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Review of

JAAKKO HINTIKKA (ED.), *FROM DEDEKIND TO GÖDEL. ESSAYS ON THE DEVELOPMENT OF THE FOUNDATIONS OF MATHEMATICS*

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The title of the book is modeled on van Heijenoort's *From Frege to Gödel*, suggesting that the works of Dedekind and other mathematicians also lead to the logician Gödel. Moreover, the content of the book as well as its preface hint that the works of Husserl and other philosophers lead to Gödel too. These remarks amount to saying that Gödel's work in mathematical logic not only originates from the development of mathematical logic but also from the development of mathematics and the mathematically-oriented philosophical discussions concerning concepts and proofs.

The volume gathers 16 lectures which were given at Boston University as a part of the 1991-92 program of the *Boston Colloquium in the Philosophy of Science*. They all deal with mathematicians, logicians and philosophers such as Weierstrass, Dedekind, Kronecker, Hilbert, Borel, Hermann Weyl, Husserl, Wittgenstein, *etc.*, whose works are significant of the history of the foundations of mathematics from 1850 to 1930 and differ more or less sharply from Frege's and Russell's logicism. The editor of the volume, Jaakko Hintikka, gives three reasons for collecting papers on individuals defending opinions of such a variety.

(1) As well as Frege and Russell, these mathematicians and philosophers also constitute a part of the background which gave rise to such great logicians as Gödel and Tarski.
(2) They have expressed viewpoints on the foundations of mathematics that have not yet received the philosophical attention they deserve. Moreover, Frege's and Russell's contributions to foundational studies were not of the same order of magnitude as their contribution to the development of logic.

(3) Studying these viewpoints also affects the way we understand the history of mathematics from 1850 to 1930.

Now, what is understood by "foundational studies"? According to Harold M. Edwards, whose paper deals with Kronecker's views on the foundations of mathematics, "foundational studies" today means "set theory and mathematical logic". However, this obviously constitutes only a part of the meaning of the phrase, as it is understood by the editor and the other contributors of the volume (as a matter of fact, Kronecker's views are alien to set theory and logic). According to Hintikka's preface, "foundational studies" roughly means studies on *ideas about mathematics*, as they are expressed by working mathematicians, logicians or philosophers. From a philosophical point of view, many of these ideas deserve greater attention than they received in the past. Hintikka mentions especially ideas about the concepts of arbitrary function and arbitrary sequence, the anticipation of model-theoretical ideas by Hilbert's axiomatical approach and the precise role of set theory in the foundations of mathematics, *etc.* We should add, among other points, the standard *vs.* nonstandard distinction, which is the theme of Hintikka's own contribution to the volume and also that of Mathieu Marion's paper on Wittgenstein and Ramsey.

We cannot summarize all the aspects of the very interesting papers which are contained in *From Dedekind to Gödel*. They frequently give new insights and firsthand information. Some of them offer a very detailed study of neglected aspects of such or such mathematician or philosopher, especially the paper "The mysteries of Richard Dedekind" as well as the one which displays Wittgenstein's discussion of the famous problem of trisecting the angle ("On saying what you really want to say... "). Other papers introduce recent ideas, such as the paper on the modern theory of magnitudes, including non-Archimedean magnitudes. The history of mathematical logic is present thanks to papers on Frege, Russell, Gödel and Tarski. Among the contributors are some world-known historians or philosophers, such as Judson Webb, Harold M. Edwards, Gregory H. Moore, Dagfinn Føllesdal, *etc.*

Everybody interested in philosophy or history of mathematics should have this book in his personal library or, at least, keep in mind that he can refer to it as to a collection of papers, most of them excellent, on fundamental issues discussed by philosophers and historians of mathematics.

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