PACIFIC JOURNAL OF MATHEMATICS
Vol. 65, No. 1, 1976

COMPACTLY COGENERATED LCA GROUPS

D. L. ARMACOST

In this paper we seek to describe and investigate a class of LCA groups which appropriately generalizes the class of finitely cogenerated abelian groups. Of three possible generalizing classes we finally choose one, which we refer to as the class of compactly cogenerated LCA groups, as being the most suitable. It turns out that this class is considerably more complicated than the corresponding class of compactly generated LCA groups. We give various criteria for an LCA group to be a member of this class, and we describe several important subclasses. As a result of our investigations we show that a divisible LCA group which is indecomposable is either compact and connected, or else is isomorphic to the group of real numbers, a quasicyclic group, or a p-adic number group.

1. Introduction. Within the category of abelian groups the finitely generated groups play an important rôle. The natural generalization of this class within the category of locally compact abelian (LCA) groups is the class of compactly generated groups, about which much detailed information is available (see, for example, [3, § 9] including the well-known structure theorem [3, 9.8]. Dual to the class of finitely generated groups within the category of abelian groups is the class of finitely cogenerated groups (see [2, pp. 109–111]). It is the purpose of this paper to investigate possible generalizations of this class within the category of LCA groups.

Throughout, all groups will be assumed to be LCA Hausdorff topological groups. The LCA groups which we mention frequently are the circle T, the real numbers R, the integers Z, the cyclic groups $Z(n)$, the rationals Q, the quasicyclic groups $Z(p^\infty)$, the p-adic integers J_p and the p-adic numbers F_p. Precise definitions of all these groups may be found in [3]. Topological isomorphism will be denoted by "$\cong$".

Let us recall the definition of a finitely cogenerated abelian group. A subset S of an abelian group G is called a *system of cogenerators* of G if for every abelian group H and homomorphism $f: G \to H$ we have $\ker(f) \cap S \subseteq \{0\} \Rightarrow f$ is a monomorphism. An abelian group is then called *finitely cogenerated* if it contains a finite system of cogenerators. It is shown in [2, Theorem 25.1] that G is finitely cogenerated if and only if the subgroups of G satisfy the minimum condition, in which case G is the direct sum of finitely many cocyclic groups.

In light of the above, there are at least three possible ways to proceed. We could investigate the class of LCA groups whose closed subgroups satisfy the minimum condition. Or, we could examine the LCA groups containing a finite system of cogenerators (replacing homomorphisms by continuous ones). Finally, we could investigate the LCA groups which contain a *compact* system of cogenerators. This last condition is the one most parallel, from the formal point of view, to the generalization of the finitely generated abelian groups to the category of LCA groups. Moreover, it is the one which we shall finally adopt. Nevertheless, the first two conditions have some interest in their own right. Still, as a result of our investigations of these two conditions, we shall be led to the adoption of the third condition as the most natural generalization of the finitely cogenerated groups to the category of LCA groups.

2. Two possible generalizations. We say that the closed subgroups of an LCA group G satisfy the *minimum condition* provided every descending chain of closed subgroups is stationary after a finite number of steps. There is an analogous definition for the maximum condition, and it is apparent by duality that the closed subgroups of G satisfy the minimum condition if and only if the closed subgroups of the dual group $\hat{G}$ satisfy the maximum condition.

THEOREM 2.1. *The closed subgroups of G satisfy the minimum condition iff $G \cong D \times T^n$, where D is discrete and finitely cogenerated and n is a nonnegative integer.*

Proof. Since the condition does not hold for R, it follows from the structure theorem for LCA groups [3, 24.30] that G contains a compact open subgroup U. Now since the closed subgroups of U satisfy the minimum condition, the subgroups of the discrete group $\hat{U}$ satisfy the maximum condition, so $\hat{U}$ is finitely generated [2, Theorem 15.5]. Hence U has the form $T^n \times F$, where n is a nonnegative integer and F is finite. Since T^n is open in U, it is also open in G, so $G \cong T^n \times D$, where D is discrete [3, 25.31]. Since the subgroups of D must also satisfy the minimum condition we conclude that D is finitely cogenerated. It is not difficult to verify that the converse holds as well.

COROLLARY 2.1. *The closed subgroups of G satisfy the maximum condition iff G has the form $D \times H$, where D is discrete and finitely generated and H is a product of finitely many p-adic integer groups* (here, as throughout, with their usual compact topology).

Proof. This follows from the theorem by duality and the structure theorems for finitely generated and cogenerated groups.

The two previous results taken together immediately imply the following:

COROLLARY 2.2. *The closed subgroups of G satisfy both the maximum and the minimum conditions iff G is finite.*

Theorem 2.1 indicates that the minimum condition on closed subgroups is too restrictive for our purposes. Various minor modifications of the chain conditions are possible (one could restrict attention to chains of open or compact subgroups, for example), but all such modifications which we have examined again lead to very narrow classes of LCA groups.

We now turn to the second possible generalization mentioned. For this we state a formal definition.

DEFINITION 2.1. A subset S of a group G is said to be a *system of cogenerators* of G iff for each group H and continuous homomorphism $f: G \to H$, $\ker(f) \cap S \subseteq \{0\} \Rightarrow f$ is one-one.

A *finitely cogenerated* group is simply one containing a finite system of cogenerators. In sharp contrast to the *finitely generated* groups (in the topological sense, that is, groups containing a finite system of elements which generate a dense subgroup, such as the monothetic groups), the finitely cogenerated groups turn out to be discrete, but to show this we need a simple lemma.

LEMMA 2.1. *Let G be finitely cogenerated. If $G = A \times B$, where A and B are closed subgroups of G, then both A and B are finitely cogenerated.*

Proof. Let $\{(a_1, b_1), \cdots, (a_n, b_n)]$ be a finite system of cogenerators of G. One may then show that $\{a_1, \cdots, a_n\}$ and $\{b_1, \cdots, b_n\}$ are systems of cogenerators of A and B respectively. We omit the details.

THEOREM 2.2. *A finitely cogenerated group is necessarily discrete.*

Proof. Let S be a finite system of cogenerators of the group G. Label the nonzero elements of S as $x_1, \cdots, x_n$. For each x_i find γ_i in $\hat{G}$ such that $\gamma_i(x_i) \neq 1$. Define $f: G \to T^n$ by the rule $f(x) = (\gamma_1(x_1), \cdots, \gamma_n(x_n))$ for each x in G. Clearly f is a continuous homomorphism such that $\ker(f) \cap S \subseteq \{0\}$. Hence f is one-one, so the transpose map $f^*: Z^n \to \hat{G}$ has dense image [3, 24.41]. Thus $\hat{G}$ is finitely generated (in the topological sense) and hence, by an application of [3, 5.14], compactly generated, so $\hat{G}$ has the form $R^n \times Z^m \times K$, where n and m are nonnegative integers and K is compact [3, 9.8]. Then G has the form $R^n \times T^m \times D$, where D is discrete. However, neither R nor T is finitely cogenerated, as is easy to check, so by Lemma 2.1 we conclude that G is discrete.

This result is perhaps surprising, in that one might reasonably expect to find nondiscrete groups possessing a finite system of cogenerators, just as there are many nondiscrete groups possessing a finite system of generators (again, of course, we mean this in the topological sense), such as the infinite compact monothetic groups, to name just a few. When we replace "finite" by "compact", however, the situation changes radically, and nondiscrete LCA groups possessing a compact system of cogenerators occur in great abundance.

3. The main definition.

Having investigated two possible generalizations, we now turn to the one which we shall finally adopt.

DEFINITION 3.1. We call a group G *compactly cogenerated* (abbreviated c.c.) iff G contains a compact system of cogenerators.

LEMMA 3.1. *Every element of a* c.c. *group* G *is compact.*

Proof. If x in G is not compact, then (x), the cyclic subgroup of G generated by x, is topologically isomorphic to the discrete group of integers [3, 9.1]. If S is a compact system of cogenerators of G, then $(x) \cap S$ is compact and hence finite, so there is a positive integer n such that $(nx) \cap S \subseteq \{0\}$. The natural projection $\pi: G \to G/(nx)$ is not one-one, even though $\ker(\pi) \cap S = \{0\}$, which is a contradiction.

DEFINITION 3.2. A subgroup E of G is called *topologically essential* in G iff each nonzero closed subgroup of G has nonzero intersection with E.

REMARK 3.1. It is easy to check that if E is a closed subgroup of G, then E is topologically essential in G iff E is a system of cogenerators of G.

REMARK 3.2. If E is an open topologically essential subgroup of G, then E is also essential in the algebraic sense: that is, if H

is any nontrivial subgroup of G, closed or not, then H has nontrivial intersection with E, as is easy to check.

The following theorem gives two characterizations of the compactly cogenerated LCA groups. Condition (ii) is the analogue of the characterization of the finitely cogenerated groups as essential extensions of finite groups. Recall that the *socle* of an abelian group G (written $S(G)$) is the subgroup of all x in G such that the order of x is a square-free integer.

THEOREM 3.1. *The following are equivalent for an* LCA *group* G:

(i) G *is* c.c.

(ii) G *contains a compact, open essential subgroup* E.

(iii) *Every element of* G *is compact, and the socle of* G *is relatively compact.*

Proof. Assume (i) and let S be a compact system of cogenerators. Embed S in an open compactly generated subgroup E of G [3, 5.14]. Using Lemma 3.1 we conclude from [3, 9.8] that E is itself compact. Since E is clearly a system of cogenerators of G, we conclude from Remarks 3.1 and 3.2 that E is essential in G, so that (i) $\Rightarrow$ (ii). It is clear that (ii) $\Rightarrow$ (i). It is moreover an easy exercise to show that a subgroup E of an abelian group G is essential in G iff E contains the socle of G and G/E is a torsion group [2, Ex. 10 on p. 87]. Hence, if (ii) holds, we have in particular that E contains $S(G)$, so that $S(G)$ is relatively compact. The implication (ii) $\Rightarrow$ (i) and Lemma 3.1 show that every element of G is compact, thus giving (ii) $\Rightarrow$ (iii). Finally, if (iii) holds, let E be any compact open subgroup of G containing $S(G)$ [3, 5.14 and 9.8]. Since every element of G is compact, we have that G/E is a discrete torsion group, so that E is essential in G, again by the exercise referred to above. Thus (iii) $\Rightarrow$ (ii), which completes the proof.

From this theorem we can draw a number of corollaries which serve to show that the class of compactly cogenerated LCA groups is very extensive, perhaps too much so for a manageable structure theorem. In particular, the following is immediate from part (iii) of the theorem:

COROLLARY 3.1. *If* G *is torsion-free, then* G *is* c.c. *iff every element of* G *is compact.*

For the next corollary we need a definition.

DEFINITION 3.3. If G is an LCA group, then by G^* we mean the minimal divisible extension of G topologized as in [3, 25.32]. In this topology, G is an open subgroup of G^*.

COROLLARY 3.2. *If G is* c.c. *so is G^*.*

Proof. Let E be a compact open essential subgroup of G. Since G is essential in G^* [2, Lemma 24.3], it follows that E is itself essential in G^*, so G^* is c.c. by part (ii) of the theorem.

4. Some simple properties of c.c. groups. We now present a few easy results about compactly cogenerated LCA groups.

PROPOSITION 4.1. *Every proper closed subgroup of a* c.c. *group is again* c.c.

Proof. This follows immediately from part (iii) of Theorem 3.1.

REMARK 4.1. The converse of this result is true as well, although we have found it necessary to postpone its proof until later (see Proposition 6.3).

PROPOSITION 4.2. *Let G be topologically isomorphic to $A \times B$, where A and B are closed subgroups of G. Then G is* c.c. *iff A and B are* c.c.

Proof. Let E and F be compact open essential subgroups of A and B respectively. Then $E \times F$ is a compact open essential subgroup of $A \times B$, so G is c.c. The converse follows from Proposition 4.1.

REMARK 4.2. The quotient of a c.c. group by a closed subgroup need not be c.c. For example, let G be the (compact) product of the p-adic integer groups J_p, one for each prime p, and let G^* be the minimal divisible extension of G (see [3, 25.32d] for a description of G^*). By Corollary 3.2, G^* is c.c. However, G^*/G is isomorphic to the discrete group Q/Z, which is not finitely cogenerated.

PROPOSITION 4.3. *Let K be a compact subgroup of G. If G/K is* c.c. *then so is G.*

Proof. Pick an open subgroup $H \supseteqq K$ of G such that $\pi(H)$ is a compact open essential subgroup of G/K, where π is the natural map from G onto G/K. We claim that H is a compact essential subgroup of G. Since both H/K and K are compact, we know that H is compact [3, 5.25]. It only remains to show that H is essential in G. To this end, let A be a nonzero subgroup of G. We must show that $A \cap H \neq \{0\}$. If $A \subseteq K$ there is nothing to prove, since $K \subseteq H$. Otherwise, $\pi(A)$ is a nonzero subgroup of G/K, and so there is a nonzero element $a + K$ in $\pi(A) \cap \pi(H)$. We conclude that a is in H and $a \neq 0$, which completes the proof.

REMARK 4.3. We have not succeeded in proving this result when K is assumed merely to be c.c.

PROPOSITION 4.4. *Every* c.c. *group is the quotient of a torsion-free* c.c. *group by a compact subgroup.*

Proof. If G is c.c., then Lemma 3.1 and [3, 24.18] imply that $\hat{G}$ is totally disconnected. It is easy to see that the minimal divisible extension $(\hat{G})^*$ of $\hat{G}$ is also totally disconnected. Let H be the dual of $(\hat{G})^*$. Since $(\hat{G})^*$ is divisible and totally disconnected, we have that H is torsion-free and every element of H is compact, so that H is c.c. by Corollary 3.1. Since $\hat{G}$ is an open subgroup of $\hat{H}$, we conclude that G is isomorphic to the quotient of H by a compact subgroup, namely, the annihilator in H of $\hat{G}$ (see [3, 23.29]).

Since every c.c. group is a quotient of a torsion-free c.c. group, it is perhaps appropriate to state the following result about torsion-free c.c. groups.

PROPOSITION 4.5. *In a torsion-free* c.c. *group, every open subgroup is essential. In particular, an open subgroup of a torsion-free group G in which every element is compact has the same rank* (necessarily infinite) *and cardinality as G.*

Proof. If O is an open subgroup of G, then O contains $S(G)$ trivially; it is moreover clear that G/O is torsion, so O is essential. The rest of the assertion follows from standard results in abelian group theory.

5. Dual characterizations of c.c. groups. We begin with a definition.

DEFINITION 5.1. A closed subgroup A of a group G is called *effective* iff there exists a proper closed subgroup B of G such that $A + B$ (the subgroup of G generated by A and B) is dense in G. A closed subgroup of G which is not effective is called *ineffective.*

REMARK 5.1. If A is a *compact* effective subgroup of G, then there exists a proper closed subgroup B of G such that $G = A + B$. This follows from [3, 4.4].

PROPOSITION 5.1. *Let H be a closed subgroup of the group G. Then H is topologically essential in G iff the annihilator $A(\hat{G}, H)$* (see [3, 23.23]) *of H in $\hat{G}$ is an ineffective subgroup of $\hat{G}$.*

Proof. Suppose that H is topologically essential in G, and let K be a proper closed subgroup of $\hat{G}$. Then $A(G, (A(\hat{G}, H) + K)) = A(G, A(\hat{G}, H)) \cap A(G, K) = H \cap A(G, K) \neq \{0\}$ since $A(G, K) \neq \{0\}$. Hence $A(\hat{G}, H) + K$ could not be dense in G, so that $A(\hat{G}, H)$ is ineffective in $\hat{G}$. The other direction is obtained simply by reversing the argument.

COROLLARY 5.1. *G is* c.c. *iff $\hat{G}$ contains a compact open ineffective subgroup.*

Proof. This follows from the previous result and part (ii) of Theorem 3.1.

Recall that an LCA group G is called *densely divisible* iff G contains a divisible dense subgroup. It is known [4, Theorem 5.2] that an LCA group G is densely divisible iff $\hat{G}$ is torsion-free. Hence from Proposition 4.5 we have:

COROLLARY 5.2. *Every compact subgroup of a densely divisible totally disconnected* LCA *group is ineffective.*

For another dual characterization of compactly cogenerated groups we make the following definition:

DEFINITION 5.2. A proper closed subgroup H of G will be called a *maximal closed subgroup* of G iff no proper closed subgroup of G properly contains H. The *Frattini subgroup* $F(G)$ is then defined as the intersection of all the maximal closed subgroups of G.

REMARK 5.2. If G has no maximal closed subgroups, then $F(G)$ is taken to be all of G. It is easy to show that $F(G) = G$ iff is densely divisible.

It is not difficult to prove the following result:

PROPOSITION 5.2. *The annihilator in $\hat{G}$ of $S(G)$ is $F(\hat{G})$.*

COROLLARY 5.3. *G is* c.c. *iff $\hat{G}$ is totally disconnected and has* *open Frattini subgroup.*

Proof. This follows immediately from part (iii) of Theorem 3.1 and Proposition 5.2.

COROLLARY 5.4. *Every open subgroup of a group G is effective unless G is both totally disconnected and has open Frattini subgroup.*

Proof. Let O be an open subgroup of G. If O is not effective, then $A(\hat{G}, O)$ is a compact essential subgroup of G, so $\hat{G}$ is c.c. An application of Corollary 5.3 now completes the proof.

6. **Some special c.c. groups.** We now present some results dealing with special subclasses of the class of compactly cogenerated LCA groups.

PROPOSITION 6.1. *A torsion-group is* c.c. *iff it is the direct sum of finitely many p-groups, each having compact socle.*

Proof. Assume that G is torsion and c.c. Then G is totally disconnected [3, 24.21]; since every element of G is compact, we have that both G and $\hat{G}$ are totally disconnected, so by [1, III, Théorème 1] G is a local direct product of its p-primary components. Now each of these p-components is c.c. by Proposition 4.1, so each has relatively compact, and hence in this case compact, socle. Let us now show that only finitely many p-components are nontrivial. This will be true if $S(G)$ is of bounded order. To see this, let E be a compact open essential subgroup of G. It follows from [3, 25.9] that E has bounded order, and since E is essential in G, $S(G)$ must have bounded order as well. This completes the proof in one direction. The other direction is an immediate consequence of Theorem 3.1 and Proposition 4.2.

If G is c.c. and elementary (that is, every element has square-free order), more can be said. For in this case, G coincides with its socle and is hence compact. Therefore by the Proposition, G is the direct sum of finitely many compact groups G_p, where G_p is an elementary p-group. Finally, each G_p must be a compact product of groups $Z(p)$ [3, 25.29].

Next we observe that the c.c. groups J_p and F_p have the property that each nontrivial closed subgroup is topologically essential (in fact, essential, since each such subgroup is already open [3, 10.16a]). The next result shows that these are the only nondiscrete LCA groups with this property.

PROPOSITION 6.2. *Let G be nondiscrete. Then each nontrivial closed subgroup of G is topologically essential in G iff G is either a p-adic integer group or a p-adic number group.*

Proof. We first claim that a group G with the property in question must be torsion-free. For if G contained elements of finite order, G would be finitely cogenerated and therefore discrete, by Theorem 2.2. Hence G is torsion-free. If G^* is the minimal divisible extension of G, each nontrivial closed subgroup of G^* is topologically essential in G^* as well. Hence, in particular, G^* cannot be decomposed as the direct sum of two of its proper closed subgroups, so by the structure theorem for divisible torsion-free groups [3, 25.33], G^* is either R, $\hat{Q}$, Q, or the minimal divisible extension of a product of p-adic integer groups. Clearly R and Q are impossible. Moreover, since Q contains effective subgroups, such as the dyadic rationals, we see by Proposition 5.1 that not every nonzero closed subgroup of $\hat{Q}$ is topologically essential. Hence G^* must be the minimal divisible extension of a product of p-adic integer groups, and it is clear that only one such group can be involved. Hence by [3, 25.32b] $G^* \cong F_p$ for some prime p, so either $G \cong J_p$ or $G \cong F_p$. The converse is straightforward.

Our next result concerns the indecomposability of c.c. groups. We call an LCA group *indecomposable* iff it cannot be written as the direct sum of two of its proper closed subgroups. First let us recall that an indecomposable discrete abelian group is either cocyclic or torsion-free [2, Corollary 27.4]. Hence a compact indecomposable group either has the form $Z(p^n)$ or J_p, where p is a prime, or else is compact and connected. Now let G be a divisible c.c. group which is indecomposable, and let E be a compact open essential subgroup of G. We claim that E must also be indecomposable. For if not, write $E = E_1 \times E_2$, where E_1 and E_2 are proper compact subgroups of E. Let E_i^* be the minimal divisible extension of E_i for $i = 1, 2$. It is clear that $E_1 \times E_2$ is an open essential subgroup of $E_1^* \times E_2^*$. Since G is divisible, the injection f mapping $E_1 \times E_2$ into G may be extended to a homomorphism $\bar{f}: E_1^* \times E_2^* \to G$ [2, Theorem 21.1]. Since f is one-one on the essential subgroup $E_1 \times E_2$, $\bar{f}$ must be one-one as well. Because f is continuous on the open subgroup $E_1 \times E_2$, $\bar{f}$ is a continuous homomorphism from $E_1^* \times E_2^*$ to G. But G is a minimal divisible extension of E [2, Lemma 24.3], so $\bar{f}$ must be surjective. It is finally easy to show that $\bar{f}$ is an open mapping, so $\bar{f}$ is a topological isomorphism from $E_1^* \times E_2^*$ onto G, which contradicts the indecomposability of G. We are now in a position to prove:

LEMMA 6.1. *A divisible indecomposable* c.c. *group is either compact and connected, quasicyclic, or a p-adic number group.*

Proof. Let E be a compact open essential subgroup of such a group G. As we have shown above, E must be indecomposable, so either E is compact and connected or else has the form $Z(p^n)$ or J_p for same prime p. If the first case occurs, it it easy to see that E must coincide with G. If $E \cong Z(p^n)$, then G is finitely cogenerated and hence, by Theorem 2.2, discrete; it is then clear that $G \cong Z(p^\infty)$. Finally, if $E \cong J_p$, it is straightforward to verify that $G \cong F_p$.

We now come to the main result of this section:

THEOREM 6.1. *An indecomposable divisible* LCA *group is either compact and connected, or is topologically isomorphic to either R, $Z(p^\infty)$, or F_p, where p is a prime.*

Proof. By the structure theorem for LCA groups [3, 24.30], either $G \cong R$ or else G contains a compact open subgroup O. In the latter case, let O^* be the minimal divisible extension of O. As above, the natural injection $f: O \rightarrow G$ may be extended to a continuous open monomorphism $\bar{f}$ from O^* onto a subgroup H of G containing O. Since H is an open divisible subgroup of G, it is a topological direct factor of G [3, 6.22b]. But G is indecomposable, so $H = G$. However, $H \cong O^*$, which is c.c. by Corollary 3.2. An application of Lemma 6.1 now completes the proof.

With the aid of this theorem we are now able to provide a proof of the result alluded to in Remark 4.1. We find it surprising that we have not been able to find a more direct proof of this result.

PROPOSITION 6.3. *The group G is* c.c. *iff each proper closed subgroup of G is* c.c.

Proof. One direction has already been proved as Proposition 4.1. Conversely, assume that each proper closed subgroup of G is c.c. Then every element of G is compact, so it only remains to show that $S(G)$ is relatively compact. If $S(G)$ is not dense in G, then the closure of $S(G)$ is c.c. by hypothesis, and is hence compact. We are therefore left to deal with the case in which $S(G)$ is dense.

To handle this case, we first observe that each open subgroup of G must have dense socle as well, and so each proper open subgroup of G, being c.c., must be compact. This means that every nontrivial compact subgroup of $\hat{G}$ is open. If $\hat{G}$ has elements of finite order, then $\hat{G}$ is discrete, so G is compact, and we are done. Thus we may assume that $\hat{G}$ is torsion-free. Let H be the minimal divisible extension of $\hat{G}$, topologized as usual. It is easy to see that each nontrivial compact subgroup of H is open in H. Moreover, H is totally disconnected, since $\hat{G}$ is (recall that every element of G is compact). Therefore by the structure theorem for divisible torsion-free groups [3, 25.33] we conclude that $H \cong Q^{M*} \times E$, where M is a cardinal number and the asterisk denotes the weak direct product, and E is the minimal divisible extension of a product of p-adic integer groups. However, since each nontrivial compact subgroup of H is open, there can be at most one such p-adic integer group involved. Therefore $H \cong Q^{M*} \times F_p^i$, where i is 0 or 1. This means that G, being a quotient of the divisible group $\hat{H} \cong \hat{Q}^M \times F_p^i$, must itself be divisible. If G can be decomposed as the direct sum of two proper closed subgroups, then G is c.c. by Proposition 4.2. On the other hand, if G is indecomposable, we invoke Theorem 6.1 to complete the proof.

Added in proof. The group Q has been inadvertently omitted from the list of groups appearing in Theorem 6.1. It arises because the compact open subgroup O in the proof could be trivial, in which case G is discrete. This change should also be noted in the abstract.

References

1. J. Braconnier, *Sur les groupes topologiques localement compacts*, J. Math. Pures Appl., N. S. **27** (1948), 1-85.

2. L. Fuchs, *Infinite Abelian Groups*, Vol. 1, Academic Press, New York, 1970.

3. E. Hewitt and K. Ross, *Abstract Harmonic Analysis*, Vol. 1, Academic Press, New York, 1963.

4. L. Robertson, *Connectivity, divisibility, and torsion*, Trans. Amer. Math. Soc., **128** (1967), 482-505.

Received September 26, 1974.

AMHERST COLLEGE