NOTE ON THE "EVALUATION OF AN INTEGRAL OCCURRING IN SERVOMACHANISM THEORY"

R. H. BATTIN

In a recent paper [1] W. A. Mersman considers the evaluation of the integral

$$I = \frac{1}{2\pi i} \int_{-\infty i}^{\infty i} \frac{g(x)}{h(x)h(-x)}\, dx$$

where $g(x)$ and $h(x)$ are polynomials in x of order $2n-2$ and n, respectively. Because of the importance of Mersman's result the present writer wishes to call attention to an alternate and somewhat more direct evaluation of this integral.

We shall utilize Mersman's notation in the main and begin with his equation (3). By division it is clear that

$$\frac{h(x)}{x - x_k} = \sum_{j=1}^{n} B_{jk} x^{n-j}$$

where it is important to observe that each of the quantities B_{jk} will, in general, depend upon k except the first which is simply $B_{1k} = a_0$. Then

$$\frac{h(x)}{x - x_k}\, h(-x) + \frac{h(-x)}{-x - x_k}\, h(x) = \sum_{s=0}^{n} \sum_{j=1}^{n} [(-1)^{n-s} + (-1)^{n-j}] a_s B_{jk} x^{2n-s-j}$$

$$= 2(-1)^n \sum_{r=1}^{n} \sum_{j=1}^{2r} (-1)^j a_{2r-j} B_{jk} x^{2(n-r)}$$

In the above expression it is understood that $a_s = 0$ for $s < 0$ or $s > n$ and $B_{jk} = 0$ for $j > n$. Mersman's equation (3) then becomes

$$\sum_{r=1}^{n} g_r x^{2(n-r)} = 2(-1)^n \sum_{r=1}^{n} x^{2(n-r)} \left\{ \sum_{k=1}^{n} A_k \sum_{j=1}^{2r} (-1)^j a_{2r-j} B_{jk} \right\}$$

Received September 20, 1953.

Pacific J. Math. 5 (1955), 481-482

For simplicity we define

$$F_j = \frac{(-1)^{j+1}}{a_0} \sum_{k=1}^{n} A_k B_{jk} \qquad j = 1, 2, \cdots, n$$

so that $F_1 = I$. There results the following set of n linear algebraic equations:

$$a_{2r-1} I + \sum_{j=2}^{2r} a_{2r-j} F_j = (-1)^{n+1} \frac{g_r}{2a_0} \qquad r = 1, 2, \cdots, n$$

Using Cramer's rule we may now solve directly for I to obtain Mersman's result as expressed by his equation (6).

Reference

1. W. A. Mersman, *Evaluation of an integral occurring in servomechanism theory*, Pacific J. Math. **2** (1952), 627-632.

Massachusetts Institute of Technology