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METRIZABILITY AND COMPLETENESS IN NORMAL MOORE SPACES

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B. Fitzpatrick, Jr. and D. R. Traylor proved [Theorem 1, Pac. J. Math., to appear] that if there is a normal, nonmetrizable Moore space then there is one which is not locally metrizable at any point. The primary purpose of this paper is to extend the stated result to include normal, complete Moore spaces. That is, it is established that there is a normal, complete, Moore space which is not locally metrizable at any point, provided there exists a normal, complete, nonmetrizable Moore space. Indeed, it is further established that, provided there exists a nonmetrizable, normal, complete Moore space, then there is one which is also connected, locally connected, not locally metrizable at any point, and, using a result of Younglove's [Theorem 1, "Concerning metric subspaces of nonmetric spaces," Fund. Math., 48 (1949), 15–25], which contains a dense metrizable subset.

F. B. Jones [Bull. Amer. Math. Soc. 43 (1937), 671–677] showed that if $2^{\aleph_0} < 2^{\aleph_1}$, then every normal separable Moore space is metrizable. It is established in this paper that if each normal, separable, connected space satisfying Axioms 0, 1, and 2 of [R. L. Moore, Foundations of Point Set Theory, Amer. Math. Soc. Colloq. Pub. No. 13 Providence, R. I. 1962] is metrizable, then each normal separable Moore space is metrizable.

Other theorems of this ilk are included in this paper.

The statement that S is a Moore space means that there exists a sequence of collections of regions satisfying Axiom 0 and the first three parts of Axiom 1 of [5]. Such a sequence is said to be a development and should a Moore space have a development which also satisfies the fourth part of Axiom 1 of [5], that space is said to be a complete Moore space.

All other definitions and terms are as in [1, 2, 3, 4, 5, 6, 7].

LEMMA. *If each normal, connected, locally connected Moore space is metrizable, then each normal Moore space is metrizable.*

Proof. The proof is similar to that used to establish Theorem 3 of [6].

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Suppose that S is a normal Moore space. Denote by w a well-ordering of the points of S and by G the collection to which the ordered pair x, y belongs if and only if x precedes y in w. If $\overline{\overline{G}}$ is the power of G, then $\overline{\overline{G}}$ is the power of S.

Denote by Z a space of power $\overline{\overline{G}}$ with the discrete topology such that Z and S are mutually exclusive. Suppose that Z' is the cartesian product of Z with the unit interval of the real line. There is a reversible transformation T from G onto Z. If $T(x, y)$ is t, let (x, y) denote $t \times (0, 1)$.

Denote by S' a space in which P is a point if and only if P is a point of S or a point of Z'. If $G_1, G_2, \cdots$ is a development of S, then g is a region of G_j in S' if and only if there is a region R of G_j such that P belongs to g if and only if either

A. (i) for some points x and y of R, x, y belongs to G and P is x, P is y, or P is a point of (x, y), or

(ii) there is a point x in R, a point y in $S - R$ such that x, y belongs to G, and P is x or P is a point of $t \times (0, 1/j)$, or

(iii) there is a point y of R, a point x in $S - R$ such that x, y belongs to G, and P is y or P is a point of $t \times (1 - 1/j, 1)$, or

B. for some pair x, y of G, no region of G_j contains $(x + y)$ and there is a point Q of (x, y) such that P is a point of an open subsegment E of (x, y) of length less than $1/j$ and E contains Q.

That S', with the development of $G'_1, G'_2, \cdots$, satisfies the lemma follows from an argument similar to that given in Theorem 3 of [6].

THEOREM 1. *If each normal, complete, connected, locally connected Moore space is metrizable then each normal, complete Moore space is metrizable.*

Proof. If S is a normal, complete Moore space, denote by S' a space as defined in the proof of the lemma. If $G'_1, G'_2, \cdots$ is the development defined in the lemma, let G_1 denote G'_1 and for each positive integer $n > 1$, denote by G_n the subcollection of G'_n to which the region R belongs if and only if $\bar{R}$ is a subset of some region of G_{n-1}. To prove that S' is complete, suppose that $M_1, M_2, \cdots$ is a sequence of closed point sets such that for each positive integer n, M_{n+1} is a subset of M_n and of the closure of some region of G_{n+1}, say g_{n+1}.

Either (i) there is a positive integer N_1 such that if k is greater than N_1 then g_k intersects S, or (ii) if N_2 is any integer then there is an integer k greater than N_2 such that $\bar{g}_k$ is a subset of $S' - S$. If (i) is the case, the completeness of S guarantees that there is a point P common to each of the sets $\bar{g}_n$. Since each M_n is closed it follows immediately that P is a point of each M_n. If (ii) is the case, then some $\bar{g}_k$ is a subset of (x, y) for some x, y of G. But then $\bar{g}_k$ is a closed subset of $t \times (0,1)$ for some t, and from the completeness properties of the real line, it follows that $\bar{g}_k \cdot [t \times (0,1)]$ is complete. Thus there must be a point common to each of the sets M_n.

It perhaps should be noted that this has established that there exists a nonmetrizable, connected, normal space satisfying Axiom 0, Axiom 1, and Axiom 2 of [5], provided that there exists a complete, normal, nonmetrizable Moore space.

THEOREM 2. *If each normal, separable, complete, connected space satisfying Axioms* 0, 1, *and* 2 *of* [5] *is metrizable, then each normal, separable Moore space is metrizable.*

Proof. Fitzpatrick and Traylor [2, Theorem 2] have established that if there is a normal, separable, nonmetrizable Moore space, then there is one which is also locally compact. Since each locally compact Moore space satisfies all of Axiom 1, then there must exist a normal, separable, complete Moore space S which is not metrizable. Denote by K a countable dense subset of S. If w is a well-ordering of the points of K, denote by G the collection to which the ordered pair x, y belongs if and only if x precedes y in w. Then $\overline{\overline{G}} = \overline{\overline{K}} = \aleph_0$. Define S' as in the lemma. If $G_1, G_2, \cdots$ is a development of S, then g is a region of G'_j in S' if and only if there is a region R of G_j such that P belongs to g if and only if either

(e) P is a point of $S - K$ in R, or

(f) either A or B of the lemma.

It follows immediately that S' is normal, separable, connected, and locally connected. Let H_1 denote G'_1 and for each integer $n > 1$, denote by H_n the subcollection of G'_n to which R belongs if and only if $\bar{R}$ is a subset of some region of H_{n-1}. The existence of S' with the development $H_1, H_2, H_3, \cdots$ established the theorem.

THEOREM 3. *If each normal, pointwise countably paracompact, connected, and locally connected Moore space is metrizable, then each normal, pointwise countably paracompact Moore space is.*

Proof. Suppose that S is a normal, pointwise countably paracompact Moore space and S' is as defined in the proof of the lemma. Denote by H' an open covering of S' and by H the collection of open sets, in S, to which h belongs if and only if there is an element h' of H' such that h is $h' \cdot S$. Then H is an open covering of S and there exists a refinement V of H such that no point of S belongs to uncountably many elements of V. Each element v of V is a subset of some element, say h_v of H'. For each element v of V, denote by v' the subset of S' to which x belongs if and only if there is a region g' in S' such that $g' \cdot S$ is a subset of v, g' is a subset of h_v, and g' contains x. If V' is that collection to which h belongs if and only if there is an element v of V such that h is v' then V' is a collection of open sets in S' such that no point belongs to uncountably many elements of V'.

Now if P is a point of $T = S' - V^* \cdot S'$ then there is an ordered pair x, y of G such that P is a point of (x, y). Denote by U an open refinement of H' such that each element of U is a subset of (x, y) for some x, y of G and such that U covers T. Since $(0, 1)$ is pointwise countably paracompact, then each (x, y) is. Thus there exists a refinement U' of U such that no point of T belongs to uncountably many elements of U'. Then $(U' + V')$ is a refinement of H' such that no point of S' belongs to uncountably many elements of $(U' + V')$ and the proof is complete.

COROLLARY. *If each normal, complete, pointwise countably paracompact, connected, locally connected Moore space is metrizable, then each normal, complete, pointwise countably paracompact Moore space is metrizable.*

Proof. The theorem follows from Theorem 1 and Theorem 3.

Note. The proof of Theorem 4 depends heavily on the proof given for [2, Theorem 1]. Indeed, the notation used in the proof of Theorem 4 is that which is used to construct the space which established Theorem 1 of [2].

THEOREM 4. *If there is a normal, complete, nonmetrizable Moore space (S^0, Ω^0) then there is one, say (S, Ω), which is not locally metrizable at any point.*

Proof. Denote by (S^0, Ω^0) a normal, complete nonmetrizable Moore space and by (S^W, Ω^W) the space defined exactly as in the proof of Theorem 1 of [2]. In that theorem, it is established that (S^W, Ω^W) is a normal Moore space which is not locally metrizable at any point. Should (S^0, Ω^0) be complete, there is no reason to expect that (S^W, Ω^W) is also complete. It is necessary then, to "complete" (S^W, Ω^W) without losing either the property of normality or the property that the completed space is not locally metrizable at any point.

A. "Completing" the space (S^W, Ω^W).

If (S^W, Ω^W) is not complete there exists a sequence $M_1, M_2, \cdots$ of closed point sets such that each M_{i+1} is a subset of M_i and for each positive integer n, M_n is a subset of the closure of a region, say g_n, of G_n^W and there is no point common to each M_i. In this case it is true that for no positive integer n, does each $\bar{g}_i$ intersect S^n. This follows since S^0 is complete and S^n consists of copies of S^0. Indeed, if for each integer i, $\bar{g}_i$ were to intersect S^0, the completeness of S^0 would force a point common to each M_n. If infinitely many of the regions g_n of G_n^W intersect S^1, then since only finitely many may intersect S^0, it is immediate that for some integer k, it is true that there is a copy of S^0, in S^1, such that that if $m > k$, then $\bar{g}_m \cdot S^1$ is a subset of that particular copy. Again the completeness of S^0 assures the existence of a point common to each of the M_n. Similarly, it follows that if only finitely many of the regions g_i intersect S^i, then since S^{i+1} consists only of copies of S^0, only finitely many of the g_i may intersect S^{i+1}. Otherwise the completeness of S^0 would force a point common to each M_n. Thus it is necessary to introduce new points into the space.

The statement that x is an element of V means that there exist an infinite point sequence $P_0, P_1, \cdots$ such that each P_i is a point of S^i in S^W, an infinite sequence of integers $n_1, n_2, \cdots$ such that for each i, P_{i+1} is a point of $S^0 \times (P_i) \times n_i = S^{i+1}_{p_i, n_i}$, the sequence $1/n_1$, $1/n_2, 1/n_3, \cdots$ converges to 0, and x is the point sequence $P_0, P_1, P_2, \cdots$. It perhaps should be noted that there exists an infinite sequence or regions $R_1, R_2, \cdots$ such that for each i, R_i is in G_i^W, R_{i+1} intersects R_i, P_i is a point of R_i, for some infinite sequence of integers $m_1, m_2, \cdots$, $R_{m_{i+1}}$ is a subset of R_{m_i}, for each positive integer M there is a positive integer n such that R_n does not intersect S^M and no point is common to each of $\bar{R}_1, \bar{R}_2, \cdots$.

The statement that P is a point of S means that P is a point of S^W or P is an element of V. The statement that R is a region G_n in (S, Ω) means that there is a region R^W of G_n^W of (S^W, Ω^W) such that the point x of S belongs to R if and only if

(i) x is a point of R^W in S^W, or

(ii) x is an element of V, $x = P_0, P_1, \cdots$ such that R^W. $[(P_0) + (P_1) + \cdots]$ is an infinite set.

B. Space is a Moore space.

It is clear that Axiom 0 and the first two parts of Axiom 1 are satisfied. To consider the third part of Axiom 1, suppose that R is a region, A is a point of R, B is a point of R, and B is not A.

Note. If R is a region in S, then $R \cdot S^W$ is $\sum_{i=0}^{\infty} R_m^{k+i}$ for some integer m, some integer k, and some sequence $R_m^k, R_m^{k+1}, \cdots$ such that

(i) R^k_m is a region of G^k_m,

(ii) $S^{k+j-1}\cdot R^{k+j}_m = R^{k+j-1}_m$,

(iii) $R^k_m \subset R^{k+1}_m \subset R^{k+2}_m \subset \cdots$,

(iv) $R^{k+1}_m \subset S^{k+i}$ and $R^{k+i}_m \not\subset S^{k+i-1}$,

(v) In S^W, the boundary of $R\cdot S^W$ is a subset of S^k if the boundary exists,

(vi) If R is a region of G_n, P is a point of S^{k-1} for some integer k, and j is a positive integer such that R contains P and intersects $S^0 \times (P) \times (j) = S^k_{p,j}$, then n is less than or equal to j and R contains $S^k_{p,j}$.

(vii) If R is a region which intersects S^k but not S^{k-1} and g is a region which intersects S^j but not S^{j-1} and k is less than j, then R contains g if R intersects g.

Now let x denote a point of $S - S^W$ such that x is not in R. Then $x = P_0, P_1, \cdots$ and since x is not in R, at most finitely many of the P_i belong to $\sum_{i=0}^{\infty} R^{k+i}_m$. Denote by N a positive integer such that if $t > N$ then P_t is not a point of $\sum_{i=0}^{\infty} R^{k+i}_m$. It is no restriction to assume that t is greater than k. But P_t is a point of S^t and, for some integer n_{t-1}, is a point of $S^0 \times (P_{t-1}) \times (n_{t-1})$. It follows that there is some region $g = \sum_{i=0}^{\infty} g^{t+i}_n$ in S^W such that u is greater than t, g does not intersect S^{t-1}, g does intersect S^t, and g contains x. To show that x is not a limit point of $R\cdot(S - S^W)$, assume that $Q = Q_0, Q_1, \cdots$ is a point of $[R\cdot(S - S^W)]\cdot g$. Then $(R\cdot S^W)\cdot(g\cdot S^W)$ must exist. There is a least integer j such that there is a positive integer n_{j-1} and a point c_{j-1} of S^{j-1} such that $S^{j-1}_{c_{j-1}, n_{j-1}}$ contains a point of $(R\cdot S^W)\cdot(g\cdot S^W)$. Since u is greater than $k+1$ it follows that $j-1$ is greater than k. But R must contain $S^{j-1}_{c_{j-1}, n_{j-1}}$ if R intersects it, by property (vi) listed above. If g were to intersect S^{j-1}, then g would contain the point c_{j-1} and, in that case, j would not be the least integer as described. Thus g does not intersect S^{j-1} and it follows from the definition of region that R contains g. This establishes a contradiction since R does not contain x. It is immediate that if R is a region of S then $R\cdot(S - S^W) = \bar{R}\cdot(S - S^W)$.

Case 1. A belongs to S^W and B belongs to S^W.

There is a positive integer n such that if h is a region of G^W_n containing A then $\bar{h}$ is a subset of $R\cdot S^W$ and does not contain B. But for each h of G^W_n for which there is a region g of G_n such that $g\cdot S^W - h$, $\bar{g}$ is a subset of R and does not contain B, from the preceding note.

Case 2. A belongs to S^W and B belongs to $S - S^W$.

For some integer t, A is a point of S^t but not a point of S^{t-1}. Then for some integer n_{t-1} and some point P_{t-1} of S^{t-1}, A is a point of $S^0 \times (P_{t-1}) \times (n_{t-1})$. Now B is $b_0, b_1, \cdots$ where each b_i is in S^i and for some integer n_{i-1}, b_i is a point of $S^0 \times (b_{i-1}) \times (n_{i-1})$. But in S^W, there is an integer k such that if h is a region of G_k^m containing P_{t-1} then $\bar{h}$ does not contain b_{t-1} and $\bar{h}$ is a subset of $R \cdot S^W$. Thus if g is a region of G_k which contains A then $\bar{g}$ is a subset of R and does not contain B since $\bar{g}$ does not contain b_{t-1} but does contain P_{t-1}.

Case 3. A is a point of $S - S^W$ and B is a point of $S - S^W$.

Then A is $a_0, a_1, \cdots$ and B is $b_0, b_1, \cdots$ with the usual notation. There is a least integer N such that a_n is not b_n. Then in S^W there is an integer k such that if h is a region of G_k^W containing a_n, then $\bar{h}$ is a subset of $R \cdot S^W$ and $\bar{h}$ does not contain b_n. Hence, let g be any region of G_k that contains A. It follows that $\bar{g}$ is a subset of R and does not contain B.

Case 4. A is a point of $S - S^W$ and B is a point of S^W.

This follows from the arguments given for the previous three cases.

C. Space is normal.

To see that S is normal, denote by H and K mutually exclusive closed subsets of S. For each positive integer n, denote by H_n the subset of $H \cdot (S - S^W)$ such that y belongs to H_n if and only if it is true that if g is a region of G_n containing y and h is a region of G_n intersecting g, then h does not intersect K. For each positive integer n, the subset K_n of K is defined in a similar fashion. Denote by P some point of $H \cdot (S - S^W)$. It needs to be proved that P is a point of some H_n. Suppose the contrary; that P does not belong to any H_n. Then if n is any positive integer, there exists a point y_n of $K \cdot (S - S^W)$, a region of g_n of G_n, a region h_n of G_n such that g_n intersects h_n, g_n contains P, h_n contains y_n, and $\bar{h}_n$ does not intersect H. Since P is in H, P is not a limit point of K. Thus there is a positive integer N such that no region of G_N containing P intersects K. Let g denote one region of G_N such that g contains P. Then there exist a sequence $g_m, g_{m+1}, \cdots$, a sequence $h_m, h_{m+1}, \cdots$, and a sequence $y_m, y_{m+1}, \cdots$ such that for each i, g_{m+1} contains P, $\bar{g}_{m+1}$ is a subset of $g \cdot g_{m+i-1}$, h_{m+i} intersects g_{m+i}, $\bar{h}_{m+i}$ does not intersect H, and y_{m+i} is a point of $K \cdot (S - S^W)$ in h_{m+i}. It is clear that each h_{m+i} intersects g. Now $g \cdot S^W$ is $\sum_{i=0}^{\infty} R_N^{t+i}$ with the usual notation. It is no restriction to assume that there is an integer q such that q is greater than N and h_q intersects g. Note that g intersects S^t but not S^{t-1}. Thus there are a point c of S^{t-1} and a positive integer d such that $S \cdot S^t$ is a subset of $S^0 \times (c) \times (d)$. Let v the least integer such that h_q intersects $g \cdot S^v$. Since, for each n, h_n does not intersect S^{n-1}, it is no restriction to assume that q is greater than t. Thus v is greater than $t+1$ and h_q does not intersect S^{q-1}. Hence, h_q does not intersect S^t. Then there is a point e_0 of S^{v-1} and a positive integer j_0 such that $h_q \cdot S^v$ is a subset of $S^0 \times (e_0) \times (j_0)$. It is clear that j_0 is greater that or equal to q and since q is greater than N, then j_0 is greater than t. Indeed, if g intersects $S^0 \times (e_0) \times (j_0)$, then g must contain $S^0 \times (e_0) \times (j_0)$ since t is less than q and j_0 is greater than N. It remains to be proved that g does intersect $S^0 \times (e_0) \times (j_0)$. Since g does intersect h_q, let z denote the least integer such that $g \cdot h_q$ intersects S^z. Then z is greater than t and z is greater than or equal to v. Indeed, there exist a sequence of points $e_1, e_2, \cdots, e_{z-v}$ and a sequence of integers $j_1, j_2, \cdots, j_{z-v}$, each of which is greater than or equal to q such that each e_i is a point of S^i, e_{i+1} is a point of $S^0 \times (e_i) \times (j_i)$, and $S^0 \times (e_{z-v}) \times (j_{z-v})$ contains a point of $h_q \cdot g \cdot S^z$. But since z is greater than t and j_{z-v} is greater than q, g must contain $S^0 \times (e_{z-v}) \times (j_{z-v})$. In like fashion, g contains $S^0 \times (e_i) \times (j_i)$ for each i and indeed, then g must contain $S^0 \times (e_0) \times (j_0)$ for the same reason. Then g contains h_q. But h_q contains a point of K and this is impossible. Therefore g does not intersect K.

The argument above has established: If P is a point of $H \cdot (S - S^W)$ there exists a positive integer N such that if g and g' belong to G_N such that g contains P and g intersects g', then g' does not intersect $K \cdot (S - S^W)$.

Therefore, denote by H_n the subset of $H \cdot (S - S^W)$ to which x belongs if and only if n is the N above and x is P and by K_n the corresponding subset of $K \cdot (S - S^W)$. Then $\sum_{n=1}^{\infty} H_n = H \cdot (S - S^W)$ and $\sum_{n=1}^{\infty} K_n = K \cdot (S - S^W)$. For each n and each point x of H_n, let g_x be a region of G_n containing x and for each point y of K_n, let g_y be a region of G_n containing y. Note that, by the definition of region, neither g_x nor g_y intersect S^n. Let $D = \sum_{x \in H} g_x$ and $E = \sum_{y \in K} g_y$. Then D does not intersect E.

Since S^W is normal, there exist domains in S^W, say D_1 and E_1, containing $S^W \cdot H$ and $S^W \cdot K$ respectively, such that $\bar{D}_1$ does not intersect $\bar{E}_1$ in S^W. For each point x of $K \cdot E_1$ let h_x be a region containing x such that $\bar{h}_x$ is a subset of E_1 and let $E_2 = \sum_{x \in K} h_x$. Suppose that P is a point of $H \cdot (S - S^W)$. Then P is not a limit point of K and there is a region g containing P such that $\bar{g}$ contains no point of K. It remains to be proved that P is not a limit point of E_2. Now

$g \cdot S^W = \sum_{i=0}^{\infty} R_m^{k+i}$ for some m and some k. That is, for some point Q of S^{k-1} and some integer r, g intersects $S^0 \times (Q) \times (r)$ but g does not intersect S^{k-1}. Suppose that some h_x of E_2 intersects g. Then $h_x \cdot S^W = \sum_{i=0}^{\infty} R_t^{j+i}$ for some integer t and some integer j. If $j < k$ and $t > r$, it is impossible for h_x to intersect g. Indeed, since $g \cdot S^{k-1}$ does not exist, it follows that m is greater than or equal to $r + 1$. If $j < k$ and $t \leqq r$, and h_x intersects g then h_x must contain g and this is impossible. Similarly, if $j > k$ and $t \geqq r$ then g must contain h_x if they intersect. Since $h_x \cdot S^W$ is $\sum_{i=0}^{\infty} R_t^{j+i}$, there is a point y of S^{j-1} and an integer u such that h_x intersects $S_0 \times (y) \times (u)$ but not S^{j-1}. Then t must be greater than or equal to u. Thus g does not intersect h_x. It follows that P is not a limit point of E_2. In a similar fashion, there is a domain D_2 containing H such that D_2 is a subset of D_1 and no point of $K \cdot (S - S^W)$ is a limit point of D_2. Then $D' = D + D_2$ and $E' = E + E_2$ are mutually exclusive domains containing H and K respectively. This establishes that S is normal.

That (S, Ω) is complete follows immediately from the definition of point and region. Clearly, S is not locally metrizable at any point since each region contains a copy of a nonmetrizable space.

THEOREM 5. *If each locally connected, connected, complete, normal Moore space is locally metrizable at some point, then each complete, normal Moore space is metrizable.*

Proof. Suppose that S is a complete, normal, nonmetrizable Moore space. By Theorem 1, S is topologically equivalent to a subset of a connected, locally connected, normal, complete Moore space, say (S^0, Ω^0). Using the method of Theorem 4, there exists a normal, complete Moore space (S_1^0, Ω_1^0) which is not locally metrizable at any point and which consists of copies of (S^0, Ω^0). Again, using the construction of Theorem 1, there is a locally connected, connected, normal, complete Moore space, (S^1, Ω^1), which contains a subset that is topologically equivalent to (S_1^0, Ω_1^0). Indeed, continuing this process indefinitely, for each positive integer n, (S^n, Ω^n) is a connected, locally connected, normal, complete Moore space which contains a subset that is topologically equivalent (by the construction of Theorem 1) to the normal, complete space $(S_1^{n-1}, \Omega_1^{n-1})$ which is not locally metrizable at any point, and (S_1^n, Ω_1^n) is the space defined as in Theorem 4, based on (S^n, Ω^n) and such that (S_1^n, Ω_1^n) is normal, complete, and not locally metrizable at any point. This defines a sequence $S_1^1, S_1^2, S_1^3, \cdots$, each of which is normal, complete, not locally metrizable at any point and S^{n+1} consists of copies of S^n in the sense of Theorem 4.

Referring again to the proof of [2, Theorem 1], if for each n, S^n

of that proof is replaced by S_1^n, the resulting space S^W has the properties described by that theorem. That space, S^W, if "completed" as in Theorem 4 of this paper, results in a Moore space which is normal, complete, connected, locally connected, and not locally metrizable at any point.

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