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ON k-QUOTIENT MAPPINGS

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Two natural generalizations of Arhangelskiĭ's compact-covering mappings are introduced, k-quotient and k'-quotient mappings. These mappings may be used to replace the much stronger perfect mappings in improving some mapping theorems concerning the invariance of topological structures. The defining k-systems and defining k_1-systems of Arhangelskiĭ are fundamentally related to the k-quotient and k'-quotient mappings. Functional characterizations of various k-spaces as domains and ranges of certain mappings are presented. Examples are included to illustrate the results of this study.

It is the purpose of this paper to introduce two natural generalizations of the compact-covering mappings of Arhangelskiĭ [3], the k-quotient and k'-quotient mappings, and to present some of their applications. These mappings are fundamentally related to the defining k-systems and defining k_1-systems of Arhangelskiĭ.

A set $H \subset X$ is said to be k-closed if $H \cap K$ is closed in X for each compact set K. A mapping $f: X \rightarrow Y$ is called *k-quotient* provided: H is k-closed in Y if and only if $f^{-1}(H)$ is k-closed in X. A mapping $f: X \rightarrow Y$ is said to be *k'-quotient* if: For each $p \in \mathrm{cl}(H) \subset Y$ such that there exists a compact set T such that $p \in \mathrm{cl}(H \cap T)$, there exists a compact set $K \subset X$ such that $f^{-1}(p) \cap \mathrm{cl}(f^{-1}(H \cap T) \cap K) \neq \varnothing$.

Some of the properties of these mappings are presented in §1 and in §2 these mappings are used to improve mapping theorems, for invariance of topological structures, which were originally proven with the much stronger perfect mappings. Functional characterizations of spaces with compactly generated topologies, as both domains and ranges, are presented in §3. The fundamental relationships between the k-quotient (k'-quotient) mappings and the defining k-systems (k_1-systems) of Arhangelskiĭ are also presented in §3. The results of this paper are clarified by the discussion of the examples in §4.

These mappings are the compact analogies of the sequentially quotient mappings defined by Frank Siwiec and myself [5]. A mapping $f: X \rightarrow Y$ is *sequentially quotient* provided: H is sequentially closed in Y if and only if $f^{-1}(H)$ is sequentially closed in X.

All mappings are assumed to be continuous surjections and all spaces are assumed to be Hausdorff. The proofs are omitted for those theorems which are easily proven in a routine manner.

1. Properties of k-quotient mappings.

The hierarchy for compact-covering, k'-quotient and k-quotient mappings is given by the first theorem. This is an extension of the implication diagram in [10]. Examples 4.1 and 4.2 show that neither implication can be reversed.

PROPOSITION 1.1. *Compact-covering* $\Rightarrow$ *k'-quotient* $\Rightarrow$ *k-quotient.*

Although the k-quotient mappings fail to have a pseudo-open type property which is possessed by k'-quotient and sequentially quotient mappings, which will be discussed in Example 4.2, they are hereditary.

PROPOSITION 1.2. *Every k-quotient mapping is hereditarily k-quotient.*

Examples 4.3 and 4.4 show that the notions of sequentially quotient and k-quotient mappings are generally independent. For countable to one mappings, the following relationship holds.

PROPOSITION 1.3. *Every countable to one k-quotient mapping is sequentially quotient.*

Proof. Let $f: X \to Y$ countable to one and k-quotient, and let H be any subset of Y such that $f^{-1}(H)$ is sequentially closed in X. If S is a convergent sequence in Y and K is a compact subset of X, then $f^{-1}(S) \cap K$ is compact and countable. Hence $f^{-1}(S) \cap K$ is a closed sequential subspace of X. Since $f^{-1}(H)$ is sequentially closed, $f^{-1}(H) \cap (f^{-1}(S) \cap K) = f^{-1}(H \cap S) \cap K$ is sequentially closed in $f^{-1}(S) \cap K$. Accordingly, $f^{-1}(H \cap S) \cap K$ is closed in X. Thus $f^{-1}(H \cap S)$ is k-closed, so $H \cap S$ is k-closed. Since S is compact, $H \cap S$ is closed. Hence H is sequentially closed. This completes the proof.

One might expect that most of the theorems in [5] for sequentially quotient mappings would have valid compactly generated analogies. For the most part this is false. The "presequential type" theorems and characterizations of [5] do not have translations to convergent (ultra) nets, filters, etc. Example 4.2 illustrates these differences quite clearly.

Sequentially quotient mappings are the convergent sequence analogies of the bi-quotient mappings of Michael [7], in the sense that the notion of a bi-quotient mapping is equivalent to the notion of a limit lifting mapping of Hájek [6]. The presequential characterizations of sequentially quotient mappings [5, Theorem 4.5] are the convergent sequence translations of the definition of limit lifting mappings. Professor Hájek defines a mapping $f: X \to Y$ to be *limit lifting* if: $y_\alpha \to y$ in Y implies there exists a subnet $\{y_\beta\}$ and $x_\beta \to x$ in

X such that $f(x_\beta) = y_\beta$ and $f(x) = y$.

The limit lifting mappings guarantee the existence of a sufficient number of convergent nets in the domain to adequately describe closures of preimages of sets. They do not lift convergent nets in the sense that the nets are covered. The sequentially quotient and k-quotient mappings are analogous to the limit lifting mappings in the sense that the existence of a sufficient number of convergent sequences or compact sets in the domain is guaranteed, to describe the closures of the preimages of sets by means of these types of sets.

2. **Invariance of topological structures.** The k-quotient mappings are substantially weaker than the perfect mappings or the k-mappings of Arhangel'skiĭ [2]. However, k-quotient mappings provide a sufficient existential constraint on the domain to improve some theorems on invariance of topological structures which before have employed the much stronger perfect mappings. As an example, consider the following theorem. (The following two theorems improve Theorems 3 and 5 of [4]. The necessary definitions are contained in [4].)

THEOREM 2.1. *The closed k-quotient image of a space with property* (k) *has property* (k).

Proof. Let $f: X \to Y$ be closed and k-quotient. Suppose X has property (k). Let $\mathscr{F} = \{F_\alpha: \alpha \in A\}$ be a discrete collection of closed subsets of Y. Then $\{f^{-1}(F_\alpha): \alpha \in A\}$ is a discrete collection of closed subsets of X. Let $\{V_\alpha: \alpha \in A\}$ be a compact-finite collection of open sets such that $f^{-1}(F_\alpha) \subset V_\alpha$ for each $\alpha \in A$ and $f^{-1}(F_\alpha) \cap V_\beta = \emptyset$, $\alpha \neq \beta$. Since f is closed, for each $\alpha \in A$ there exists an open set $U_\alpha \subset Y$ such that $f^{-1}(U_\alpha) \subset V_\alpha$ and $F_\alpha \subset U_\alpha$. Let $p \in Y$, and let $q \in f^{-1}(p)$. Then $q \in V_\alpha$ for each α such that $p \in U_\alpha$. Since $\{V_\alpha: \alpha \in A\}$ is compact-finite, $p \in U_\alpha$ for at most finitely many $\alpha \in A$. Thus $\{U_\alpha: \alpha \in A\}$ is point-finite. Assume $\{U_\alpha: \alpha \in A\}$ is not compact-finite. Let K be a compact set such that $U_\alpha \cap K \neq \emptyset$, for infinitely many $\alpha \in A$. Let $A' = \{\alpha \in A: U_\alpha \cap K \neq \emptyset\}$, and for each $\alpha \in A'$ let $y_\alpha \in U_\alpha \cap K$. Then $T = \{y_\alpha: \alpha \in A'\}$ is infinite, because $\{U_\alpha: \alpha \in A\}$ is point-finite. Thus T has a cluster point in K, say y. Then $T' = T - \{y\}$ is not k-closed. Thus $f^{-1}(T')$ is not k-closed. Thus there exists a compact set H such that $f^{-1}(T') \cap H$ is not closed. Since $f^{-1}(\{y_\alpha\})$ is closed, for each $\alpha \in A'$ and $f^{-1}(T') \cap H$ is not closed, $A'' = \{\alpha \in A': f^{-1}(\{y_\alpha\}) \cap H \neq \emptyset\}$ is infinite. Since $f^{-1}(\{y_\alpha\}) \subset V_\alpha$, for each $\alpha \in A''$, $V_\alpha \cap H \neq \emptyset$ for each $\alpha \in A''$. Thus $\{V_\alpha: \alpha \in A''\}$ is not compact-finite. From this contradiction we have $\{U_\alpha: \alpha \in A\}$ is compact-finite. This completes the proof.

Since a normal space is mesocompact if and only if it is metacompact and has property (k) [4], and the closed image of a metacompact space is metacompact [11], we have the following theorem.

THEOREM 2.2. *The closed k-quotient image of a normal mesocompact space is a normal mesocompact space.*

3. **Functional characterization of k-spaces and defining k-systems.** In this section the theorems are either compactly generated analogies to theorems in [5], improvements of theorems in [9] or modifications of theorems in [3]. The notions of defining k-systems and defining k_1-systems of Arhangelśkiĭ are fundamentally involved with the mappings of this paper. Professor Arhangelśkiĭ defines, in [3], a *defining k-system* in a space X to be a collection of compact sets $\mathscr{K}$ such that $M \subset X$ is closed whenever $M \cap K$ is closed for each $K \in \mathscr{K}$. Also, he calls a collection of compact sets $\mathscr{K}$ a *defining k_1-system* provided: if $p \in \mathrm{cl}(M)$ then there exists a $K \in \mathscr{K}$ such that $p \in \mathrm{cl}(M \cap K)$.

The following theorem can be established easily from the definitions.

THEOREM 3.1. *If X is a k-space (k'-space), then every quotient (pseudo-open) mapping defined on X is k-quotient (k'-quotient).*

Example 4.5 shows that the converse of both parts of Theorem 3.1 and the sufficiency of Theorem 5.2 of [5] are *false* for range spaces which are T_1-spaces. If no separation axioms are assumed on the range spaces, then the converse of this theorem can be established in the following manner. Suppose X is not a k-space (k'-space). Then there is a set H which is k-closed and not closed. (Then there is a set H and a point $p \in \mathrm{cl}(H)$ such that $p \notin \mathrm{cl}(H \cap K)$ for any compact $K \subset X$.) Let f be a mapping that identifies H to a point. Then f is quotient (pseudo-open) but not k-quotient (k'-quotient).

The remaining theorems in this section are all related to range characterizations and defining k-systems. The next theorem is the compact analogy of Theorems 5.3 and 5.4 of [5] and improves Theorems 2.1 and 2.2 of [10].

THEOREM 3.2. *Y is a k-space (k'-space) if and only if every k-quotient (k'-quotient) mapping onto Y is quotient (pseudo-open).*

Proof. Let $f: X \rightarrow Y$ any k-quotient (k'-quotient) mapping onto a k-space (k'-space) Y. Let $H \subset Y$ be such that $f^{-1}(H)$ is closed in X. Since f is k-quotient, H is k-closed in Y. Since Y is a k-space,

H is closed in Y. (Let $p \in \mathrm{cl}(H)$. Then there is a compact set $T \subset Y$ such that $p \in \mathrm{cl}(H \cap T)$. Since f is k'-quotient, there is a compact set $K \subset X$ such that $f^{-1}(p) \cap \mathrm{cl}(f^{-1}(H \cap T) \cap K) \neq \emptyset$. Since $f^{-1}(p) \cap \mathrm{cl}(f^{-1}(H)) \neq \emptyset$, f is pseudo-open.) To prove the converse, consider the mapping from the disjoint sum of the compact subsets of Y onto Y, defined in the obvious manner. This mapping is compact-covering, thus k-quotient (k'-quotient). By hypothesis this is a quotient (pseudo-open) mapping from a locally compact space onto Y. Accordingly, Y is a k-space (k'-space). This completes the proof.

In Example 4.2 a k-quotient mapping onto a compact space is given which is not pseudo-open. Thus the direct compact analogy of Theorem 5.4 in [5], which would be Y is a k'-spacc if and only if every k-quotient mapping onto Y is preudo-open, is *false*. This example also shows that Theorem 4.6 of [9] and 2.4 of [10] can not be improved by using k-quotient mappings alone. In particular, it is *false* that if Y is a strongly k'-space [9] then every k-quotient mapping onto Y is countably bi-quotient, and it is *false* that if Y is locally compact then every k-quotient mapping onto Y is bi-quotient. Range characterizations of this type probably can be obtained by using suitably stronger variations of k'-quotient mappings.

The next theorem is the sufficiency of Arhangelśkiĭ's Theorems 10 and 11, which were stated in [3]. We state it here for completeness. The necessity of these theorems are false, as can be observed in Example 4.7. We will establish that the k-quotient and k'-quotient mappings are precisely the notions needed to yield valid versions of these theorems.

THEOREM 3.3. [Arhangelśkiĭ] *If Y is a k-space (k'-space) and $f: X \to Y$ is such that the images of the compact subsets of X form a defining k-system (defining k_1-system) then f is quotient (pseudo-open).*

The following two theorems indicate the fundamental connection between k-quotient (k'-quotient) mappings and the defining k-systems (defining k_1-systems) of Arhangelśkiĭ. They also constitute a correct alternate form of Theorems 10 and 11 of [3].

THEOREM 3.4. *Let Y be a k-space. The mapping $f: X \to Y$ is a k-quotient mapping if and only if the images of compact subsets of X form a defining k-system in Y.*

Proof. Let f be k-quotient, and let M be such that $M \cap f(K)$ is closed for each compact subset $K \subset X$. Then $(f^{-1}(M \cap f(K))) \cap K = f^{-1}(M) \cap K$ is closed in X for each compact set $K \subset X$. Thus $f^{-1}(M)$ is k-closed in X, and since f is k-quotient, M is k-closed in Y. Since

Y is a k-space, M is closed in Y. Hence the collection of images of compact sets is a defining k-system. To prove the converse, let M be a non$k$-closed set in Y. Then there exists a compact set $K \subset X$ such that $M \cap f(K)$ is not closed. If $K \cap f^{-1}(M)$ is closed, thus compact, then $f(K \cap f^{-1}(M)) = f(K) \cap M$ is compact, thus closed. Hence $K \cap f^{-1}(M)$ is not closed. Thus $f^{-1}(M)$ is not k-closed. Accordingly, f is k-quotient and this completes the proof.

THEOREM 3.5. *Let Y be a k'-space. The mapping $f: X \to Y$ is a k'-quotient mapping if and only if the images of compact subsets of X form a defining k_1-system in Y.*

Proof. Let f be k'-quotient, and let $p \in \mathrm{cl}(H) - H$. Since Y is a k'-space, there exists a compact set T such that $p \in \mathrm{cl}(H \cap T)$. Also, since f is k'-quotient, there exists a compact set $K \subset X$ such that $f^{-1}(p) \cap \mathrm{cl}(f^{-1}(H \cap T) \cap K) = f^{-1}(p) \cap \mathrm{cl}(f^{-1}(H) \cap (f^{-1}(T) \cap K)) \neq \varnothing$. Hence, $p \in f(\mathrm{cl}(f^{-1}(H) \cap (f^{-1}(T) \cap K))) \subset \mathrm{cl}(f(f^{-1}(H) \cap (f^{-1}(T) \cap K)) = \mathrm{cl}(H \cap (f(f^{-1}(T) \cap K)))$. Since $f^{-1}(T) \cap K$ is a compact set in X, there exists a compact set in X such that $p \in \mathrm{cl}(H \cap f(f^{-1}(T) \cap K))$. Thus the collection of images of compact subsets of X is a defining k_1-system in Y. To prove the converse, let $p \in \mathrm{cl}(H) \subset Y$. Since the collections of images of compact subsets of X form a defining k_1-system in Y, there exists a compact set $K \subset X$ such that $p \in \mathrm{cl}(H \cap f(K))$. Since $H \cap f(K) = f(f^{-1}(H) \cap K) \subset f(\mathrm{cl}(f^{-1}(H) \cap K))$ and

$$f(\mathrm{cl}(f^{-1}(H) \cap K))$$

is closed, $\mathrm{cl}(H \cap f(K)) \subset f(\mathrm{cl}(f^{-1}(H) \cap K))$. Thus $p \in f(\mathrm{cl}(f^{-1}(H) \cap K))$, and $f^{-1}(p) \cap \mathrm{cl}(f^{-1}(H \cap f(K)) \cap K) \neq \varnothing$. Hence f is k'-quotient and this completes the proof.

4. Examples.

EXAMPLE 4.1. A k'-quotient mapping which is not compact-covering.

The mapping $f: E \to F$ in Michael's Example 3.1 in [8] is open, E is a σ-compact metric space, F is a compact metric space and $f^{-1}(y)$ is locally compact for each $y \in F$. The mapping f is not compact-covering, but since it is a pseudo-open mapping defined on a k'-space, it is k'-quotient, by Theorem 3.2.

EXAMPLE 4.2. A k-quotient mapping from a locally compact space onto a compact space which is not k'-quotient.

Let $Y = [0, \Omega]$ be the ordinal space, where Ω is the first uncountable ordinal. Let $X_1 = Y - \{\Omega\}$, and let X_2 be the subspace of Y consisting of all limit ordinals. Let X be the disjoint topological sum of X_1 and X_2, and let $f: X \rightarrow Y$ be the identification, $f(\alpha) = \alpha$ for each $\alpha \in X$. This is Michael's Example 8.5 in [7]. The mapping f is quotient, but not pseudo-open, and hence not k'-quotient, by Theorem 3.2. Since X is locally compact, f is k-quotient.

Many of the characterizations and applications of sequentially quotient mappings in [5] were the result of the presequential properties of sequentially quotient mappings. This example shows that the compactly generated analogs, using convergent nets, filters and filterbases, to the presequential characterizations are not possible for k-quotient mappings. In particular, the net of nonlimit ordinals $\{\alpha\}$ converges to Ω in Y. However, the net $\{f^{-1}(\{\alpha\})\}$ in X has no subnet that converges to any point in $f^{-1}(\{\Omega\})$. Thus f is not limit lifting. In fact, $\mathrm{cl}\,(f^{-1}(Z)) \cap f^{-1}(\{\Omega\}) = \varnothing$, where Z is the set of nonlimit ordinals in Y.

EXAMPLE 4.3. A k-quotient mapping which is not sequentially quotient.

Consider the perfect mapping $f: \beta N \rightarrow N^*$, from the Stone-Čech compactification of N onto the one-point compactification of N, defined by $f(n) = n$ for each $n \in N$, and $f(p) = \infty$ for each $p \in \beta N - N$. Since N is not sequentially closed in N^* and $f^{-1}(N)$ is sequentially closed in βN, f is not sequentially quotient.

EXAMPLE 4.4. A continuous open sequentially quotient mapping, onto a compact space, which is not k-quotient.

Let $Y = [0, \Omega]$ be the ordinal space, where Ω is the first uncountable ordinal. For each $n \in N$, let $X_n = \{(\alpha, 1/n): \alpha \in [0, \Omega)\}$. Let $X = \{\Omega\} \cup (\bigcup_{n \in N} X_n)$ have the topology generated by the following neighborhood bases. For each $p \in X - \{\Omega\}$, $p = (\alpha, 1/k)$ for some $\alpha \in [0, \Omega)$ and for some $k \in N$. The neighborhood base as p will be all sets of the form $\{(\beta, 1/k): \beta \in U\}$, where U is any basic open neighborhood of α in the order topology on $[0, \Omega)$. The neighborhood base at Ω will be all sets of the form, $\{\Omega\} \cup (\bigcup_{\beta \in U'} (\beta, 1/n): n \geqq n_0)$ where $U' = U - \{\Omega\}$ and U is any basic open neighborhood of Ω in order topology on $[0, \Omega]$, and $n_0 \in N$. Thus each X_n is a copy of $[0, \Omega)$ and the basic open neighborhoods of Ω are unions of residual subsets of residual columns.

X is not a k-space, bacause $X - \{\Omega\}$ is k-closed but it is not closed. That $X - \{\Omega\}$ is k-closed, follows from the fact that Ω is not a cluster point of any compact set in X. To verify this, let K be any compact set in X. Since K is compact and X_n is closed for each $n \in N$, $K \cap X_n$ is compact for each $n \in N$. For each $n \in N$ such that $K \cap X_n \neq \emptyset$, there exists some $\beta_n \in [0, \Omega)$ such that $K \cap X_n \subset \{(\alpha, 1/n): \alpha \leqq \beta_n\}$. If $K \cap X_n = \emptyset$, let $\beta_n = 0$. Since $\beta = \sup\{\beta_n: n \in N\} < \Omega$, the neighborhood $\{\Omega\} \cup (\bigcup_{\lambda > \beta} \{(\lambda, 1/n): n \in N\})$ of Ω does not intersect $K - \{\Omega\}$. Accordingly, Ω is not a cluster point of any compact set in X.

Let $f: X \to Y$ be defined by $f(\alpha, 1/k) = \alpha$ for each $(\alpha, 1/k) \in X - \{\Omega\}$ and $f(\Omega) = \Omega$. The mapping f is continuous and open, because both images and preimages of basic open sets are open under f. The mapping f is sequence covering [9], and thus f is sequentially quotient [5, Theorem 4.3(a)]. The set $[0, \Omega)$ is not k-closed in Y, but $f^{-1}([0, \Omega)) = X - \{\Omega\}$ is k-closed in X. Hence f is not a k-quotient mapping.

EXAMPLE 4.5. A space X such that every quotient mapping, from X onto a T_1-space, is compact-covering and sequence covering, but X is not a k-space.

Let X be the subspace of the ordinal space $[0, \Omega]$, where Ω is the first uncountable ordinal, defined by $X = [0, \Omega] - \{\lambda: \lambda$ is a limit ordinal and $\lambda < \Omega\}$. Let $f: X \to Y$ be any quotient mapping from X onto a T_1-space Y. For each $y \in Y - \{f(\Omega)\}$, $f^{-1}(y)$ is countable. Let $\alpha_y \in f^{-1}(y)$, for each $y \in Y - \{f(\Omega)\}$ and let $Z = \{\alpha_y: y \in Y - \{f(\Omega)\}\}$. Then the subspace $Z \cup \{\Omega\}$ is homeomorphic to Y.

The restriction of f to $Z \cup \{\Omega\}$, f^*, is the homeomorphism. Clearly, f^* is a one-to-one continuous mapping from $Z \cup \{\Omega\}$ onto Y. Since $\{y\}$ is open in Y, for each $\{\alpha_y\} \subset Z$, $f^*(\{\alpha_y\}) = \{y\}$ is open. Let $U_\beta = (Z \cup \{\Omega\}) \cap [\beta, \Omega]$ be any basic open neighborhood of Ω. Then $f^*(U_\beta) = Y - \{y: \alpha_y < \beta\}$. Since $f^{-1}(f^*(U_\beta)) = X - \bigcup \{f^{-1}(y): \alpha_y < \beta\}$ there exists $\gamma < \Omega$ such that $[\gamma, \Omega] \cap X \subset f^{-1}(f^*(U_\beta))$. Thus $f^*(U_\beta)$ is open. Hence f^* is also an open mapping. Accordingly, f is compact-covering and sequence covering, but X is not a k-space.

EXAMPLE 4.6. The k-quotient image of a k-space is not necessarily a k-space.

Let A be the set of points in the plane $\{(1/n, 1/m): n, m \in N\} \cup \{(0, 0)\}$. Let each singleton $\{(1/n, 1/m)\}$ be open, and let the neighborhood base at $(0, 0)$ be the collection of sets of the form $\{(1/n, 1/m): n > k$ and $m \geqq m_n\} \cup \{(0, 0)\}$. That is, a neighborhood of $(0, 0)$ contains the union of residual subsets of residual columns. This space A is the well-known space of Arens [1]. The compact sets in A are finite. Let B be the set A with the discrete topology. The identity mapping, e, from B onto A is continuous compact-covering, but the image is not k-space. Clearly, e can not be a quotient mapping.

EXAMPLE 4.7. The collection of images of compact sets, under an open mapping onto a compact metric space, need not form a defining k-system.

Let $Y = \{0\} \cup \{1/n: n \in N\}$ with the usual topology. The mapping $f: A \to Y$, where A is Arens space as described in Example 4.6, defined by $f((0, 0)) = 0$ and $f((1/n, 1/m)) = 1/n$, for each $n, m \in N$, is an open mapping onto the compact metric space Y. The images of the compact sets in A are finite subsets of Y. Any defining k-system in Y must contain a set which contains a residual subset of $\{1/n: n \in N\}$. Hence the images of the compact sets in A do not form a defining k-system in Y.

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