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REVIEW

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The volume originates from the Florence LMPS Congress, as mentioned in the subtitle. It is not a proceedings volume in the strict sense; it consists of papers selected from a more numerous list of talks given at the Congress in Sections 1-5 and 10. Contributions to sections, devoted to methodology and philosophy of science have already been presented in three volumes, two of which have appeared, the first in 1997, *Logic and Scientific Methods* and *Structures and Norms in Science*, edited by M. L. Dalla Chiara, K. Doets, D. Mundici and J. van Benthem (Synthese Library, vols. 259, 260); and in 1999, *Language, Quantum, Music*, edited by M. L. Dalla Chiara, R. Giuntini and F. Laudisa (Synthese Library, vol. 281).

It is well known that large meetings of this sort result in many-volume proceedings if the demand for completeness (at least for invited talks) is satisfied. The present volume contains only twenty brief and dense papers that the editors have judiciously selected from the set of contributions to the 1995 Congress in the areas of proof theory, constructive formal systems, recursion theory, modal logic and other associated areas of logic. Some motivations for the choice of the twenty papers are hinted at in a short Preface, some remain implicit. The editors' interests are reflected in this choice, which offers a large and stimulating picture of lines of research still in progress today. Here, only a short summary of the contents of each paper will be sketched before an equally short closing remark of a general character is given.

1. M. Forti, F. Honsell, M. Lenisa, "Operations, Collections and Sets within a General Axiomatic Framework", 1-24. The paper is a crisp

synthesis of the foundational approach inspired by Ennio De Giorgi, who since 1985 and especially through his seminars at the Scuola Normale, Pisa, worked out an original program, articulated into "Basic Theories". The program is centered on four desiderata for a foundation of mathematics: (1) to avoid the reduction of all of mathematics to only one kind of collection (*i.e.*, to sets), by admitting qualitatively different entities and focusing in particular on the concept of operation as non-extensional, (2) to be self-referential, (3) to be open-ended, (4) to admit (semi-formal) axiomatisation. The authors give mainly a list of axioms and definitions together with their motivations, to be located in a constructive theory of operations aimed at applications in computer science. A few results are mentioned, one of which establishes the relative consistency of the "Framework Theory" (based on a set of the listed axioms) with respect to ZF. The sense of quantifiers in the axioms may be problematic in consideration of the above desiderata, but further justification can possibly be found in De Giorgi's writings.

2. G. Hellman, "Some Ins and Outs of Indispensability: A Modal-Structural Perspective", 25-40. By a moderate admission of indispensability arguments, and a scrupulous analysis of their previous formulations, the author achieves a more precise evaluation of the inference from the indispensable rôle of a specific mathematical principle within natural sciences to its truth or relevance for foundational purposes. After mentioning the presumed virtues of both mereological modalisation (applied to nominalistic versions of a good deal of impredicative number theory) and weak subsystems of PA^2, the case is made for a moderate holism, respecting the specificity of the evidence for mathematical statements. Objections by P. Maddy and E. Sober to indispensability arguments are faced and parried – perhaps they are rejected too strongly, in the sense that appeal to common sense is skirting triviality. Finally, the Ochkamist value of ACA_0 in the perspective of reverse mathematics is stressed. The number of different topics collected within this paper leaves many steps implicit and the style falls short of Cartesian clarity.

3. T. Fruchart, G. Longo, "Carnap's Remarks on Impredicative Definitions and the Genericity Theorem", 41-56. This paper provides justification in terms of "prototypical proofs" for the inference from $\varphi(a)$ to $\forall x\varphi(x)$ with extreme clarity. Thus it intends to vindicate Carnap's claim that (second order) impredicative definitions can be safe provided they rely on a proof (of that kind) and that this is all that is needed. What Carnap missed was that the justification has to resort to constructive logic in the form of polymorphic lambda-calculus. To

this aim the authors add to Girard's System F the "Axiom C", which allows proving the *generic* nature of types, by using the Church-Rosser Property as the only non-logical assumption: two polymorphic terms M and N, *i.e.* of type $\forall X.\sigma$, that are equal on one single input type are equal on any other. A notable regularity is thus achieved, one that provides "more solid grounds" to mathematical practice, in a spirit close to category theory, where proofs have uniformity built in.

4. W. Sieg, J. Byrnes, "Gödel, Turing, and K-Graph Machines", 57-66. Turing's Thesis is here divided into a three-stage argument, which the authors suggest makes the Thesis much more plausible. (1) Any mechanical procedure is a *bounded-and-local, deterministic* computation, where the bound concerns both the number of symbolic configurations and the states to be taken into account. Localisation concerns the tape's squares and configurations, centered on the observed one to be taken into account. Moreover, any machine computation is extensionally equivalent to one of that kind. (2) Any mechanical procedure is a computation of some *string machine*; moreover, such a notion is reducible to that of a *letter machine.* (3) Finally, and here lies the original content of the paper, any mechanical procedure is a computation by a K-graph machine (K for Kolmogorov) and any such machine is equivalent to a letter machine. The conceptual analysis by Sieg and Byrnes is stimulating: as a key component in Turing's notion of an algorithm they spot a topological assumption for the neighbourhoods of any given configuration (in the scanned square). Obviously, the intended space is neither discrete nor indiscrete. Among the many options to examine, one is that K-graph machines make use of finite graphs. But here local-and-bounded is not synonymous with finite, unless the latter notion is reinterpreted.

5. A. Berarducci, B. Intrigila, "Linear Recurrence Relations are Δ_0 Definable", 67-82. The paper investigates Δ_0-definability of subsets of $\mathbb{N}^{k+1}$, as graphs of functions $\mathbb{N}^k \longrightarrow \mathbb{N}$. By only admitting polynomially bounded quantifiers, focus is concentrated on those properties of Δ_0-functions which are provable in arithmetic with the induction scheme similarly restricted to Δ_0-formulae. The question is: Which kinds of recursive definitions do not lead outside of Δ_0? As a partial answer, the authors show, by means of graph-theoretic techniques that, at least for a special class of recurrence relations F, the function $n \longmapsto F(n)$ has a Δ_0-graph and that this result is extendable to a corresponding class of matrices. A crucial step is that the number of solutions of a diophantine equation $c_1x_1+\ldots+c_kx_k$ is Δ_0-definable for *fixed* coefficients; the problem of whether this number is a Δ_0-function of coefficients is also faced.

6. G. Jäger, R. Kahle, T. Strahm, "On Applicative Theories", 83-92. A great deal of recent work in proof theory stems from Feferman's exploration of a system of "explicit mathematics", as a two-sorted (types and operations) intensional ground for Bishop's constructive analysis. To grant (at least partial) self-application in principle, most of the applicative theories, considered as developments of combinatory logic, deal with one-sorted domain of operations. The purpose of the present paper is to survey proof-theoretic results on applicative theories having, as the underlying logic, Beeson's (classical) logic of partial terms, in which the only atomic formulae are $N(t)$, $t\downarrow$, and $s = t$, for terms s, t defined by 'basic' operations. Then a set of axioms is added which characterizes the theory "BON" (by Feferman and Jäger), based on a partial combinatory algebra with pairing, projections, natural numbers, definition by cases and primitive recursion on $\mathbb{N}$. The result is a subtheory of PRA, but using relatively weak induction principles allows the derivation of a proof-theoretic equivalent to PRA and even PA. What more is needed to recover as much as possible of classical mathematics in applicative theories, while preserving, as far as possible, predicative constructivity? In comparing a range of formal systems of increasing power, the authors refer to results about polynomial time computable arithmetic obtained by one of the editors. In fact, A. Cantini (see his *Logical Frameworks for Truth and Abstraction*, Amsterdam, North-Holland, 1996) has provided a relevant contribution to this field, in particular for what concerns Aczel's notion of *Frege structures*, as a way of enriching applicative theories with types. Some of the proof-theoretic results mentioned in the paper appear to be also of interest for theoretical computer science.

7. U. Kohlenbach, "The Use of a Logical Principle of Uniform Boundedness in Analysis", 93-106. The paper offers a proof-theoretic analysis of how to extract bounds on Π_2-sentences in subsystems of arithmetic in all finite types. To avoid commitment with arithmetical comprehension over functions, an axiom (F) is introduced which allows the proof that all functions $[0,1] \longrightarrow \mathbb{R}$ represented by a functional are uniformly continuous and possess a modulus of uniform continuity. After recalling that the existence of suitable bounds searched for is provable in a subsystem of PA^2, in line with the program of reverse mathematics advocated by H. Friedman, the author lists some applications, showing the gain obtained by his approach in simplifying the proofs of classical theorems of analysis. Unfortunately, these well-known results, once logically dressed as suggested, seem hardly as recognisable and usable as they are in mathematical practice.

8. G. Mints, S. Tupailo, "Epsilon-Substitution Method for the Ramified Language and Δ^1_1-Comprehension Rule", 107-130. The ϵ-symbol, by which quantifiers are explicitly definable, was used by Hilbert in order to transform infinitary proofs into finitary ones, provided choice is turned into a logical notion. The method suggested by him was substitutional: given a set E of number-theoretic sentences, a 'solution' for E is obtained once all critical formulae for the E-language, *i.e.*, those of the form $\varphi(t) \longrightarrow \varphi(\epsilon x \varphi x)$, are true under the substitution of numerals for closed ϵ-terms. Searching for such a solution provides a refined sequence of substitutional approximations, satisfying the condition that if none of the first $i+1$ substitutions $s_0, \ldots, s_i$ solves E, and F is the first formula in E responsible for the failure, then $s_{i+1}(\epsilon x \varphi) =$ the least $n \leq t$ such that $s_{i+1}(\varphi(n))$ is true. How can one ensure, for any E, and in particular for the sentences of analysis, that the sequence $\langle s_i \rangle$ terminates in a finite number of steps? The authors address this question a subsystem of "ramified analysis" (intended as a second-order arithmetic with a Δ^1_1-comprehension rule), conveniently reformulated with ϵ-terms. By refining and extending the substitution method, the termination asked for is achieved. However, as might be expected, the construction is non-effective. The fact that the underlying logic remains classical is another drawback.

9. X. Caicedo, "The Abstract Compactness Theorem Revisited", 131-142. The paper concerns the relationship between Łós-like theorems for infinitary languages and compactness. In elementary logic, the compactness theorem follows from Łós's Theorem on ultraproducts. In 1983, J. Makowsky and S. Shelah proved the Abstract Compactness Theorem, which states that an analogous equivalence holds for the $[\kappa, \lambda]$-compactness of any model-theoretic logic. Whereas they used ultrapowers and suitable expansions of the given language, the author emphasises the "purely topological" nature of the result, by deriving it from the preservation of $[\kappa, \lambda]$-compactness under products (of appropriate arity) of topological spaces. The size of the cardinal κ was already known to be essential for this preservation property. The 'reference' spaces to be considered are analogous to that induced by the elementary classes.

10. W. Veldman, "On Sets Enclosed Between a Set and its Double Complement", 143-154. From an intuitionistic perspective, the paper investigates the notion of weak stability, concerning subsets that are included in between Y and $\neg\neg Y$ (all within a universe set X). The import of this notion is examined for Cantor space and intuitionistic $\mathbb{R}$. The key definition is that of a subset, named Perhaps(Y, Z), defined in terms of an apartness $\#$ as $\{x \in X : \exists y \in Y (x \# y \to x \in Z)\}$.

Y is *weakly stable* (in X) iff Y *coincides* with Perhaps(Y, Y), where coincidence is non-apartness (an equivalence relation) and is implicitly extended from elements to subsets of X. Some results are listed, starting with the correlation, in metric spaces, of Perhaps(Y, Z) with sequences of special stability, to arrive at spreads and well-founded trees in connection with which the property of bounded "perhapsity" is examined. It is proved that this property does not hold for the set of real numbers with a binary expansion.

11. J. Lambek, "Relations. Binary Relations in the Social and Mathematical Sciences", 155-164. With typical elegance, Lambek gives here a quick picture of the usefulness of the calculus of binary relations, once the three operations $\otimes$, $/$ and $\backslash$ are defined as adjunctions. Some applications in various fields are surveyed: from the formulation of kinship systems, as subtle as the Trobriand's, to models of noncommutative linear logic. Then, in order to generalise the relational property of the "connecting homomorphism" emphasized by Saunders Mac Lane in homological algebra, the concept of *potential* relation is introduced within a suitable "operational" category. The paper also offers a crystal-clear, purely algebraic analysis of recursive functions in terms of the above calculus.

12. G. Battilotti, G. Sambin, "Basic Logic and the Cube of its Extensions", 165-186. The *labyrinth* of logics is somewhat frustrating to contemplate as soon as one's own philosophical biases are suspended and one is not content with the mere proliferation of new formal systems. In this paper a unification (at the propositional level) is achieved through few simple and powerful guiding ideas, in terms of which not only an essential atlas of fundamental families of logics is given, but also a new logic **B** is identified as the "basic logic": below (in provability order) intuitionistic, linear and ortho- logic, resp., thus occupying a position dual to classical logic. In fact, the formulation of **B**, here given as a sequent calculus, allows the determination of the two remaining vertices of a cube of logics. The authors carefully prove that any two logics so identified are not equivalent to each other. A corresponding comparative analysis at the semantic level is still lacking; however, as the second author is well-known for the development of *formal topology*, connections between **B** and such a semantic framework can be confidently expected to emerge.

13. H. Ono, "Some Observations on Noncommutative Substructural Logics", 187-194. A purely syntactic interest in logics with no exchange rules or axioms is shared by this paper too, which avoids any appeal to motivations for Lambek calculi or relevant entailment. The starting point is a sequential system **FL** ("full Lambek logic") with no

structural rule. Proof-theoretic extensions of **FL**, obtained by adding exchange ($\mathbf{FL}_e$), weakening ($\mathbf{FL}_w$) and contraction ($\mathbf{FL}_c$), are investigated, but the core of the paper carefully focuses on only the implicational fragments of such extensions, $(\mathbf{FL}_e)_{\rightarrow}$, now presented in Hilbert-style. Among the results proved by the author is that cut-elimination holds for **FL** extended with a rule called "global" contraction, whereas it does not hold for $(\mathbf{FL}_c)_{\rightarrow}$. The decision problem for both systems is discussed in comparison to previous results about relevant logics.

14. D. Westerståhl, "On Predicate Logic as Modal Logic", 195-208. Modal interpretations of satisfaction are discussed in this paper, mainly devoted to the $\exists/\Diamond$ analogy, in view of Tarski's definition – $M, \alpha \models \exists x\varphi$ iff *there is a* β *such that* $\alpha {=_x} \beta$ *and* $M, \beta \models \beta$, for assignments α, β to $\mathrm{Dom}(M)$, where $\alpha {=_x} \beta$ means that for all variables z except possibly x, $\alpha(z) = \beta(z)$; and Kripke's definition of $\Diamond$ (*there is a ... such that ... and ...*). Under this analogy, J. van Benthem had suggested the possibility of identifying sublogics of first-order predicate logic without identity. To better assess the correspondence on the semantic side, this paper compares the strength of different hypotheses on the class of frame-models; rather than by enriching the language, the issue of atomic formulae is approached by imposing constraints on models, such as the *locality principle* and the *path principle.* The special status of generalised assignment frames (GA) is emphasised. The author shows that a formal sub-system of predicate logic (now to be seen as a modal propositional logic!) axiomatises GA-truth. Finally, a few conditions (on GA-frames) sufficient to collapse modalities are listed.

15. A Chagrov, "A First-Order Effect and Modal Propositional Formulas", 209-218. A different kind of comparison of first-order with modal propositional logic is examined in this paper in a more standard model-theoretic way. As a modal analogue to a classical result for first-order theories, the following property is introduced: (*) a formula has an infinite-rooted frame whenever it has arbitrary finite-rooted frames. (Note that tense logic provides counterexamples.) The author essentially devotes the paper to the proofs of two results: Theorem 1. There exists a finitely axiomatised normal modal logic with property (*); Theorem 2. Any normal extension of $\mathbf{K4} = \mathbf{K} + \Box p \longrightarrow \Box\Box p$ has property (*).

16. M. Fitting, "Herbrand's Theorem for a Modal Logic", 219-226. The problem faced in this paper is indeed one of general interest. In contrast with other results about first-order classical logic, Herbrand's Theorem turned out to be difficult to transfer to non-classical logics. Here, Fitting sketches how it works for the modal system **K**, provided

its syntax is expanded by *predicate (lambda) abstraction* in order to match the challenge raised by Skolemisation in a modal context (non-rigidity of singular terms and its effect on the scope ambiguity of $\Box$). In general, due to the presence of abstraction terms, validity of the Herbrand expansion of a modal formula is "not entirely a propositional issue" (p. 224) as the author admits, but he has proved that validity of quantifier-free closed modal formulae is decidable (by tableau methods). Predicate abstraction is suggested here as an essential tool for a satisfactory first-order modal logic.

17. F. Wolter, M. Zakharyaschev, "Intuitionistic Modal Logic", 227-238. Among the different possible ways to develop an intuitionistic modal logic, that of respecting the mutual independence of $\Box$ and $\Diamond$ is the one adopted in this paper as a guiding principle. The resulting approach owes much to the path-breaking work, since the mid-seventies by G. Fischer-Servi, who found a system, known as **FS**, that connects $\Box$ and $\Diamond$ in such a way as to support a 'nice' translation into first-order intuitionistic logic. A notable extension of **FS** is the system named **MIPC**, already devised by Prior. The paper examines these and other axiomatic systems for normal intuitionistic modal logic, by adapting notions already at work in the duality between modal frames and modalised Heyting algebras. In particular, K. Fine's theorem on persistence for the classical case is generalised to the intuitionistic one. The authors also discuss other approaches to develop a relational semantics for these kinds of systems and provide a useful list of completeness results for them.

18. H. Jervell, "Dynamic Datastructures", 239-248. Within the paper, the issue of data structures with an inner computational 'activity' is approached, using Girard's dilators as reference background. For this purpose, an idea proposed by Feferman is suitably modified in considering not just the category **LIN** of linear orders and order preserving maps, but also two subcategories: the category **WF** of well-founded orderings and **FIN** of finite orderings. The basic intuition is thinking of a dynamic data structure (on natural numbers) as an endofunctor F on **WF**, with $F(x)$ = the set of ordinals given by the data structure, provided a couple of conditions are satisfied: F takes ordinals to ordinals and preserves increasing functions. Among the endofunctors on **LIN**, some are selected in virtue of their preservations properties (*e.g.*, of direct limits) and characterised in terms of the "denotation set" for an ordering x, to be taken as, roughly speaking, the collection of all finite approximations of x.

19. A. Leitsch, "Resolution and the Decision Problem", 249-270. After a few very useful historical notes on the *Entscheidungsproblem*

for first-order logic, the paper offers a concise survey of model-theoretic results on reduction classes. The line adopted here is, however, proof-theoretic and one specifically centered on the resolution algorithm, originally based on clause-forms. (Note that these provide a reduction class.) The author extends an idea by W. Joyner about resolution strategies in automated theorem proving, in order to deal with clause-form classes that cannot be obtained from prenex form classes (in the presence of function terms). The advantage of the resolution method lies in that for any non-derivable sentence it produces actual counter-models. 'A-ordering' refinements and hyper-resolution techniques are extensively compared in the final sections of this paper, which stands out for its particularly clear and well-organised exposition conveying the 'sense' of a whole area of research.

20. S. Guerrini, S. Martini, A. Masini, "Modal Logic, Linear Logic, Optimal Lambda-Reduction", 271-282. The last paper of the volume is motivated by the interest in proof-nets for linear logic – actually, the only modalities considered are ? and ! , the aim being that of extracting useful information on reduction of (lambda-)terms. Under the correspondence of cut-elimination with β-reduction, the fundamental idea is that of working out an assignment of (local) levels to formulae in proof-nets, in conjunction with J.-J. Lévy's result that there is one optimal recursive *parallel* strategy for normalisation of lambda-terms. The concept of *sharing level structure of links* is proposed to internally control the operation of reindexing. Finally, the authors mention that, in the presence of absorption, they have found an internal (non-semantical) solution to the problem of obtaining the graph of term N' from that of N, when N reduces to N'.

The book is carefully edited. The number of detected misprints (and gaps in references) is very small and they never prevent understanding. As always with a proceedings volume, the scientific quality of the contributions, their readability and their lasting usability vary widely. And, as expected, some papers assume familiarity by the reader with their specific subjects. But as a whole, the content of the book offers a faithful picture of the state of the art in logical research at the end of 20th century. Foundations of mathematics are less faithfully represented. In fact, no paper deals properly with standard set-theoretic or category-theoretic foundations, though some basic hints can be found in a few contributions (such as #3, #6, #11). Clearly, the book's origins should be borne in mind.

A final note of a different character must be added. The 1995 Florence Congress testified to the international credit achieved by the research group in logic at the University of Florence. This group was founded by Ettore Casari, who from the sixties devoted his efforts to returning logical and foundational studies in Italy to the highest standards; and for many years the group was inspired by his presence (before he moved to the Scuola Normale of Pisa). Andrea Cantini and Pierluigi Minari, co-editors of the book were, in fact, former students of Casari. Both the organisation of the Congress and now this volume bear his imprint, if at long range. Also the reviewer personally owes him a great deal.

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