirce, Harvard's Professor of Mathematics and Astronomy and one of America's greatest mathematicians, had the benefit of a home on campus, provided by the University. Benjamin held his classes in the Peirce house, known by his students as "Function Hall" in honor of Benjamin Peirce's great attachment to $\phi\nu$. It was appropriate that so many of the papers presented at the Congress, perhaps as many as one sixth, dealt with some aspect of Peirce's mathematical or logical contributions. It is to be regretted somewhat, however, that no sessions of contributed papers were devoted to Peirce's technical contributions to mathematics except as it pertains to logic or to foundations. The only session devoted fully to Peirce's mathematics was mainly philosophical and consisted of contributed papers by Claudine Engel-Tiercelin, Pesi R. Masani, R. Valentine Dusek, and Angela Ales Bello. Of more central interest to mathematicians, especially for historians of mathematics, was Carolyn Eisele's plenery lecture, "Peirce and Mathematics," and Helena Pycior's response, and the presidential address of Hillary Putnam (president of the congress) which dealt with Peirce's unique conception of continuity as Peirce had presented it in his Harvard Lectures of 1903.

The logic program, however, was quite full and covered much ground, a fitting tribute to the first, and perhaps still the only, scholar to list his profession as logician for *Who's Who*. Peirce was also the first scholar admitted to the National Academy of Science for his work in logic. Willard Van Orman Quine is the second. As co-organizer (with Don D. Roberts) of some of the logic sessions, I was asked to prepare a report on the Congress, concentrating on papers that would likely be of most interest to readers of *Modern Logic*. Toward that end, I have collected and organized the abstracts of the more formal or mathematical logic papers, but it should be born in mind that some abstracts were not available so that those included below constitute only a sample (albeit a large one) of the logical proceedings of the congress. None of the plenary lectures are abstracted, although several of these would certainly have been of interest. One plenary session that should be mentioned was on "Peirce, Probability, and Decision Theory," with Isaac Levi as the principal speaker and with Joseph S. Ullian as discussant. Joseph Dauben's paper on "Peirce and History of Science," with Peter Skagestad as discussant, was as interesting for the historian of logic and mathematics as it was for the historian of science. For many logicians, the high point of the congress may have been Willard V. O. Quine's plenary talk, "Peirce and Logic," not so much perhaps because of any new revelations as for his acknowledgement of Peirce's historical importance. Quine's devotion to Frege's place in the history of logic is well-known, so it was a special treat for the Peirce enthusiasts to hear him pay some homage to Peirce for his contributions to quantification theory (although Quine was careful to remind us that even though it was through Peirce that quantification came to logic and mathematics, still Frege--independently--had it first). One of the most substantive and important papers of the congress was Randall Dipert's response to Quine. The plenary papers will be the first collection of congress papers to be published (by Texas Tech University Press) and will appear under the title, *Peirce and Contemporary Thought*. A number of books will follow (to be published either by Texas Tech University Press or by Indiana University Press), including at least one collection on Peirce's logic. A collection entitled *Studies in the Logic of Charles S. Peirce,* which will contain many of the papers abstracted below, is presently being considered by Indiana University Press for its Peirce Studies Series.

Some interesting non-plenary papers, for which abstracts were not available, are the following: Robert Burch's "Peirce's Reduction Theorem Explicated and Proved," Alan J. Iliff's "The Role of Matrix Representation in Peirce's Development of Quantifiers," Henry E. Kyburg, Jr., "Peirce and Statistical Deduction," Ilkka Niiniluoto's "Peirce's Theory of Statistical Explanation," and Lauro Frederico Barbosa da Silveria's "Mathematics and Necessity from Kant to Peirce." Henry Hiż gave an exciting account of "Peirce's Influence on Logic in Poland," in which he outlined several of Peirce's important contributions and his impact on such figures as Łukasiewicz, Leśniewski, Tarski, and Wajsberg. Peirce's influence on Polish logicians appears to have been mainly in the following areas: investigation of the use of implication as the only primitive connective of propositional logic, the analysis of negation as an interpretation of expressions containing only implications, investigation of Peirce's law (CCCpqpp) as the demarcation between positive and classical logic, the use of logical values as propositional terms, suggestions for the development of many-valued logics, quantification developed on the analogy (more or less) with infinite product and infinite sum in mathematics, and the logical theory of relations.

Of special interest were the comments of Ivor Grattan-Guinness, who responded to twenty papers in four logic sessions, and capped those sessions with a retrospective commentary on the logic program and a general survey of the relations between logic(s) and mathematics during the 19th century up to Peirce's time. The abstract of Grattan-Guinness's response, as prepared for the congress proceedings, is included below.

Finally, I should point out that for Peirce logic is a normative science, a branch of philosophy, that deals with signs in general, with necessary and sufficient conditions for representing, with truth and reference, and with the formal structures and methods of different kinds of reasoning. A great deal that would not likely be of general interest to readers of *Modern Logic* appropriately falls under logic in Peirce's scheme, and as a result I have elected to omit some twenty or more abstracts of papers read in the logic sessions. Those papers covered such topics as inductive and abductive (the logic of hypothesis formation--logic of discovery) reasoning, Peirce's theory of proper names, the algebraical structure of meaning phenomena, and non-monotonic logic--subjects which seemed to me to be more philosophical than mathematical. Nevertheless, in two or three cases I have included abstracts that may be of more interest to philosophers than to historians of mathematical logic.

I would like to thank the authors and the congress organizer, Kenneth Laine Ketner, for permission to publish the following abstracts.

Peirce between Logic and Mathematics (Ivor Grattan-Guinness, Middlesex Polytechnic): Some considerations are offered on the context and background to Peirce's contributions to algebraic logic; similarities and differences between this tradition and that of mathematical logic are indicated. Peirce's views on the relationship between (his sort of) logic and mathematics are appraised, and finally a letter of 1870 written by his father and introducing him to de Morgan is reproduced.

Peirce's Axiomatization of Arithmetic (Paul Shields, Lake Forest College): In 1881 Peirce published the first successful set of axioms for the natural number system. Let N be a set, R a relation on N, and 1 a distinguished element of N; also assume definitions of "minimum", "maximum", and "immediate predecessor" with respect to R and N. Then Peirce's axioms may be stated:

i) N is simply ordered by R
ii) N is closed with respect to immediate predecessors
iii) 1 is the minimum element in N; N has no maximum
iv) Mathematical induction holds for N.

These axioms are equivalent to the better known axiom systems of Dedekind and Peano, which were not published until the end of the decade.

Peirce's paper, "On the Logic of Number," was published in the *American Journal of Mathematics,* vol. 4, in 1881. It stands on its own as a remarkable document in the history of the foundations of mathematics in the nineteenth century. It also demonstrates vividly the increasing power of Peirce's logic of relatives and introduces many themes which recur in Peirce's later thought. Thus Peirce went on to sketch the basic arithmetic of natural numbers, providing one of the earliest known examples of recursive definitions for addition and multiplication, along with proofs of associative, commutative, and distributive laws for these operations. Peirce also introduced the notion of cardinal numbers, defined in the modern way by reference to initial segments of ordinals. Based on this latter definition, Peirce described an (ordinary) finite set as one for which the corresponding initial ordinal segment had a maximum.

His paper is perhaps best known, however, for containing the first purely cardinal definition of a finite set--as a set for which De Morgan's syllogism of transposed quantity is valid. This is the property that is often referred to today, perhaps unjustly, as being "Dedekind-finite."

Peirce's Philosophical Conception of Sets (Randall R. Dipert, SUNY-Fredonia): The relatively small differences among 20th-century set theories (NBG, ZF, NF, with or without *Urelementen* , various axioms of infinity, choice, and so on) that have now coalesced into the popular "iterative" notion of set most closely identified with Zermelo-Fraenkel set theory, have served to hide, and even obscure, complex historical discussions about the ontological and epistemological status of various finely-differentiated notions of a collection. A modern day set-theoretician who turns to the works of Peirce is almost certain to be disturbed--if not completely perplexed--by subtle attempts to contrast notions such as collection, class, multitude, *Menge,* and so on, together with difficult discussions on individuality, unity, and other metaphysical and methodological issues. Such discussions appear in many of Peirce's works, but become increasingly more sophisticated in his correspondence with Schröder, work of the later 1890's and the Lowell lectures of 1903. I argue that in Peirce we see one of the last historically and metaphysically informed logicians to worry seriously about questions that had vexed medieval and later, 19th-century logicians such as Boole and Schröder, and that the discussions concerning the precise nature and justification of sets has since Peirce lapsed into a more-or-less casual "pragmatic" argument of the worst sort: it "works" doesn't it?---when in fact what counts as 'working' is far from clear, and it is far from obvious that quite different, and more philosophically sophisticated, basic notions of a collective could not be made to 'work' better. My primary interest in the paper is understanding the philosophical and historical issues that animated Peirce's difficult discussions of various sorts of collections.

Peirce's Pre-Logistic Account of Mathematics (Angus Kerr-Lawson, University of Waterloo): A close analogy is found between Charles Peirce's philosophy of mathematics, and the position held by a great many of today's practising mathematicians. This shared view finds few difficulties with the epistemology of mathematics, in contrast to the various foundational schools which sought, in the intervening years, to ground mathematics safely in logic, or in mental constructions, or in sure combinatorial manipulations. It contrasts also with the views of many contemporary philosophers of mathematics, who insist that if mathematical entities are to be allowed, they must be given the fullest existential status. Peirce with his possibilities, and mathematicians with their structures, admit a level of existence secondary to the full-fledged existence proper to actual things. This paper tries to bring out the close bond between allowing a second level of existence, and a relaxed attitude towards epistemological questions.

Peirce on the Interconnections between Mathematics and Logic (Stephen H. Levy, Fanwood, New Jersey): Peirce defined mathematics as "the science which draws necessary conclusions," and logic as "the science *of* drawing necessary conclusions." Later, he made his "first real discovery" of two types of necessary reasoning--theoremic and corollarial--and stressed their importance for mathematical reasoning. In this paper, I refine the distinction and show that it illuminates not only the nature of mathematical reasoning, but the relationship between mathematics and logic as well.

Peirce often argued that although logic depended on mathematics, mathematics did not depend on logic. However, when focussing on other logico-mathematical questions, Peirce was considerably more insightful into the relationships between the two subjects and repeatedly expressed a different, truer view.

Analysis of the "foreign idea" characterizing theoremic reasoning shows that mathematics depends on logic just as much as the reverse. In Peirce's distinction, we have substantial support for the widely held contemporary view that logic and mathematics are interdependent.

Mathematics and Logic in Peirce's Description of a Notation for the Logic of Relatives (James Van Evra, University of Waterloo): As works in 19th century algebraic logic go, Peirce's DNLR is extraordinary. Initially described as an extension of Boole's logical algebra to include relative terms, the work instead rapidly breaks continuity with the preceding tradition by incorporating a wide variety of mathematical notions which have no obvious logical bearing. I explain this turn of events as an instance of what happens to an analogically-based science when the host science (i.e. algebra) undergoes significant change. The result in this case is a powerful work of a different order from that of its forbears.

Relations and Quantification in Peirce's Logic, 1869-1885 (Daniel D. Merrill, Oberlin College): This paper deals with the relationship between Peirce's two most important contributions to logic: the logic of relations and the theory of quantification. It emphasizes the contrast between Peirce's algebraic logic of relations (ALR) of 1870 and his quantificational logic of relations (QLR) of 1885; and it asks why Peirce preferred the QLR to the ALR.

The first sections of the paper point out the strengths of the ALR to be found in Peirce's 1870 memoir on the logic of relatives. In contrast to some interpretations, it claims that Peirce's logic of "relatives" is, in fact, a true logic of relations. The rich and varied quantificational resources to be found in the ALR of 1870 are also outlined.

The paper concludes with a systematic discussion of Peirce's reasons for preferring the QLR to the ALR. This question is approached from the standpoints of both the deductive adequacy and the expressive adequacy of the two logics. Within each of these categories, attention is devoted to three types of considerations: those of power, convenience, and analytical depth. The conclusion is that considerations of convenience and analytical depth seem to have been more important for Peirce than those of power.

From the Algebra of Relations to the Logic of Quantifiers (Geraldine Brady, Chicago): Peirce's work in mathematical logic before 1883 was motivated to an important degree by the problem of giving a systematic treatment of cases of the syllogism involving the nonemptiness of some of the classes considered. In his paper of 1870 on the algebra of relations he worked largely within Boole's program of developing logic as an equational calculus (generalized to the partial ordering representing the notion of subset). In his paper of 1880, in which he introduced ideas about the connection between inference and material implication, he was still very concerned with the problem of handling the theory of the syllogism. A turning point in Peirce's work occurred when, stimulated by a paper of his student, O.H. Mitchell, he developed his system of quantificational logic. By 1885 Peirce had developed two systems of logic: the algebra of relations (1870) and quantificational logic (1885). This work was clarified and extended by Schröder (1895), and the first-order version of Peirce's system provided the framework within which certain important papers of Löwenheim (1915) and Skolem (1920, 1923) were written. This line of development, being model-theoretic, was quite separate from that of Frege and Whitehead-Russell, which was deduction-theoretic. Thus Peirce had a major influence on the development of mathematical logic in the twentieth century.

Tarski's Development of Peirce's Logic of Relations (Irving H. Anellis, Iowa State University): "Tarski's Development of Peirce's Logic of Relations" will examine in particular the historical and mathematical connections between the work of C. S. Peirce and A. B. Kempe in the 1880-1890s on relative triples or linear triads as products of two binary relations between logical atoms and Tarski and Givant's formalism **L**3 as a three-variable fragment of first-order logic. This paper will also establish a partial positive reply to Schröder's question of whether the algebra of relatives can express all statements about relations as equations of the calculus of relations, since Tarski-Givant's axiomatic system presents set theory and number theory as sets of equations between predicates constructed from two atomic predicates denoting the identity and the set-theoretic elementhood relations. However, it is also shown in particular that Peirce's reduction thesis, according to which all tetradic and greater polyadic relations are expressible as products of monadic, dyadic, and triadic relations, may not hold, since Tarski and his colleagues have provided numerous examples of propositions about tetradic relations which are expressible in first-order logic but which are not expressible as equations in three-variable fragment **L**3 of first-order logic equivalent to the algebra of relations. One such example is the proposition that *there exist four elements*. Consequently, Peirce's reduction thesis will hold without modification only in case it can be shown that the systems of Peirce and Tarski are not logically equivalent.

New Light on Peirce's Iconic Notation for the Sixteen Binary Connectives (Glenn Clark, Mount Union College): In 1983 a 16 by 16 table constructed by Peirce in 1902 was noticed in a "Fragments" folder at the Peirce Edition Project in Indianapolis. Since the marginal indices and the 256 cells contained the binary connective signs introduced by Peirce in MS 431 ("The Simplest Mathematics") and used extensively on certain pages therein, some connection with that manuscript was conjectured. It was soon determined that the table is the one referred to on page 92 of MS 431A and in paragraph 271 of the *Collected Papers* , Vol. 4 (4.271)†, although it does not appear in either place. It is a key to reconstructing Peirce's procedure in finding more that 24,000 tautologies in twelve related forms, each involving five binary connectives, as recorded on pages 92-93 of MS 431A. For each form he specified three of the connectives by use of the table but did not say how the table was constructed; the other two connectives were specified without explanation. The symmetry properties of Peirce's iconic signs for the binary connectives have been used to verify all of the above-mentioned tautologies and 2092 others claimed by Peirce in five simpler forms treated in MS 431A (*Collected Papers* 4.268-270, 272-273).

†*Collected Papers of Charles Sanders Peirce,* vol. 4 (Harvard University Press, 1933).

Untapped Potential in Peirce's Iconic Notation for the Sixteen Binary Connectives (Shea Zellweger, Mount Union College): The iconic notation for the sixteen binary connectives that Peirce devised in 1902 (MS 431) is still very much alive today. The untapped potential that it contains deserves to be examined very carefully. This potential, in fact, is robust enough so that, as an exercise at least, it will force us to *rethink* the propositional calculus. This could lead to a watershed clarification.

Start with 2-valued logic and the semantics of truth tables. Generate three series of numbers. The first is for propositions: 1, 2, 3, 4 ... n. The second is for the 1st-order truth tables: 2, 4, 8, 16 ... 2^n The third is for the 2nd-order truth tables: 4, 16, 256, 65,536 ... $2^{(2^n)}$. What follows extends to and incorporates a fourth series of numbers.

Introduce and let a matching set of *negation tables* (1st order) at (n + 1) act on the 2nd-order truth tables, when an asterisk (*) stands for a generalized connective. The (+1) is introduced when negation acts on the connectives themselves (N*). For example, when all eight (2^{n+1})combinations of (NA, N*, NB) act on the sixteen binary connectives (A * B), this generates a table of (8 x 16) transformations that determine *2nd-order relations,* namely, equivalence relations (≡) between the binary connective relations (*) between (A,B). As it turns out, it takes all combinations of only three (n + 1) symmetry operations to generate an exact count of these 128 negation-driven interrelations among the sixteen binary connectives, themselves treated as objects at a higher level of subject matter.

In like manner, when (n = 4), it takes only five (n + 1) symmetry operations to generate the corresponding transformational table that contains (32 x 65,536) or 2,097,152 of these interrelations among the quaternary connectives. In general, the semantics of truth tables is such that negation tables generate a corresponding series of numbers for these negation-driven sets of equivalence interrelations: 16, 128, 4096, 2,097,152 ... $2^{(n+1)} \times 2^{(2^n)}$.

Now focus on notation in general, such that a good notation will have a syntax that will isomorphically match all and any of these exponentially explosive sets of equivalence interrelations. As it turns out, such a notation requires a precise custom-designed coordination of iconicity, symmetry, adapted matrices, and the algebra of certain abstract groups. In 1902 Peirce already had the main beginning frame of what is required. It takes only five simple refinements to bring mental economy to the untapped potential. The results are surprising, elegant, and loaded with perspicuity.

Peirce on the Application of Relations to Relations (Robert Burch, Texas A & M): This paper explicates the operation on logical terms that Peirce calls "the application of a relation" in his 1870 work "Description of a Notation for the Logic of Relatives, resulting from an Amplification of the Conceptions of Boole's Calculus of Logic." The paper argues that Peirce combines in the notion of "the application of a relation" two different through closely affiliated operations; one of these is a binary operation analogous to algebraic multiplication, while the other is a unary operation. All of Peirce's remarks about "the application of a relation" in his 1870 work are argued to be easily understandable once one sees that both the binary and the unary operation are incorporated into this notion. The binary and the unary operation are presented both in a graphical syntax similar to the syntax of Peirce's existential graphs and in a syntax akin to the syntax of first-order predicate logic with identity. The paper also argues that it makes very little difference whether we explicate Peirce as concerned with "relatives" or as concerned with "relations." Peirce's logical system in his 1870 work, it is suggested, is a fully-developed logic of relations that is at least as powerful in expressive capability as first-order predicate logic with identity. Moreover, it is suggested, in Peirce's notion of "the application of a relation" the germ of his so-called "reduction thesis" is contained.

Teridentity and Genuine Triads (Jacqueline Brunning, University of Toronto): Peirce's doctrine of categories is a claim that all reality is partitioned into three mutually exclusive and jointly exhaustive classes: monads, dyads and triads. Peirce claimed that a study of the algebra of relations would justify his theory.

Peirce's third category, triads, seems indefensible. The reduction of polyads to dyads is commonplace in present logical theory. Moreover, the definitional resources of his algebras are so powerful, they allow the reduction of triads to dyads. A closer look at the algebras indicates that, in spite of apparent similarities, Peirce's notions of definition and triad are subtly different from present usage. Examination of these differences may disarm objections to his third category.

Peirce's algebras suggested the theory of categories. Peirce also claimed the categories suggested the existential graphs. So the algebras are related to the graphs via the categories. The comma notation of the algebras is the ancestor of teridentity of graphs. Peirce's algebras failed to provide a compelling explicit demonstration that the third category was necessary. It is teridentity of the existential graphs that both provides the justification for the third category and an understanding of a Peircean triad.

Peirce's Graphical Decision Procedure (Don D. Roberts, University of Waterloo): In 1903 Charles Peirce tried to develop a decision procedure for his system of existential graphs, which is a complete functional calculus of first order (FCFO). His procedure was to consist of two parts: the first would reduce any graph containing quantifiers to a graph without quantifiers; the second would determine whether the resulting graph was a tautology, a contingency, or a contradiction. Peirce did not complete the first part of his method, but he remained confident that it could be done, contrary to what Church proved in 1936 (that the decision problem for FCFO is unsolvable). Peirce did succeed with respect to the second part, however, which means that he solved the decision problem for the propositional calculus.

Lewis Carroll's Method of Trees: Its Origins in *Studies in Logic* (Francine Abeles, Kean College)†: In 1894, Charles L. Dodgson (Lewis Carroll) developed a mechanical method to test the validity of complicated multiliteral statements using a *reductio ad absurdum* argument. The basic ideas are similar to those in Beth's "Semantic Tableaux" and their seeds can be found in papers by Peirce's students, C. Ladd and A. Marquand, that appeared in *Studies in Logic* (1883) edited by Peirce. Dodgson named his approach, the Method of Trees. It was virtually unknown until 1977 when W. Bartley published the second part of Dodgson's *Symbolic Logic,* a book thought to have been lost. In this paper we examine the Method of Trees closely and establish the semantic connections between it, Ladd's "inconsistent triad" and Marquand's logic machines.

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† Abeles's paper appears in this issue of *Modern Logic*.

Applications of Existential Graphs In Artificial Intelligence and Linguistics (John F. Sowa, IBM Systems Research Institute): With his dependency grammar, Lucien Tesnière introduced graph notations into linguistics. His graphs had a strong influence on the semantic networks developed for machine translation, computational linguistics, and artificial intelligence. Until the 1970s, however, those systems could not express all of the operators and quantifiers of first-order logic. Peirce's existential graphs, developed 80 years earlier, formed a logical system that was more complete, more elegant, and simpler than any of them. Furthermore, Peirce's nesting of contexts was isomorphic to the structures independently developed by Hans Kamp for discourse representation. Conceptual graphs are a synthesis of Peirce's graphs with semantic networks. The result is a complete system of logic that maps to natural languages more simply and directly than the predicate calculus.

Peirce and Russell: The History of a Neglected Controversy (Benjamin S. Hawkins, Jr., Columbus College): Early this century Charles Peirce (1839-1914) and Bertrand Russell (1872-1970) were combatants in a *guerre de plume* which has so far received scant attention. It is that controversy (a missed opportunity in the History of Logic) that is examined in the present paper.

The character of Peirce's and Russell's difference is outlined; issues relating to work of Gottlob Frege (1848-1925) and to Peirce's review of Russell's *Principles of Mathematics* are discussed, recourse being substantially made to annotations by Peirce in a copy of *Principles of Mathematics* in the Houghton Library at Harvard; efficacy of the Peirce-Russell 'controversy' is evaluated, and objections are proffered to an accepted interpretation of Peirce on Russell.

Peirce and Philo (Jay Zeman, University of Florida): Even a casual examination of Peirce's algebras of logic reveals an approach to the conditional which emphasizes its importance to him. Although at various times he employs other connectives than his '—<' as what we would call "primitive," the significance of the conditional in his thought emerges in a variety of ways throughout his work.

Perhaps the commonest wisdom about Peirce's approach to the conditional draws on his identification of himself as "a Philonian" and his development within the algebras of logic of a conditional function which is our good old classical ("material") implication. This interpretation might identify Peirce the Philonian as the "nominalist," in lifelong dialogue with Peirce the "realist"; in that dialogue-struggle, the "nominalist self surrenders his last stronghold, that of Philonian or material implication" (Fisch 196)[†] only quite late in his life, around the time of the *Prolegomena to an Apology for Pragmaticism.*

I suggest that this picture of Peirce and the conditional is incomplete. With his development of the Existential Graphs, he had at hand a notation which permitted a more iconic approach to modality, and with it, to representation of a conditional function which we might call "strict." But as early as 1880 and quite consistently thereafter he argues for a conditional with what we might call "an extension in thirdness"; the familiar "*consequentia simplex de inesse*" is second to that third, and never (in the post-"Fixation" days anyway) a substitute for it. The Gamma Graphs, especially the "multiple sheet" versions, provided a notation for concepts **which had been integral to Peirce's thought for some two decades.**

A twenty-minute paper can only begin to sketch out the lines of this, and to point out other relevant matters in Peirce's work on the conditional, such as axioms (from about 1911) which factor what we can recognize as distributive ($\cap$-$\cup$ distributive!) properties of the conditional from non-distributive properties. All in all, this work suggests an integration of Peirce's "nominalist" and "realist" selves earlier than is commonly thought.

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†Fisch, Max H. *Peirce, Semiotic, and Pragmaticism,* ed. Kenneth Laine Ketner and Christian J. W. Kloesel, Bloomington: Indiana University Press, 1986.

Conclusion

To the modern reader, the wide range of Peirce's contributions as described in the above abstracts or as mentioned elsewhere in this report may, at first, seem exaggerated--perhaps too much is being claimed for Peirce. A sceptical reaction is to be expected when someone whose work has not been widely acclaimed in recent times, is presented as though he were a modern-day Leibniz, or even a modern-day Aristotle. It will not relieve doubts to learn that Peirce ranked himself with Leibniz, although it may give the doubter some pause to learn that no less a scholar than Schröder compared Peirce favorably with Aristotle. Perhaps a more sober assessment was that made by Łukasiewicz in 1922 when he listed his "fellow workers" in the "still tiny group of philosophers and mathematicians who have chosen mathematical logic as the subject or the basis of their investigations." ("On Determinism," *Selected Works*, p. 111.) Only five mathematicians cum logicians (other than himself) were chosen by Łukasiewicz for this select group, and Peirce was among them. The others were Leibniz, Boole, Frege, and Russell. But testimonials alone will never, and should never, secure for Peirce a permanent place among such company. Only through the strength of his work can he ever merit such a rank. It is surprising that so many years after his death, mathematicians and logicians are starting to study Peirce's writings with a new-found interest and excitement. What was revealed at the Peirce Congress is that what we may expect to find in Peirce is not just of historical significance; we may expect to find important new ideas and insights for a number of fields of contemporary mathematical and logical study. What rank Peirce will achieve in the history of logic or, more generally, in the history of thought, can only be guessed, but that need not concern us. History will take care of itself. But if neglect of Peirce has deprived us of useful ideas, that is another matter--one we must now rectify.