Modern Logic
Volume 8, Number 3 & 4 (May 2000-October 2001), pp. 95–98.

Review of

DOUGLAS M. JESSEPH, *BERKELEY'S PHILOSOPHY OF MATHEMATICS*

Chicago/London: University of Chicago Press, 1993
xii + 322 pp. ISBN 0-226-39897-8

IRVING H. ANELLIS

Although this book is primarily about philosophy of mathematics, it is noteworthy to historians of logic interested in the history of non-standard analysis for the comment made about the significance of the history of the philosophy of mathematics in the work of recent and contemporary workers and adherents of modern nonstandard analysis such as Abraham Robinson.

After writing (p. 131) that "contemporary model theory allows for the development of a consistent theory of infinitesimals," Jesseph goes on to assert that "[T]he relevance of current accounts of the infinitesimal to issues in the seventeenth and eighteenth centuries is rather minimal ..." This is *wrong* and would most assuredly have come as much as a surprise to Robinson as to anyone familiar with Robinson's work, in particular with §I of his article 'The Metaphysics of the Calculus" [7]).

Robinson opens his article by saying [7, p. 53]:

> From the end of the seventeenth century until the middle of the nineteenth, the foundations of the Differential and Integral Calculus were a matter of controversy. While most students of Mathematics are aware of this fact they tend to regard the discussions which raged during that period entirely as arguments over technical details, proceeding from logically vague (Newton) or untenable (Leibniz) ideas to methods of Cauchy and Weierstrass which meet modern standards of rigor. However, a closer study of the history of the subject reveals that those who actually took part in this dialogue were motivated or influenced quite frequently by basic philosophical attitudes. To them the problem of the foundations of the Calculus was largely a philosophical question....

> Thus, d'Alembert states in a passage from which I have taken the title of this address:
>
> 'La théorie des limites est la base de la vraie Métaphysique du calcul différentiel.'
>
> It will be my purpose today to describe and analyse the interplay of the philosophical and technical ideas during several significant phases in the development of the Calculus. I shall carry out this task against the background of Non-standard Analysis as a viable Calculus of Infinitesimals. This will enable me to give a more precise assessment of certain historical theories than has been possible hitherto.

If we take Robinson at his word in claiming that his examination and evaluation of seventeenth- through nineteenth-century discussions of the calculus gives a more precise assessment of some historical theories of the calculus, then the relevance of current accounts of the infinitesimal to issues in the seventeenth and eighteenth centuries is really *far* from minimal. Jesseph, let it be noted, did not cite the article of Robinson from which I quoted.

In "The Metaphysics of the Calculus," Robinson dealt with Berkeley by noting [7, p. 159] that:

> Like the proponents of the new theory, its critics also were motivated by a combination of technical and philosophical considerations. Berkeley's 'Analyst' ... constitutes a brilliant attack on the logical inadequacies both of the Newtonian Theory of Fluxions and of Leibnizian Differential Calculus. In discrediting these theories, Berkeley wished to discredit also the views of the scientists on theological matters. But beyond that, and more to the point, Berkeley's distaste for the Calculus was related to the fact that he had no place for infinitesinials in a philosophy dominated by perception.

It is curious that Jesseph would have missed or ignored Robinson's treatment when Robinson, in this brief statement, supports Jesseph's thesis that "Berkeley's rejection of the infinitesimal calculus... should come as no surprise. The epistemological constraints which led him to deny the thesis of infinite divisibility would obviously rule out the much stronger doctrine of infinitesimal magnitudes or the theory of fluxions" (Jesseph, p. 152).

Jesseph is also inaccurate on several scholarly matters: he gives the date of publication of Robinson's *Non-standard Analysis* as 1965 rather than 1966; H. Jerome Keisler's name is given by Jesseph as "Gerald Keisler", and the publisher of Keisler's *Elementary Caculus: An Infinitesimal Approach* is given to be Academic Press rather than Prindle, Weber & Schmidt.

Under the circumstances, we are indubitably fortunate that Jesseph chose not to deal in more detail with nonstandard analysis than to tell us in a footnote (n. 6, pp. 131-132) that '[I]n present-day accounts, infinitesimals appear as "hyperreal" numbers in certain nonstandard models of arithmetic, and contemporary accounts of hyperreal numbers define the product of two hyperreals as a hyperreal number.'

We know that there were, of course, other, more recent, workers in philosophy of mathematics who had raised similar objections to Berkeley's against infinitesimal calculus. One of these was Bertrand Russell (see, *e.g.*, Anellis [2]), and he tied it — of more direct and explicit interest to historians of set theory and logic — to the problems which he saw in Cantorian set theory (as outlined in Anellis [1] and sketched more fully in Anellis [3, 4]). Lest some readers feel inclined to note and complain of the absence of Russell or others from Jesseph's account, it should be noted that Russell's complaints had philosophical origins other than Berkeleyism (see Anellis [5]; [6, p. 193]), and there is no record of Russell's even having read Berkeley during the period in question (see [8]). Moreover, Jesseph's treatment of "The Aftermath of the *Analyst*" extends only to contemporary reactions to Berkeley's writings on the subject, up to the mid-eighteenth century. So on this score, at least, we cannot appropriately complain of Jesseph's "omission".

References

[1] Anellis, I. H., "Bertrand Russell's earliest reactions to Cantorian set theory, 1896 - 1900," in J. E. Baumgartner, D. A. Martin & S. Shelah (eds.), *Axiomatic set theory*, *Contemporary Mathematics* **31** (1984), pp. 1-11.

[2] ———, "Russell's problems with the calculus," in C. Binder (ed.), *Proceedings of the Austrian symposium on history of mathematics, 9-15 November 1986, Neuhofen an der Ybbes* (Vienna, OGGNW, 1986), pp. 124 -128; reprinted with revisions, in V. L. Rabinovich (ed.), *Abstracts, LMPS '87, Moscow, USSR, 17-22 August 1987, Eighth International Congress of Logic, Methodology and Philosophy of Science* (Moscow, Acad. Sci. USSR, 1987), vol. 3, §13, pp. 16-19.

[3] ———, "Russell's earliest interpretation of Cantorian set theory, 1896 -1900," *Philosophia Mathematica (2)* **2** (1987), pp. 1-31.

[4] ———, "Bertrand Russell's theory of numbers, 1896 -1898," *Epistemologia* **10** (1987), pp. 303-322.

[5] ———, "Russell and Engels: two approaches to a Hegelian philosophy of mathematics," *Philosophia Mathematica (2)* **3** (1987), pp. 151-179.

[6] ———, “Review of *Bertrand Russell, Philosophical papers 1896-99*,” edited by Nicholas Griffin and Albert C. Lewis. Volume 2 of *The Collected Papers of Bertrand Russell*, *Modern Logic* **3** (1993), pp. 192-200.

[7] Robinson, A., “The metaphysics of the calculus,” in J. Hintikka (ed.), *The Philosophy of Mathematics* (London: Oxford University Press, 1969), pp. 153-163; reprinted from: I. Lakatos (ed.), *Problems in the Philosophy of Mathematics* (Amsterdam: North-Holland, 1967), pp. 28-40.

[8] Russell, B., “What shall I read?,” (1891-1902) ms., Russell Archives. Published in K. Blackwell, A. Brink, N. Griffin, R. Rempel & J. Slater (eds.), *Cambridge essays, 1889 - 99*, vol. I of *The Collected papers of Bertrand Russell* (London: Unwin Hyman, 1983), pp. 138-146.

Des Moines Area Community College, Boone Campus, and Modern Logic Publishing 2408 1/2 W. Lincoln Way (Upper Level) Ames, IA 50014-7217

E-mail address: F1.MLP@ISUMVS.IASTATE.EDU