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ON T_1-COMPACTIFICATIONS

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In a recent paper [4], Reed constructed a class of T_1-compactifications which generalized the well known correspondence between T_2-compactifications, proximity relations and families of maximal round filters. This class includes the Wallman compactification and the one point compactification of a locally compact T_1-space. In this paper the first two problems posed by Reed are solved. In particular we prove that in a nearness space the Reed compactification is equivalent to a cluster compactification. Use is made of the duality between filters and grills as developed by Thron [5].

1. Preliminaries. Here we give briefly some relevant definitions and known results. For more details see Naimpally and Warrack [3] and Reed [4]. All spaces are T_1.

An *extension structure* Φ on a topological space (X, τ) is a family of open filters on X which include all the neighborhood (nbhd) filters. Φ is said to be T_1 iff no filter in Φ contains another filter in Φ. For each $A \subset X$, set $A^\wedge = \{\mathscr{F} \in \Phi : A \in \mathscr{F}\}$. Then the family $\{G^\wedge : G \in \tau\}$ is a base for the topology $\tau^\wedge$ of Φ such that $(j, (\Phi, \tau^\wedge))$ is a principal extension of (X, τ) where $j(x) = \mathscr{N}_x$, the nbhd filter at $x \in X$.

An extension structure Φ is *totally bounded* iff each ultraclosed filter on X contains a member of Φ. Φ is said to be *covered* iff each member of Φ is contained in an ultraclosed filter on X. Further $P <_\Phi Q$ iff for each $\mathscr{F} \in \Phi$ if P belongs to some ultraclosed filter $\mathscr{H}$ containing $\mathscr{F}$, then $Q \in \mathscr{F}$. Denoting by $\mathscr{H}^i$ the open hull of $\mathscr{H}$, Φ is *regular* iff for each $\mathscr{F} \in \Phi$, $\mathscr{F} = \Phi(\mathscr{H}^i)$ for each ultraclosed filter $\mathscr{H} \supset \mathscr{F}$, where $\Phi(\mathscr{H}^i) = \{A \subset X : F <_\Phi A \text{ for some } F \in \mathscr{H}^i\}$. A *compactification structure* is an extension structure that is totally bounded, covered and regular. The principal extension obtained from a compactification structure on a T_1-space (X, τ) is a T_1-compactification of X, and we call it the *Reed Compactification* [4].

A *stack* $\mathscr{S}$ on a nonempty set X is a nonempty family of nonempty subsets of X such that if $A \in \mathscr{S}$ and $A \subset B$, then $B \in \mathscr{S}$. A *grill* $\mathscr{G}$ on X is a stack on X such that $(A \cup B) \in \mathscr{G}$ iff $A \in \mathscr{G}$ or $B \in \mathscr{G}$. A family of subsets of X is a grill iff it is a union of ultrafilters (Thron [5]). If $\mathscr{S}$ is a stack on X,

$$\begin{aligned} c(\mathscr{S}) &= \{E \subset X \mid X - E \notin \mathscr{S}\} \\ &= \{E \subset X \mid E \cap S \neq \emptyset \quad \text{for each} \quad S \in \mathscr{S}\} \end{aligned}$$

is called the *dual* of $\mathscr{S}$. $\mathscr{F}$ is a filter on X iff $c(\mathscr{F})$ is a grill on X, and $\mathscr{F} = c(\mathscr{F})$ iff $\mathscr{F}$ is an ultrafilter (Thron [5]). Note also that $c(c(\mathscr{F})) = \mathscr{F}$ and $\mathscr{F} \subset c(\mathscr{F})$ for each filter $\mathscr{F}$.

If (X, δ) is a LO-proximity space, then a *clan* σ on X is a grill such that if $A, B \in \sigma$, then $A\delta B$. A *bunch* σ is a clan such that $A \in \sigma$ iff $\bar{A} \in \sigma$. A *cluster* σ on X is a bunch such that if $A \notin \sigma$, then there is a $B \in \sigma$ such that $A\delta B$. Every cluster is a maximal bunch; the converse holds in an EF-proximity space (see [1], [3], [5]). It was shown in [1] that the space of all maximal bunches of a separated LO-space (X, δ) is a T_1-compactification, which we call a maximal bunch compactification.

Thc proofs of the following results are easy and hence omitted.

LEMMA 1.1. (i) *If σ is a bunch in (X, δ), then $c(\sigma)$ is an open filter on X.*

(ii) *If $\mathscr{H}$ is an ultraclosed filter on X, then*

$$b(\mathscr{H}) = \{A \subset X : \bar{A} \in \mathscr{H}\}$$

is a bunch containing $\mathscr{H}$. Also $\mathscr{H} \subset c(\mathscr{H}) \subset b(\mathscr{H})$ and $c(\mathscr{H})$ is a clan.

Lemma 1.1, in particular, enables us to give an example of an open filter which is not contained in any ultraclosed filter.

EXAMPLE 1.2. Consider three distinct sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$ and three distinct points a, b, c. Let $\{x_n\}$ converge to $\{b, c\}$, $\{y_n\}$ to $\{c, a\}$ and $\{z_n\}$ to $\{a, b\}$. Let X be the union of $\{a, b, c\}$ and the ranges of the three sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$. Then X is a T_1-space and has a compatible LO-proximity δ_0 namely,

$$A\delta_0 B \quad \text{iff} \quad \bar{A} \cap \bar{B} \neq \emptyset . \tag{1.3}$$

Then $\sigma = \{A \subset X : A \text{ is infinite}\}$, is a maximal bunch which is not a cluster. (This was first privately communicated to the first author by Professor A. J. Ward.) It is easy to see that σ does not contain any ultraclosed filter $\mathscr{H}$; for if $\mathscr{H} \subset \sigma$, then the closed set $\{a, b, c\}$ would intersect every member of $\mathscr{H}$ and hence would be in $\mathscr{H}$ and consequently in σ, a contradiction. It follows that $c(\sigma)$ is an open filter that is not contained in any ultraclosed filter. This shows that Φ need not be covered.

2. Equivalence of Reed and maximal bunch compactifications. In this section we obtain conditions under which the Reed and maximal bunch compactifications are equivalent.

We first construct a LO-proximity from an extension structure (see also Thron [5]).

THEOREM 2.1. *Let Φ be an extension structure on a T_1-space (X, τ). Then the relation $\delta = \delta(\Phi)$ defined by:*

2.2. *$A\delta B$ iff there is an $\mathscr{F} \in \Phi$ such that $A, B \in c(\mathscr{F})$ is a compatible separated LO-proximity on X.*

Proof. Since each member $\mathscr{F} \in \Phi$ is open it follows that $A \in c(\mathscr{F})$ iff $\bar{A} \in c(\mathscr{F})$, and so $A\delta B$ iff $\bar{A}\delta\bar{B}$. The fact that δ is a basic proximity is easily verified; hence δ is a LO-proximity. For each nbhd filter $\mathscr{N}_x$, $c(\mathscr{N}_x) = \sigma_x$ the point cluster. Since all the nbhd filters are included in Φ, δ is compatible with τ.

COROLLARY 2.3. (i) *For each $\mathscr{F} \in \Phi$, $c(\mathscr{F})$ is a bunch in (X, δ).*

(ii) *If $\mathscr{F} \in \Phi$ and if there is an ultraclosed filter $\mathscr{H}$ containing $\mathscr{F}$, then $b(\mathscr{H}) \subset c(\mathscr{F})$.*

(iii) *If $\mathscr{F} \in \Phi$ and if there is an ultraclosed filter $\mathscr{H}$ containing $\mathscr{F}$, then $b(\mathscr{H}) = c(\mathscr{F})$ if and only if $\mathscr{F} = \mathscr{H}^i$. Further in this case $c(\mathscr{F})$ is a cluster.*

Proof. (i) Easy.

(ii) If $E \in b(\mathscr{H})$, then $\bar{E} \in \mathscr{H}$. This shows that $X - \bar{E} \notin \mathscr{F}$ and so $\bar{E} \in c(\mathscr{F})$. Hence $E \in c(\mathscr{F})$.

(iii) Suppose $\mathscr{F} = \mathscr{H}^i \subset \mathscr{H}$. By (ii) $b(\mathscr{H}) \subset c(\mathscr{F})$. If $b(\mathscr{H}) \neq c(\mathscr{F})$, then there exists a closed set E in $c(\mathscr{F}) - b(\mathscr{H})$. Thus $X - E \in \mathscr{H}^i = \mathscr{F}$, thereby showing that $E \notin c(\mathscr{F})$, a contradiction.

Conversely, suppose $b(\mathscr{H}) = c(\mathscr{F})$. To show that $\mathscr{F} = \mathscr{H}^i$, it suffices to show that $\mathscr{H}^i \subset \mathscr{F}$. Let G be an open member of $\mathscr{H}^i$. If $G \notin \mathscr{F}$, then $X - G \in c(\mathscr{F}) = b(\mathscr{H})$. Since $X - G$ is closed, $X - G \in \mathscr{H}$, a contradiction.

Finally we note that if Φ is covered and $c(\mathscr{F}) = b(\mathscr{H})$ for each $\mathscr{F} \in \Phi$, then the proximity $\delta(\Phi)$ is δ_0 (see (1.3)). Therefore, if $A \notin c(\mathscr{F})$, then $\bar{A} \notin \mathscr{H}$. This implies the existence of an $U \in \mathscr{H}$ with $\bar{A} \cap \bar{U} = \emptyset$. Hence $A \not\delta U$, thereby showing that $c(\mathscr{F})$ is a cluster.

REMARK 2.4. (i) It is shown in [3] that if $\mathscr{F}$ is a maximal round filter on an EF-space (X, δ), then $c(\mathscr{F})$ is a cluster.

(ii) If Φ is a covered extension structure on (X, τ) and $\delta = \delta(\Phi)$, then $A \not\delta B$ implies $\bar{A} <_\Phi X - B$. For, if $\mathscr{F} \in \Phi$ and $A \in c(\mathscr{F})$ then $A \not\delta B$ implies $X - B \in \mathscr{F}$.

In what follows $c(\Phi)$ denotes the set $\{c(\mathscr{F}) : \mathscr{F} \in \Phi\}$. A space of bunches on (X, δ) means a subspace of the space of all bunches in (X, δ) with absorption topology ([1], [3]). A compactification of (X, δ)

whose members are (not necessarily all) clusters (resp. maximal bunches) is called a cluster (resp. maximal bunch) compactification of X. $\sigma_x = \{A \subset X : x\delta A\}$, the point cluster ([1], [3]).

The condition (2.6) given below provides the solution to two of Reed's problems.

LEMMA 2.5. *Let Φ be a compactification structure on (X, τ) such that $\delta = \delta(\Phi)$ satisfies*:

$$(2.6) \qquad A\delta B \quad \textit{iff} \quad \bar{A} \lessdot_{\Phi} X - B ,$$

then $c(\mathscr{F})$ is a cluster for each $\mathscr{F} \in \Phi$.

Proof. Let $\mathscr{F} \in \Phi$ and $\mathscr{H}$ be an ultraclosed filter containing $\mathscr{F}$. Then $\mathscr{F} = \Phi(\mathscr{H}^i) \subset \mathscr{H}^i \subset \mathscr{H} \subset c(\mathscr{F})$. By (2.3) (i), $c(\mathscr{F})$ is a bunch. If $E \notin c(\mathscr{F})$, then $X - E \in \mathscr{F}$. Hence there is an $F \in \mathscr{H}^i$ such that $F \lessdot_{\Phi} X - E$. Since $\mathscr{H}^i \subset \mathscr{H}$, there is a closed set $A \in \mathscr{H}$ such that $A \subset F$. Clearly $A \lessdot_{\Phi} X - E$ and consequently $A\bar{\delta}E$. Thus, $c(\mathscr{F})$ is a cluster.

LEMMA 2.7. *If Φ is an extension structure on a topological space (X, τ), then the principal extension $(j, (\Phi, \tau^{\wedge}))$ is homeomorphic to a space of bunches.*

Proof. Clearly the map $c: \Phi \to c(\Phi)$ defined by

$$c(\mathscr{F}) = \{E \subset X \mid X - E \notin \mathscr{F}\} \quad \text{for each} \quad \mathscr{F} \in \Phi ,$$

is a bijection. If $\delta = \delta(\Phi)$ then $c(\Phi)$ is a family of bunches in the LO-space (X, δ). We assign the absorption topology on $c(\Phi)$ (see [1]). We now show that c is a homeomorphism. Let $\mathscr{F} \in \Phi$ and $\mathscr{A} \subset \Phi$. $\mathscr{F} \in \mathrm{cl}(\mathscr{A})$ iff for each open set $F \in \mathscr{F}$, $F^{\wedge} \cap \mathscr{A} \neq \phi$ iff for each open set $F \in \mathscr{F}$, there is a filter $\mathscr{G} \in \mathscr{A}$ such that $F \in \mathscr{G}$ iff for each closed set $E = X - F \notin c(\mathscr{F})$, $E \notin c(\mathscr{G})$ for some filter $\mathscr{G} \in \mathscr{A}$ iff for each closed set E absorbing $c(\mathscr{A})$, $E \in c(\mathscr{F})$ iff $c(\mathscr{F}) \in \mathrm{cl}(c(\mathscr{A}))$.

REMARK 2.8. $c(\Phi)$ contains all point clusters and the map $\psi: X \to c(\Phi)$ defined by $\psi(x) = \sigma_x$ is a dense embedding of X into $c(\Phi)$. The relation $c \circ j = \psi$, then shows that the extensions $(j, (\Phi, \tau^{\wedge}))$ and $(\psi, (c(\Phi), a))$ where a is the absorption topology, are equivalent.

We now prove one of the main results of our paper.

THEOREM 2.9. *Let Φ be the Reed compactification of a T_1-space (X, τ) such that $\delta = \delta(\Phi)$ satisfies 2.6. Then Φ is equivalent to a maximal bunch compactification of (X, δ). (This need not consist of all maximal bunches in (X, δ).)*

Proof. By Lemma 2.5, $c(\mathscr{F})$ is a cluster and hence a maximal bunch for each $\mathscr{F} \in \Phi$. By Lemma 2.7, Φ is equivalent to $c(\Phi)$.

REMARK 2.10. We note that the above theorem includes the three special cases considered by Reed [4].

(i) If Φ consists of maximal round filters on an EF-space (X, δ), then it is well known that $A\bar{\delta}B$ iff $\bar{A} <_{\Phi} X - B$ (see [3]).

(ii) In case Φ is the trace system of the Wallman compactification, $c(\mathscr{F}) = b(\mathscr{H})$ for each $\mathscr{F} \in \Phi$ and hence $\delta(\Phi) = \delta_0$. Suppose $\bar{A} <_{\Phi} X - B$ but $A\delta B$. Then $\bar{A} \cap \bar{B} \neq \emptyset$ thereby showing that $A, B \in \sigma_x$, the point cluster for some $x \in X$. But then $X - B \notin \mathscr{N}_x$, a contradiction.

(iii) Next let Φ be the trace system of the one-point compactification of a noncompact locally compact T_1-space (X, τ) and let $\bar{A} <_{\Phi} X - B$. Suppose $A\delta B$. As in (ii), $\bar{A} \cap \bar{B} \neq \emptyset$ will lead to a contradiction. So the only possibility is that $\bar{A}, \bar{B} \in c(\mathscr{F})$ where $\mathscr{F}$ is the open hull of the intersection of all the nonconvergent ultraclosed filters. But then $X - \bar{B} \notin \mathscr{F}$, thereby showing $\bar{A} \not<_{\Phi} X - \bar{B}$, contradicting $\bar{A} <_{\Phi} X - B$.

The following theorem gives a characterization of clusters in $(X, \delta(\Phi))$ which are the duals of the members of Φ. We use the notation $r(\sigma) = \{A \subset X$: There is an $F \in \sigma$ such that $F\bar{\delta}(X - A)\}$.

THEOREM 2.11. *Let Φ be extension structure on a T_1-space (X, τ) and let $\delta = \delta(\Phi)$. Then for each $\mathscr{F} \in \Phi$, $c(\mathscr{F})$ is a cluster in (X, δ) iff $\mathscr{F} = r(c(\mathscr{F}))$.*

Proof. Suppose $\mathscr{F} = r(c(\mathscr{F}))$. We know that $c(\mathscr{F})$ is a bunch. If $A \notin c(\mathscr{F})$, then $X - A \in \mathscr{F}$. Hence there exists a $B \in c(\mathscr{F})$ such that $B\bar{\delta}A$ showing thereby that $c(\mathscr{F})$ is a cluster.

Conversely, suppose $c(\mathscr{F})$ is a cluster. If $A \in r(c(\mathscr{F}))$, then $B\bar{\delta}(X - A)$ for some $B \in c(\mathscr{F})$. So $X - A \notin c(\mathscr{F})$, showing thereby that $A \in \mathscr{F}$ i.e., $r(c(\mathscr{F})) \subset \mathscr{F}$. On the other hand, if $A \in \mathscr{F}$, then $X - A \notin c(\mathscr{F})$. Since $c(\mathscr{F})$ is a cluster, there is an $F \in c(\mathscr{F})$ such that $F\bar{\delta}(X - A)$. Hence $A \in r(c(\mathscr{F}))$. Thus $\mathscr{F} \subset r(c(\mathscr{F}))$ and hence $\mathscr{F} = r(c(\mathscr{F}))$.

COROLLARY 2.12. *The Reed Compactification Φ is equivalent to a cluster compactification if and only if $\mathscr{F} = r(c(\mathscr{F}))$ for each $\mathscr{F} \in \Phi$.*

We now show that the Reed Compactification is equivalent to a cluster compactification in nearness spaces. This development was suggested by the referee to whom the authors are grateful. For definition of contigual nearness we refer to [2].

LEMMA 2.13. *Let Φ be a Reed Compactification of a T_1-space X and ν_Φ the nearness generated by the duals of filters in Φ. Then ν_Φ is contigual.*

Proof. That ν_Φ is a nearness on X is easy to prove. To show that ν_Φ is contigual we have to prove that if $\mathscr{A}$ be a family of subsets of X such that every finite subfamily of $\mathscr{A}$ is in ν_Φ then $\mathscr{A} \in \nu_\Phi$. Let $\mathscr{S} = \{F: F$ is closed and $\exists \mathscr{F} \in \Phi$ and $A \in \mathscr{A}$ such that $(X - F) <_\Phi (X - \bar{A}) \in \mathscr{F}\}$. We show that $\mathscr{S}$ has the finite intersection property. Let $F_1, F_2, \cdots, F_n \in \mathscr{S}$. For each i, choose $A_i \in \mathscr{A}$ and $\mathscr{F}_i \in \Phi$ such that $(X - F_i) <_\Phi (X - \bar{A}_i) \in \mathscr{F}_i$. By the assumption of $\mathscr{A}$, $\{A_1, A_2, \cdots, A_n\} \in \nu_\Phi$ and hence there is a filter $\mathscr{F} \in \Phi$ such that $\{A_1, A_2, \cdots, A_n\} \subset c(\mathscr{F})$. Since Φ is covered, we can choose an ultraclosed filter $\mathscr{H}$ such that $\mathscr{F} \subset \mathscr{H}$. Now, for each i, $(X - A_i) \notin \mathscr{F}$ and hence $(X - \bar{A}_i) \notin \mathscr{F}$. Since $(X - F_i) <_\Phi (X - \bar{A}_i)$ it follows that $(X - F_i) \notin \mathscr{H}$. Hence $F_i \in \mathscr{H}$ and so $\bigcap_{i=1}^n F_i \neq \emptyset$. Hence $\mathscr{S}$ has the finite intersection property. Let $\mathscr{Y}$ be an ultraclosed filter containing $\mathscr{S}$. Since Φ is totally bounded, we can choose $\mathscr{G} \in \Phi$ such that $\mathscr{G} \subset \mathscr{Y}$. To prove the lemma we show that $\mathscr{A} \subset c(\mathscr{G})$.

Let $A \in \mathscr{A}$ and $A \notin c(\mathscr{G})$. Then $(X - A) \in \mathscr{G}$. Since $\mathscr{G}$ is open, $(X - \bar{A}) \in \mathscr{G}$. Since Φ is regular, $\mathscr{G} \subset \Phi(\mathscr{Y}^i)$ so we can choose an open set $V \in \mathscr{Y}$ such that $V <_\Phi (X - \bar{A})$. But, then $(X - V) \in \mathscr{S} \subset \mathscr{Y}$ which is impossible. Hence $\mathscr{A} \subset c(\mathscr{G})$.

LEMMA 2.14. *If Φ is a Reed Compactification of X then $\mathscr{F} = r(c(\mathscr{F}))$ for each $\mathscr{F} \in \Phi$.*

Proof. As shown above $r(c(\mathscr{F})) \subseteq \mathscr{F}$. Conversely, let $A \in \mathscr{F}$. Let $\mathscr{H}$ be an ultraclosed filter containing $\mathscr{F}$. We show that $\mathscr{H} \cup \{X - A\} \notin \nu_\Phi$. Let $\mathscr{G} \in \Phi$ and suppose $\mathscr{H} \subset c(\mathscr{G})$. Then $\mathscr{G} \subset \mathscr{H}$. Now since $\mathscr{F} \subset \mathscr{H}$ and Φ is regular, we have $\mathscr{F} \subset \Phi(\mathscr{H}^i)$. Let U be an open set in $\mathscr{H}$ such that $U <_\Phi A$. Then since $\mathscr{G} \subset \mathscr{H}$ we have $A \in \mathscr{G}$. Thus, $(X - A) \notin c(\mathscr{G})$. Hence $\mathscr{H} \cup (X - A) \not\subset c(\mathscr{G})$ and thus $\mathscr{H} \cup (X - A) \notin \nu_\Phi$. Now, since ν_Φ is contigual, there is a set K in $\mathscr{H}$ such that $\{K, (X - A)\} \notin \nu_\Phi$. This says that $K \bar{\delta} (X - A)$. But $K \in c(\mathscr{F})$ since $\mathscr{F} \subset \mathscr{H}$. Hence $A \in r(c(\mathscr{F}))$. Therefore, $\mathscr{F} \subseteq r(c(\mathscr{F}))$.

THEOREM 2.15. *The Reed Compactification is equivalent to a cluster compactification of the induced contigual nearness.*

Proof. Follows from Lemma 2.14 and Theorem 2.11.

3. Reed's second problem. In this section we give a solution to the second problem of Reed [4]. Let $(e, (Y, \tau'))$ be a T_1-compactification of (X, τ) with the trace system Φ. Let $\ll$ be the relation induced on X by the elementary proximity on Y, viz

$$A \ll B \quad \text{iff} \quad \mathrm{cl}(e(A)) \cap \mathrm{cl}(e(X - B)) = \varnothing \ .$$

Then as remarked in [3], $\ll^* \subset <_\Phi$. We show that if Φ satisfies (2.6) then $<_\Phi \subset \ll^*$, and hence the two relations are equal.

We observe that the following yield compatible LO-proximities on X:

$$(3.1) \qquad A\delta_1 B \quad \text{iff} \quad \mathrm{cl}(e(A)) \cap \mathrm{cl}(e(B)) \neq \varnothing$$

$$(3.2) \qquad A\delta_2 B \quad \text{iff} \quad \mathrm{cl}(j(A)) \cap \mathrm{cl}(j(B)) \neq \varnothing$$

$$(3.3) \qquad \delta_3 = \delta(\Phi) \ .$$

It is easy to see that $\delta_2 = \delta_3$ and $\delta_1 \geqq \delta_2$.

THEOREM 3.4. *If Φ satisfies (2.6) then $<_\Phi = \ll^*$.*

Proof. Since it is known that $\ll^* \subset <_\Phi$, we need prove $<_\Phi \subset \ll^*$. Suppose $A <_\Phi B$ but $A \not\ll^* B$. Then there exists a closed set $F \subset A$ such that $F \not\ll B$. Hence $\mathrm{cl}(e(F)) \cap \mathrm{cl}(e(X - B)) \neq \varnothing$. Since $\delta_2 = \delta_3$, there exists an $\mathscr{F} \in \Phi$ such that $F, (X - B) \in c(\mathscr{F})$. By (2.6) $F \not<_\Phi B$ which contradicts $A <_\Phi B$.

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