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Petr Hájek (editor)
Gödel '96: Logical Foundations of Mathematics, Computer Science, and Physics—Kurt Gödel's Legacy,
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REVIEW

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This festschrift constitutes the proceedings of a conference held in Brno, Czech Republic, commemorating the 90th anniversary of Kurt Gödel's birth. As usual with such volumes, *Gödel '96* contains some papers directly about the honoree's work, plus several more representing original contributions to research areas more or less related to Gödel. Of course, Gödel is best known for his work in mathematical logic. But his interests ranged widely, and thus so do the topics covered in this volume, as its subtitle indicates. (Although Gödel's bibliography even extends to mathematical economics, that field does not appear here, however.) Highlighting some of the nine invited addresses in this book will convey some of this variety.

Solomon Feferman, noted as an editor of Gödel's collected works as well as a logician in his own right, wrote the lead article, "Gödel's program for new axioms: Why, where, how and what?" This discusses at some length Gödel's repeated statements of the need for additional set-theoretic axioms to settle questions such as the Continuum Hypothesis or even arithmetic propositions. Gödel expressed the hope for axioms "based on hitherto unknown principles ... which a more profound understanding of the concepts underlying logic and mathematics would enable us to recognize as implied by these concepts." Feferman then goes on to present his own recent work in that direction, in particular, the application of reflection principles in an attempt to generate, in Gödel's words, "new axioms which are exactly as evident and justified as those with which you started".

G. F. R. Ellis's paper deals with a completely different side of Gödel's research. Just as Abraham Robinson worked on not only model theory and non-standard analysis, but also the aeronautics of delta wings, so too did Gödel venture into the realm of physics—although at a much more abstract level. Ellis comments on two publications of Gödel on general relativity. The first, from 1949, gave a solution of Einstein's gravitational field equations that violates causality (very loosely speaking, allows for time-travel paradoxes). A subsequent 1952 article produced a family of solutions in which the universe rotates and expands, thereby accounting for redshift. Ellis summarizes these papers and traces their impact on subsequent developments in cosmology.

The contribution by Pavel Pudlák and that by Gaisi Takeuti and Masahiro Yasumoto deal with bounded arithmetic and its connections with computational complexity. Gödel was no stranger to such considerations. As far back as 1936, his "Über die Länge von Beweisen" provided, as John Dawson has said, an "early example of what are known today as 'speed-up' theorems." And a 1956 letter from Gödel to von Neumann anticipated the $P = NP$ problem. The two aforementioned papers in this volume, however, are not surveys of Gödel's work in this field, but rather present current progress.

Boris Kushner's "Kurt Gödel and the constructive Mathematics of A. A. Markov" and Charles Parsons' "Hao Wang as Philosopher" each focus on a person other than Gödel. But in both cases the authors pay a good deal of attention to Gödel's ideas and the influences that he exerted. Other invited addresses in *Gödel '96* treat such topics as multivalued logics, Turing machines and generalizations, and modal logic.

The thirteen contributed papers in the volume exhibit a wide range, not only in content, but also in quality. Again, there are many papers, not directly related to Gödel, of a technical nature in proof theory, set theory, and so on. The article by C. Anthony Anderson and Michael Gettings does connect directly with Gödel, discussing his take on the Ontological Argument. Some dubious passages appear in the contributed papers, though. For example, Michael Stöltzner's effort states, "The most irrational number, the golden ratio, provides ... maximal stability" in orbital motions. I do not know what it means for a number to be more irrational than another. But if it could mean anything at all, surely a nice, unassuming quadratic irrational like the golden ratio would be no more irrational than a transcendental! Undoubtedly, Robert Meyer's paper wins the Most Unorthodox Award. I will let him speak for himself with his opening and closing sentences. (Note: all boldfaces are in the original, and there are plenty more in the rest of his text.) "Being mainly self-educated in beginning logic, I was alarmed to read in [Quine] that Gödel had shown that elementary number theory is either inconsistent or incomplete. 'What,' thought I to myself, 'could this possibly **mean**?' Could it be in **doubt** that $2+2=4$? Might one **multiply** 27 and 37 and get 998? What is **going on** here? More mature reflection convinces one that what is going on is a logical **dirty trick**. ... What are we to make of a statement ~(721 is provable) when this is the very statement #721? As I said at the outset, this is a **dirty trick**. And dirty tricks ought not to be confused with profound metaphysical insights about 'not'. We rest our case!" So do I.

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