ERRATA

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ERRATA: ON NON-EQUILIBRATED ALMOST MONOTONIC FUNCTIONS OF THE ZYGMUND-BARY-STECHKIN CLASS

Abstract

An error in Section 4 of [NS] is corrected.

1 Introduction.

In [NS] the proof of an auxiliary Lemma 4.2 in Section 4 contains a gap. As a consequence Theorem 4.3 based on that lemma is not valid in the general case. Fortunately, the statements of Section 4 being "things in itself" were not used in the main results in the latter part of the paper. However, Lemma 4.2 should be modified. For a potential reader of the paper we give the following corrected version of Section 4 of the paper [NS]. (The text below is to replace Section 4 of [NS].)

4 Some Properties of the Zygmund-Bary-Stechkin Class Φ.

The following lemma shows that the bounds $\delta_1 < m_\omega$ and $\delta_2 > M_\omega$ in Theorem 3.4 cannot be improved.

Lemma 4.1. *Let a nondecreasing function ω belong to Φ. If $\frac{\omega(x)}{x^\alpha}$ is almost increasing and $\frac{\omega(x)}{x^\beta}$ is almost decreasing for some $0 < \alpha \leq \beta < 1$, then $m_\omega \geq \alpha$ and $M_\omega \leq \beta$.*

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PROOF. Suppose to the contrary that $m_\omega < \alpha$. Then the function $\omega_1(x) = \frac{\omega(x)}{x^{m_\omega}}$ is also almost increasing and $\omega_1(0) = 0$ since $m_\omega < \alpha$. Therefore, $\omega_1 \in W$. But also the function

$$\frac{\omega_1(x)}{x^{\delta_1}} = \frac{\omega(x)}{x^\alpha}, \ \delta_1 = \alpha - m_\omega$$

is almost increasing. Then, by Lemma 3.3, the function $\omega_1(x)$ satisfies the (Z_0)-condition. Therefore, by Theorem 3.2, its lower index m_{ω_1} is positive $m_{\omega_1} > 0$ which is impossible since $m_{\omega_1} = m_\omega - m_\omega = 0$ by (2.2).

Similarly, the statement $M_\omega \le \beta$ is obtained. □

The next lemma makes the statement of the previous lemma more precise.

Theorem 4.2. *For any function $\omega \in \Phi$ its lower and upper indices m_ω are calculated by the formulas*

$$m_\omega = \sup\left\{\alpha \in (0,1) : \frac{\omega(x)}{x^\alpha} \ \textit{is almost increasing} \right\}, \tag{4.1}$$

$$M_\omega = \inf\left\{\beta \in (0,1) : \frac{\omega(x)}{x^\beta} \ \textit{is almost decreasing} \right\}. \tag{4.2}$$

PROOF. Let $a = \sup\left\{\alpha \in (0,1) : \frac{\omega(x)}{x^\alpha} \ \textit{is almost increasing} \right\}$. Lemma 4.1 states that $m_\omega \le a$, while from Theorem 3.4 it follows that $m_\omega \ge a$. Consequently, $m_\omega = a$ and we get (4.1).

Relation (4.2) is proved similarly. □

References

[NS] N. Samko, *On non-equilibrated almost monotonic functions of the Zygmund-Bary-Stechkin class*, Real Analysis Exchange, **30**(2) (2004-2005), 727–746.