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Free Logic. Selected Essays

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REVIEW

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Let me start with an important warning: the subtitle “Selected Essays” is rather misleading. It suggests that previously published papers have been ‘merely’ brought together, *i.e.*, reprinted in a single volume for the benefit of the reader. Such is definitely not the case. Although much of the material has in effect been published before, stretching over a forty-year period, it has gone through a thorough revision—one of the papers, *e.g.*, is the result of the integration of several published papers—and in addition, new material has been added, based on unpublished (joint) work. Perhaps “New Essays” would have been an overemphasis in the other direction, but to my mind it is indeed more ‘new’ than merely ‘selected’.

Nine essays constitute this volume, each one of them dealing with free logic, either within free logic itself or related to applications, some historical, some philosophical, some logical-technical. In short, the author covers most of the free logic landscape by selecting a small number of mountain peaks offering, nevertheless, a full view.

The first essay “Russell’s Version of the Theory of Definite Descriptions” aims to show that Russell had in effect two theories of definite descriptions, the first one expressed in the well-known classic “On Denoting” (1905), the second one presented in the even greater classic *Principia Mathematica* (1910). Lambert shows that Russell had different objectives in mind when he analyzed definite descriptions—1905 dealt with a logical analysis of ordinary language, whereas 1910 was aimed at the reduction of mathematics to logic—thus leading to different conceptions.

“Existential Import, ‘E!’ and ‘The’” is a truly foundational paper of free logic. A careful analysis of the shortcomings of several proposals to deal with *the fallacy of existential import*, *i.e.*, to deduce

$(\exists x)(Px \& Qx)$ from $(\forall x)(Px \supset Qx)$, leads nearly inevitably to the solution put forward by *free logic*. As a sixteen-page defense of free logic by one of its founding fathers, this paper is paradigmatic material.

In “The Reduction of Two Paradoxes and the Significance Thereof”, Lambert shows that two paradoxes that are usually considered as separate problems—on the one hand, “the round square is and is not a round square”, and, on the other hand, the Russell paradox—are from the perspective of free logic actually related and can therefore be treated in a uniform way. Basically what happens is that in both cases existential statements are needed to derive the paradoxes:

$$(\exists x) (x = iy (\text{RoundSquare}(y) \& \sim \text{RoundSquare}(y))),$$

where “*i*” is the well known iota-operator (although usually “*i*” is used, here “*i*” is used throughout the book), in the case of the first paradox, and

$$(\exists x) (x = \{y : y \notin y\})$$

in the case of the Russell paradox (if we agree to read $\{y : A\}$ as shorthand for $ix (\forall y)(y \in x \equiv A)$, which seems quite acceptable for we do talk about *the* Russell set). Rejecting these, as one has the liberty to do in free logic, thereby blocks the derivation. It is a beautiful illustration of the power of (the family of) free logic(s) to handle philosophical and logical problems by providing new views and solutions.

“The Hilbert-Bernays Theory of Definite Descriptions” deals with the problem of how to represent formally what Hilbert and Bernays present informally in *Die Grundlagen der Arithmetik* (1934), *viz.* that talk about “the object *x* such that ...” is acceptable if the existence and uniqueness of the object can be *proved*. This generates an intriguing problem because as a consequence the grammar of the language becomes dependent on the notion of provability that itself presupposes the availability of a grammar. How to get away from (though not necessarily solve) this problem is the topic of the remainder of this paper, focusing on the approach of Sören Stenlund, *The Logic of Description and Existence* (Uppsala: Filosofiska Studier, 1973), discussing its virtues and, near the end of the paper, mainly its faults.

In “Foundations of the Hierarchy of Positive Free Definite Description Theories” a framework is presented that allows a classification of a number of positive free logics. The most important result that comes out of it, is that Bas Van Fraassen was wrong when he suggested that positive free logics form a one-dimensional structure. Lambert shows that it is indeed two-dimensional. Of special interest, in the sense that it could very well turn out to be a useful notion outside of free logic, is

the *skeleton*-idea, although apparently the idea itself has been around for a while, first developed by Von Neumann (p. 75). The problem is basically this: say we are working within the natural numbers, then the two expressions “the x such that $x^2 = 2$ ” and “the x such that $x + 2 = 1$ ” have the same extension, *viz.* nothing. Nevertheless, you do want to make a distinction because it would be odd to claim “the x such that $x^2 = 2$ is identical to the x such that $x + 2 = 1$ ”. The solution is to bring all terms into account, thus “the x such that $x^2 = 2$ ” (also) says something about 2, so it can be seen as an expression or a function involving 2, say, $f(2)$, whereas the expression “the x such that $x + 2 = 1$ ” can be represented as a function g of two arguments, *viz.* 2 and 1. Such functions are called skeletons and they make it possible to distinguish meaningfully between expressions having no extensions. Or, to put it in other words, as I understand this idea, a name always comes together with a kind of *general* description. It serves as a context without, however, bringing into the logic the intensionality that usually goes together with contextualization.

The two papers that follow are of a more philosophical nature and deal precisely with this problem. In “Predication and Extensionality” the whole issue—how could it be otherwise in the framework of a discussion about the merits and shortcomings of free logic?—is about predication, extension, and substitutivity of identity. What Lambert wants to show is that the radical Quinean solution to eliminate all singular terms from an ideal regimented language, in order to avoid all problems having to do with non-existents and the loss of substitutivity of identity, is too harsh. A far simpler solution is in the author’s very own words “the insertion of the words ‘if any’ in the appropriate place in the statement” (p.106), *i.e.*, the sentence ‘Vulcan rotates’ is false if it is understood to mean ‘Vulcan, if any such thing exists, rotates’ and that is clearly false. An additional advantage is that, although we have to look at the world to decide whether sentences involving singular terms are true or false (which seems alright, one is inclined to think), this is not so as to logical form for sentences such as ‘Clinton travels around’ and ‘Vulcan rotates’ are treated in exactly the same logical way.

The next chapter, “Nonextensionality”, further explores these deep issues. Here two approaches are discussed: the first one dealing with an inner-outer domain of objects, the second one dealing with a special interpretation of what predicates are and could be. In the former case the “traditional” problems of such a semantics are presented. How to make a distinction on logical grounds between real things (= the inner

domain) and virtual things (= the outer domain)? And what prevents us from introducing special quantifiers, given that presumably $(\forall x)\dots$ is meant to range over the inner domain? So if $[\forall x]\dots$ stands for a quantifier over all objects, then again we are in trouble with extensionality and substitution of identity. In the latter case the basic assumption is to make a distinction between the predicate “is delightful” and the sentence “It is delightful” (although it obviously involves the predicate). In the case of the sentence one needs to know what “it” is referring to, to establish its truth-value, whereas the extension of the predicate will be a set. What Lambert shows is that (kinds of) free logics are precisely what is needed to be able to make such distinctions while still maintaining extensionality. In short, combining these two essays, it should be clear that Quine should have embraced free logic.

The core essay of the book is “The Philosophical Foundations of Free Logic”. Actually, one would have expected to have found this paper in the beginning of the book as it sketches the history and philosophy of free logic in an extremely accessible presentation. An alternative title could have been “Everything you always wanted to know about free logics, but were afraid to ask”. All essential topics are dealt with in this fifty-page presentation. The important distinction between positive, negative and nonvalent free logics is presented and analyzed. For the first one, the statement “Pegasus = Pegasus” is a false statement, for the middle one a true statement and for the third one it has no truth-value. A number of misunderstandings are cleared up and the question whether free logic is an alternative to or an extension of classical predicate logic is discussed as well. This careful dealing with the subject leads Lambert to write down such beautiful phrases like this: “In free logics, then, there may be expressions—‘Vulcan’ or ‘the man born simultaneously of nine jotun maidens’ or ‘1/0’, for instance—that are singular terms (*contra* Russell), don’t have existential import (*contra* Frege), and may (Meinong) or may not refer to some variety of nonexistent objects (Parsons)” (p. 138). A logical landscape sketched in one sentence is a sure sign of insight. Further on, there is a nice discussion about the Carnapian “null thing” n and the problems it creates. Supervaluations are introduced and evaluated and, finally, a discussion about the eliminability of the $E!$ -operator in classical predicate logic without identity is outlined. This paper is really an excellent alternative/complement to Ermanno Bencivenga’s contribution *Free Logics* in the *Handbook of Philosophical Logic*, volume 5 of the second edition, published in 2002, pp. 147-196, by Springer, New York.

The last essay in this book, “Logical Truth and Microphysics”, is a beautiful application of free logic to that branch of physics that continues to puzzle us all: quantum mechanics. The basic idea is to use the supervaluational approach of Van Fraassen to allow for the possibility that, on the one hand, one can safely claim that “It is always the case that $G(b, t) \vee \sim G(b, t)$ is true”—where $G(b, t)$ stands for “ b has position G at time t ”—and nevertheless reject the statement that is so problematic within a quantum theoretical framework, *viz.*, “It is always the case that $G(b, t)$ is true or that $\sim G(b, t)$ is true”. It is a nice solution as it takes into account the particularity of quantum mechanical statements that can be non-determined, not-fixed or what have you, and yet saves the maximum of classical logical truths, such as excluded middle. So, perhaps somewhat surprisingly, the final sentence of this paper and of the book is that “quantum mechanics does not require special logics called ‘quantum logics’”. Given the enormous enthusiasm at the present moment for quantum logics, and all its associated ideas such as lattices, toposes, category theory and even more eccentric ideas, this sounds like a strong statement indeed. Although at the same time one has to realize that Lambert is not being inconsequential, far from it. The author prefers to see free logics as an extension of classical logic and not as an alternative. He prefers to keep extensionality whenever and wherever possible, so it is not surprising that he is not really impressed by quantum logics, as these systems can only be seen as alternatives to classical logic, rejecting as they do such cherished logical laws like distributivity.

Nevertheless, as the overall conclusion one cannot deny that free logic, whether positive, negative or neutral, has been and still is an important addition to the logical garden (of Eden?), combining the two most aimed for properties of a “nice” logic: depth and simplicity. Simple solutions for complex and deep problems is what we are looking for after all and this book of essays throws in a third element that is specifically tied to its author, namely a clarity of exposition and presentation that at moments makes the book actually fun to read.

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