Modern Logic
Volume 8, Number 3 & 4 (May 2000-October 2001), pp. 113–116.

Review of

PENELOPE MADDY, *NATURALISM IN MATHEMATICS*

Oxford: Clarendon Press, 1997
viii + 254 pp. ISBN 0-198-23573-9

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Naturalism in Mathematics addresses a fundamental question that lies between Mathematics and Philosophy: what is the status of the axioms from which all our mathematical knowledge derives? Penelope Maddy has been working on this topic for several years already and she has published a multitude of papers on her work. This book is based on some of those papers, as is *Realism in Mathematics* (1990) [2], (see Frápolli [1]).

Everything we know, including our scientific theories, must rest on some kind of evidence, either direct or indirect. Having indirect evidence for a particular thesis or theory means that there is a way of reducing this theory into another for which we have independent reasons, and thus the reduced theory or thesis inherits its evidential support from the theory into which it is reduced.

It is commonly accepted that mathematics can be defined in set theoretical terms and that mathematical objects other than sets can be suppressed in favor of these abstract entities. Thus, one of the most basic philosophical questions about set theory is, "What sort of evidence can we offer to support set-theoretical axioms?" Historically, the different answers to this question have implied different epistemological and metaphysical conceptions of the world. In *Realism in Mathematics* and in Part II of *Naturalism in Mathematics*, the object of the present review, Maddy pursues one of the most appealing conceptions: mathematical realism. This view claims that mathematics works by emulating the natural sciences, in that there is an external reality, independent of human beings, which mathematical theories attempt to reflect and explain. Mathematicians discover, but do not create, the mathematical entities with which they work. They define these entities through axioms which attempt to describe the most abstract features of reality as it is. Among the list of outstanding realist mathematicians, we can count Cantor, Frege and Gödel.

In her book Maddy focuses on set theory for it offers a particularly suitable context for testing ontological and metaphysical problems. One reason for this is that set theory is recent and discussion about its axioms is still lively and controversial. The on-going debate about independent questions, which cannot be settled in the framework established by traditional set-theoretical axioms, is a proof of its liveliness. Some well- known questions concern the independence of the Continuum Hypothesis, the Axiom of Choice and, the one favored by Maddy, the Axiom of Constructibility.

In this context, the outstanding problem is this: if, in a realistic way of thinking, the axioms of set theory describe some kind of reality, while there still exist independence questions, then it would seem that set- theoretical axioms do not fully capture the portion of reality that they are designed to portray. Somewhere there should be new axioms that, together with the traditional ones, can give a complete picture, leaving no room for undecidable propositions.

Maddy argues that realism is not a good option and conceding that independence questions are legitimate does not entail accepting a realist stance. In fact, she goes on to defend a kind of mathematical anti-realism, a view which she dubs "philosophical modesty" and "mathematical naturalism" with which we will deal with later on. Before explaining and commenting on Maddy's proposal, let us first describe the structure and content of Naturalism in Mathematics. The book is divided into three parts, entitled "The Problem", "Realism", and "Naturalism". It also includes a short Conclusion, a Bibliography and an Index.

Part I: The Problem deals with the origins of set theory as it is developed in the works of Cantor, Frege and Zermelo, and explains the status of set theory as the foundation of mathematics. Here Maddy presents (i) the standard axioms and the traditional external and internal arguments in their favor and against them, (ii) the independence questions and (iii) the most debated candidates for the category of new axioms: the Axiom of Constructibility, the Axiom of Large Cardinals and the Axiom of Determinacy. She shows that the choice of any of these candidates requires a particular strategy and that all three strategies are connected in positive and negative ways. They are not compatible with one another, however, and so it is necessary to choose only one of them. And here is the crux of the problem: on what grounds can we pursue one extension of Zermelo-Fraenkel Set Theory and leave the others by the wayside? To illustrate ways in which it is possible to argue for an independence question, she chooses the example of the Axiom of Constructibility. Maddy shows that adding the Axiom of Constructibility ($V = L$) to Zermelo-Fraenkel Set Theory would restrict the development of the theory in ways that are independently valuable.

In **Part II: Realism**, Maddy examines two well-known kinds of mathematical realism, one related to Gödel and the other to Quine. Gödelian realism squares mathematics with physical sciences. The trouble with Gödel's position is the difficulty of finding that capacity in the human mind that connects the realm of mathematical reality with the subjects of knowledge. Also lacking is a way of assessing and justifying the truth of mathematical propositions.

Quinean realism is free of the difficulty that would make it advisable to reject Gödel's version. Quine defends a holistic picture of the system of human knowledge in which mathematical knowledge is only a part. Quine does not require that there be mathematical entities, although mathematical theories, forming part as they do of our general system, must be contrasted with reality just like any other kind of empirical knowledge. If our empirical theories are succesful, then mathematical theories will be confirmed, in the same way and in the same "confirmation acts" that buttress the rest of our knowledge. Quine calls this argument in favor of mathematical realism the "indispensability argument": it is not possible to test a physical theory without contrasting some portion of mathematics with it. Maddy argues against Quinean realism, saying that everything that it attempts to explain can also be explained by using Camap's distinction between internal and external existence questions. She also argues that Quine's adherence to the indispensability argument conflicts with the usual practice among scientist and mathematicians. In fact, Maddy shows that scientists and mathematicians make use of mathematical concepts and theories heuristically without deriving the existence and truth of the entities and theories involved.

In **Part III: Naturalism**, Maddy develops her view of mathematical naturalism. Following the teachings of the latter Wittgenstein, she proposes a kind of naturalism that is, in a manner of speaking, a sort of anti-philosophy. Wittgenstein thought that once philosophical problems and their sources were understood, these problem would turn out to be mere misunderstandings. He explains the apparent appeal of mathematical realism as a way of showing that mathematics does not depend on moods or vague ideas in particular minds. In his view, pure mathematics is meaningless, only applied mathematics says anything. Maddy does not go that far, but she sympathizes with the Wittgenstein spirit and concedes that whenever there is a clash between philosophy and mathematics, it is philosophy that must yield. Her naturalism consists of her support for the pre-eminence of mathematics (or science) over philosophy together with her thesis that philosophy can neither prescribe nor restrict mathematical or scientific practice.

Maddy also stresses that mathematics has aims and methods which differ from those of empirical science, and this is the reason why scientific naturalism, again related to Quine, does not fit in well with mathematical practice. For this reason, Maddy proposes Mathematical naturalism instead, a kind of naturalism that consists of the use of mathematical methods to assess mathematical problems. These cannot be solved by looking towards the philosophy of mathematics but towards "the needs and goals of mathematics itself " (p. 191).

In the last chapter she explains how her proposal works, making a case against the Axiom of Constructibility from her naturalistic point of view. Maddy shows here that the Axiom of Constructibility would exclude some other set-theoretical postulates, such as the existence of measurable cardinals, which have identifiable benefits on their own.

In spite of Maddy's "anti-philosophical" perspective, her approach to the question of the evidence for mathematical propositions is extremely appealing from a philosophical point of view, or at least from the point of view of a particular philosophical tradition that I also favor. Maddy's analysis dovetails perfectly with the history of mathematics and mathematical practice. Quite rightly, it excludes a conception of philosophy as a pre-eminent discipline, having a privileged epistemological status, with the authority to act like a jury before which all kinds of knowledge must stand to account. Mathematical naturalism puts philosophers of mathematics where they should be, side by side with mathematicians, looking at real mathematical practice.

Mathematical naturalism is a healthy proposal and Maddy's arguments in its favor are highly persuasive. I consider *Naturalism in Mathematics* commendable reading for philosophers of mathematics, set theorists and mathematicians in general.

References

[1] Frápolli, M.J., "Review of P. Maddy's *Realism in Mathematics*," *Modern Logic* **2** no. 4 (1992), pp. 388-391.

[2] Maddy, P., *Realism in Mathematics*, Oxford: Clarendon Press, 1992.

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