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REPRESENTATION OF COMPACT AND WEAKLY COMPACT OPERATORS ON THE SPACE OF BOCHNER INTEGRABLE FUNCTIONS

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If X^* has the Radon-Nikodym property, then for every compact operator $T: L_1(\mu, X) \to Y$ there is a bounded function $g: \Omega \to L(X, Y)$ that is measurable for the uniform operator topology on $L(X, Y)$ such that

$$T(f) = \int_\Omega fg d\mu$$

for all f in $L_1(\mu, X)$. The same result holds for weakly compact operators if X^* is separable Schur space. These representations yield Radon-Nikodym theorems for operator valued measures and a generalization of a theorem of D. R. Lewis.

The representation of linear operators on the Banach space $L_1(\mu, X)$ of Bochner integrable functions, has been the object of much study for the past forty years. Dunford and Pettis began this investigation in 1940 [6] with the representation of weakly compact and norm compact operators on $L_1(\mu)$ by a Bochner integral. Their work was based on an earlier paper of Pettis [9] and was complemented by the work of Phillips [11]. More recently, the theory of liftings has been used by Dinculeanu [5] and others to obtain a representation for the general linear operator on $L_1(\mu, X)$. In this paper we will use methods in the spirit of Dunford, Pettis, and Phillips to show that if X^* has the Radon-Nikodym property, then the compact operators on $L_1(\mu, X)$ are representable by measurable kernels and if X^* is a separable Schur space (i.e., weakly convergent sequences converge in norm) then the weakly compact operators on $L_1(\mu, X)$ are representable by measurable kernels. As corollaries, we obtain a Radon-Nikodym theorem for operator-valued measures and a generalization of a theorem of D. R. Lewis [4, p. 88] on weakly measurable functions that are equivalent to norm measurable functions.

Throughout this paper (Ω, Σ, μ) is a finite measure space and X, Y and Z are Banach spaces with duals X^*, Y^*, and Z^* respectively. The space of all bounded linear operators from X to Y will be denoted by $L(X, Y)$. The subspaces of $L(X, Y)$ consisting of all the weakly compact and norm compact operators from X to Y will be denoted by $W(X, Y)$ and $K(X, Y)$. The space $L_1(\mu, X)$ is the space of μ-Bochner integrable functions on Ω with values in X and

$L_\infty(\mu, X)$ is the space of X-valued μ-Bochner integrable functions on Ω that are essentially bounded. An operator $T: L_1(\mu, X) \to Y$ is representable by a measurable kernel if there is a bounded measurable $g: \Omega \to L(X, Y)$ such that

$$T(f) = \text{Bochner} - \int_\Omega f g d\mu .$$

From this, it follows shat $\|T\| = \|g\|_\infty$ [5, p. 283]. Recall that a Banach space is weakly compactly generated if it is the closed linear span of one of its weakly compact sets. Finally, note that if π is a partition of Ω into a *countable* number of disjoint elements of Σ and if f is in $L_1(\mu, X)$, then the function $E_\pi: L_1(\mu, X) \to L_1(\mu, X)$ defined by

$$E_\pi(f) = \sum_{E \in \pi} \frac{\int_E f d\mu}{\mu E} \chi_E$$

(here the convention $0/0 = 0$ is observed) is a linear operator.

Most of the first lemma is well-known so we omit the proof.

LEMMA 1. *For each countable partition π, the operator E_π is a contraction on $L_1(\mu, X)$ and $L_\infty(\mu, X)$. Moreover, if the partitions are directed by refinement, then*

$$\lim_\pi \|E_\pi(f) - f\|_1 = 0 \qquad \textit{for all } f \textit{ in } \quad L_1(\mu, X)$$
$$\lim_\pi \|E_\pi(f) - f\|_\infty = 0 \qquad \textit{for all } f \textit{ in } \quad L_\infty(\mu, X) .$$

Before stating the main theorem we require a preliminary definition. A function g in $L_\infty(\mu, L(X, Y))$ is said to have its essential range in the *uniformly* (*weakly*) *compact operators* if there is a (weakly) compact set C in Y such that $g(\omega)x \in C$ for almost all ω in Ω and x in X with $\|x\| \leqq 1$.

THEOREM 2. *Let X^* have the Radon-Nikodym property. Then there is an isometric isomorphism between the space of compact operators $K(L_1(\mu, X), Y)$ and the subpace of $L_\infty(\mu, K(X, Y))$ consisting of theose functions whose essential range is in the uniformly compact operators. In fact, T in $K(L_1(\mu, X), Y)$ and g in $L_\infty(\mu, K(X, Y))$ are in correspondence if and only if*

$$T(f) = \int_\Omega f g d\mu \qquad \textit{for all } f \textit{ in } \quad L_1(\mu, X) .$$

Proof. Let T be in $K(L_1(\mu, X), Y)$. Notice that for any partition π, f in $L_1(\mu, X)$, and g in $L_\infty(\mu, X^*) = (L_1(\mu, X))^*$, we have that

$$\int_\Omega E_\pi(f) g d\mu = \int_\Omega f E_\pi(g) d\mu .$$

It follows from this that the adjoint of TE_π is $E_\pi T^*$. Now, if the partitions π are countable, we have that

$$\lim_\pi E_\pi f = f \qquad \text{for all } f \text{ in } \quad L_\infty(\mu, X^*)$$

by Lemma 1. Since $\|E_\pi\|_\infty \leqq 1$, this limit is uniform on compact sets. By Schauder's theorem, $T^*: Y^* \to L_\infty(\mu, X^*)$ is compact and so

$$\lim_\pi E_\pi T^* y^* = Ty^*$$

uniformly for $\|y^*\| \leqq 1$. Therefore,

$$\lim_\pi E_\pi T^* = T^*$$

in the operator norm. Since $E_\pi T^* = (TE_\pi)^*$, it follows that

$$\lim_\pi TE_\pi = T$$

in operator norm.

Now, for each countable partition π, define $g_\pi: \Omega \to L(X, Y)$ by

$$g_\pi(\cdot)x = \sum_{A \in \pi} \frac{T(x\chi_A)}{\mu A} \chi_A(\cdot) .$$

Then for each partition π, ω in Ω, and x in X with $\|x\| \leqq 1$, we have that $g_\pi(\omega)x \subseteqq T\{f: f \text{ in } L_1(\mu, X), \|f\|_1 \leqq 1\}$. Since T is compact, it follows that $g_\pi(\omega)$ is in $K(X, Y)$ for each partition π and ω in Ω. Moreover, one easily sees that

$$TE_\pi(f) = \int_\Omega f g_\pi d\mu$$

for all simple functions f in $L_1(\mu, X)$ and thus for all functions f in $L_1(\mu, X)$. Hence if π_1 and π_2 are two partitions, then

$$(TE_{\pi_1} - TE_{\pi_2})(f) = \int_\Omega f(g_{\pi_1} - g_{\pi_2}) d\mu .$$

Since

$$\lim_{\pi_1, \pi_2} \|TE_{\pi_1} - TE_{\pi_2}\| = 0 ,$$

an appeal to [5, p. 283] establishes that

$$\lim_{\pi_1,\pi_2} \|g_{\pi_1} - g_{\pi_2}\|_\infty = \lim_{\pi_1,\pi_2} \|TE_{\pi_1} - TE_{\pi_2}\| = 0 .$$

Thus the net (g_π) is Cauchy in the norm of $L_\infty(\mu, K(X, Y))$. It follows that there is a g in $L_\infty(\mu, K(X, Y))$ such that

$$\lim_\pi \|g_\pi - g\|_\infty = 0$$

and so

$$\lim_\pi \int_\Omega f g_\pi d\mu = \int_\Omega f g d\mu$$

for all f in $L_1(\mu, X)$. We also have, for almost all ω, that

$$g(\omega)x \subseteqq \overline{T\{f: f \in L_1(\mu, X), \|f\| \leqq 1\}}$$

for all x in X with $\|x\| \leqq 1$. Hence the essential range of g consists of uniformly compact operators. Finally, Lemma 1 ensures that

$$T(f) = \lim_\pi TE_\pi(f) = \lim_\pi \int_\Omega f g_\pi d\mu = \int_\Omega f g d\mu .$$

Conversely, suppose that $g: \Omega \to K(X, Y)$ is a bounded measurable function such that there is a compact set $C \subset Y$ with $g(\omega)x$ in C for almost all ω in Ω and all x in X with $\|x\| \leqq 1$. Without loss of generality, we may assume $g(\omega)x$ is in C for all ω in Ω. Define

$$T(f) = \int_\Omega f g d\mu$$

for $f \in L_1(\mu, X)$. Another appeal to [5, p. 283] shows $\|T\| = \|g\|_\infty$. Let

$$f = \sum_{i=1}^n x_i \chi_{E_i}$$

be a simple function in $L_1(\mu, X)$ with $\|f\| \leqq 1$ i.e.,

$$\sum_{i=1}^n \|x_i\| \mu E_i \leqq 1 .$$

Then

$$\begin{aligned} T(f) &= \int_\Omega g f d\mu = \sum_{i=1}^n \int_{E_i} g(\omega) x_i d\mu(\omega) \\ &= \sum_{i=1}^n \|x_i\| \mu E_i \cdot \frac{1}{\mu E_i} \int_{E_i} g(\omega) \frac{x_i}{\|x_i\|} d\mu \end{aligned}$$

is in $\overline{\text{co}}\, C$ by [4, p. 48]. Since $\overline{\text{co}}\, C$ is compact by Mazur's theorem, the operator T is compact. This completes the proof.

That X^* has the Radon-Nikodym property is necessary as well as sufficient for the first part of the above proof. Indeed, if each T in $K(L_1(\mu, X), Y)$ is representable by a Bochner integrable g in $L_\infty(\mu, K(X, Y))$, then taking Y to be the scalars shows that $L_1(\mu, X)^* = L_\infty(\mu, X^*)$ which implies [4, p. 98] that X^* has the RNP. An immediate consequence of Theorem 2 is a Radon-Nikodym theorem for certain operator valued measures.

COROLLARY 3. *Let X^* have the* RNP *and let $G: \Sigma \to K(X, Y)$ be a μ-continuous vector measure of bounded variation. If, for each E_1 in Σ with $\mu E_1 > 0$, there exists E_2 in Σ with $E_2 \subseteqq E_1$ and $\mu(E_2) > 0$ such that*

$$\left\{\frac{G(E)x}{\mu(E)}: x \in X,\ E \in \Sigma,\ E \subseteqq E_2,\ \mu(E) > 0,\ \|x\| \leqq 1\right\}$$

is relatively norm compact, then there exists a Bochner integrable $g: \Omega \to K(X, Y)$ such that

$$G(E) = \int_E g d\mu$$

for each E in Σ.

Proof. By exhaustion [4, p. 70], the corollary is established if for each E_1 in Σ with $\mu(E_1) > 0$ we can find E_2 in Σ with $E_2 \subseteqq E_1$ and $\mu E_2 > 0$ and a Bochner integrable g such that

$$G(E) = \int_E g d\mu$$

for all E in Σ with $E \subseteqq E_2$. So let $E_1 \in \Sigma$ with $\mu(E_1) > 0$ and select the $E_2 \subseteqq E_1$ guaranteed by the hypothesis. Define an operator T on the simple functions in $L_1(\mu, X)$ by

$$T(f) = \sum_{i=1}^n G(A_1 \cap E_2)x_i \quad \text{if} \quad f = \sum_{i=1}^n x_i \chi_{A_i},\ A_i \quad \text{in} \quad \Sigma,\ A_i \cap A_j = \phi$$

if $i \neq j$. Notice that if $\|f\| \leqq 1$

$$\sum_{i=1}^n \|x_i\| \mu A_i \leqq 1\ ,$$

then

$$\sum_{i=1}^n \|x_i\| \mu(A_i \cap E_2) \leqq 1$$

and so

$$T(f) = \sum_{i=1}^{n} ||x_i|| \mu(A_i \cap E_2) \cdot \frac{G(A_i \cap E_2)\dfrac{x_i}{||x_i||}}{\mu(A_i \cap E_2)}$$

is in

$$\overline{\text{co}} \left\{ \frac{G(E)x}{\mu E} : x \in X,\ E \in \Sigma,\ E \subseteqq E_2,\ \mu(E) > 0,\ ||x|| \leqq 1 \right\},$$

a set which is compact by Mazur's theorem. Thus T has a compact linear extension to all of $L_1(\mu, X)$. Hence, by Theorem 2, there exists a Bochner integrable $g: \Omega \to K(X, Y)$ such that

$$T(f) = \int_\Omega fgd\mu$$

for all $f \in L_1(\mu, X)$. In particular, if E is in Σ and $E \subseteqq E_2$, then

$$G(E)x = T(x\chi_E) = \int_E gxd\mu .$$

Since g is Bochner integrable, we have, by [4, p. 47], that

$$G(E) = \int_E gd\mu$$

as required.

Our next result is a generalization of a theorem of D. R. Lewis [4, p. 88] dealing with the equivalence of weakly measurable and measurable functions. The proof uses the following result of Amir and Lindenstrauss [1, p. 43]: If X is a weakly compactly generated space and $X_0 \subseteqq X$ and $Y_0 \subseteqq X^*$ are separable subspaces, then there is a bounded projection $P: X \to X$ with separable range such that $X_0 \subseteqq P(X)$ and $Y_0 \subseteqq P^*(X^*)$.

PROPOSITION 4. *Let X^* and Y be weakly compactly generated Banach spaces. If $f: \Omega \to K(X, Y)$ is a bounded function such that for each y^* in Y^* the function $y^*f(\cdot): \Omega \to X^*$ is measurable, then there is a bounded measurable function $g: \Omega \to K(X, Y)$ such that for each y^* in Y^*, $y^*f(\cdot) = y^*g(\cdot)\mu$-a.e., (the exceptional set may depend on y^*).*

Proof. We claim that the set $A = \{y^*f(\cdot): y^* \in Y^*, ||y^*|| \leqq 1\}$ is compact in $L_1(\mu, X^*)$. If not, then there is a sequence y_n^* in the unit ball of Y^* and $\delta > 0$ such that

$$||y_n^*f(\cdot) - y_m^*f(\cdot)||_{L_1(\mu, X^*)} > \delta$$

for $m \neq n$. Choose a bounded projection $P_1: Y \to Y$ with separable range such that $P_1^* y_n^* = y_n^*$ for all n. Since each $y_n^* f(\cdot): \Omega \to X^*$ is measurable and hence essentially separably valued, there is a bounded projection $P_2: X^* \to X^*$ with separable range and sets Ω_n in Σ with $\mu(\Omega \backslash \Omega_n) = 0$ and $y_n^* f(\Omega_n) \subseteqq P_2(X^*)$ for every n. Now, since each $f(\omega)$ is a compact operator we have, for all x^{**} in X^{**}, that $f(\omega)^{**} x^{**}$ is in the natural image of Y in Y^{**} and so we may define $h: \Omega \to K(X^{**}, Y)$ by $h(\omega) x^{**} = P_1 f(\omega)^{**} P_2^* x^{**}$. We claim that for each x^{**} in X^{**}, the function $h(\cdot) x^{**}: \Omega \to Y$ is measurable. To see this, note that since P_1 has separable range, the functions $h(\cdot) x^{**}$ are separably valued and since

$$y^* h(\cdot) x^{**} = y^* P_1 f(\cdot)^{**} P_2^* x^{**} = x^{**} P_2 f(\cdot)^* P_1^* y^*$$

and each $f(\cdot) P_1 y^*: \Omega \to X^*$ is measurable, the functions $h(\cdot) x^{**}$ are weakly measurable. An appeal to the Pettis measurability theorem [4, p. 42] establishes the measurability of $h(\cdot) x^{**}$. Now if Y_0 is the Banach space obtained by taking the closed linear span of $P_1 Y$ in Y, then Y_0 is separable and h can be viewed as taking its values in $K(X^{**}, Y_0)$. Moreover, if we define $S: Y \to Y_0$ by $Sy = P_1 y$, then $h(\omega) x^{**} = S P_1 f(\omega)^{**} P_2^* x^{**}$. Thus, if y_0^* is in Y_0^*, then $h(\omega)^* y_0^* = P_2^{**} f(\omega)^{**} P_1^* S^* y_0^*$ is in $P_2 X^*$, since the range of $f(\omega)^{***}$ is in X^* and P_2^{**} extends P_2. Let $Z = \overline{P_2 X^*}$ and $B = \{T: T \text{ in } K(X^{**}, Y_0), T^* Y_0^* \subset Z\}$. We claim that B is separable. To see this, let U and V denote the closed unit balls of Z^* and Y_0^* endowed with the weak* topologies. Since Y_0 and Z are separable, U and V are compact metric spaces, and thus, so is $U \times V$. For each T in B, define a function JT on $U \times V$ by $JT(u, v) = u T^* v$. Then the map $T \to JT$ is a linear isometry of B into $C(U \times V)$ [8] and so, by [7, p. 437], B is separable. Since the values of h in $K(X^{**}, Y_0)$ lie in B and $\|h(\omega_1) - h(\omega_2)\|_{K(X^{**}, Y)} = \|h(\omega_1) - h(\omega_2)\|_{K(X^{**}, Y_0)}$ for all ω_1, ω_2 in Ω, the values of h in $K(X^{**}, Y)$ form a separable set. Now because $h(\cdot) x^{**}$ is measurable for each x^{**} in X^{**}, an appeal to [5, p. 102] establishes that h is measurable. Since h is bounded, h is Bochner integrable and so we may choose a sequence h_n of $K(X^{**}, Y)$-valued simple functions such that

$$\lim_n \int_\Omega \|h - h_n\| d\mu = 0 .$$

Define operators S_n and S from $L_\infty(\mu, X^{**})$ to Y by

$$S_n(g) = \int_\Omega g h_n d\mu \quad \text{and} \quad S(g) = \int_\Omega g h d\mu$$

for g in $L_\infty(\mu, X^{**})$. Since each h_n takes on only a finite number of values, each S_n is a compact operator. Moreover, we have that

$$\|(S - S_n)(g)\| \leqq \int_\Omega \|g\| \, \|h - h_n\| d\mu \leqq \|g\|_\infty \int_\infty \|h - h_n\| d\mu$$

for all g in $L_\infty(\mu, X^{**})$. It follows immediately that the operator S is compact. The adjoint of S is the operator $y^* \to y^*h(\cdot)$ and hence by Schauder's theorem is also compact. But $y_n^*h(\cdot)=y_n^*f(\cdot)$ a.e. This contradicts

$$\|y_n^*f(\cdot) - y_m^*f(\cdot)\|_{L_1(\mu, X^*)} > \delta$$

for $m \neq n$ and establishes that the set A is compact.

Now choose y_n^* in Y^* such that $y_n^*(\cdot)$ is dense in A. If h is constructed as above for this sequence (y_n^*), then h is measurable and so, by Egoroff's theorem, for all $\delta > 0$ there is a set E in Σ with $\mu(\Omega \backslash E) < \delta$ such that $h\chi_E$ can be approximated uniformly by simple functions. Fix $\delta > 0$ and choose such a set E. It follows that the sequence $y_n^*f(\cdot)\chi_E = y_n^*h(\cdot)\chi_E$ is relatively compact in $L_\infty(\mu, X^*)$. Since this sequence is $L_\infty(\mu, X^*)$-dense in $\{y^*f(\cdot)_{\chi_E}: \|y^*\| \leqq 1\}$, this set is relatively compact in $L_\infty(\mu, X^*)$.

Now define $T: Y^* \to L_\infty(\mu, X^*)$ by $Ty^* = y^*f(\cdot)_{\chi_E}$. Then T is compact and as an operator on $L_1(\mu, X)$, $T^*: L_1(\mu, X) \to Y^{**}$ is compact. Notice that the dominated convergence theorem ensures that T is w^* to w^* sequentially continuous. Thus, if y^{**} is in $T^*(L_1(\mu, X))$, then y^{**} is a weak* sequentially continuous functional on Y^*. But since Y is weakly compactly generated, this means y^{**} is a w^* continuous functional on Y^* [3, p. 148]. Hence, $T^*(L_1(\mu, X))$ is contained in Y. Theorem 2 now produces a Bochner integrable $g: E \to K(X, Y)$ such that

$$T^*(k) = \int_E kg d\mu$$

for all k in $L_1(\mu, X)$. But, if y^* is in Y^*, then $T^{**}y^* = y^*g$. It follows that $y^*g = y^*f$ a.e. on E. Since $\mu(\Omega \backslash E) < \delta$, this completes the proof.

Theorem 2 does not hold for weakly compact operators. To see this, let Ω be the unit interval endowed with Lebesgue measure and let $r_n(\cdot)$ be the nth Rademacher function i.e., $r_n(\omega) = \text{signum}(\sin 2^n\pi\omega)$. Consider the function $g: [0, 1] \to L(\ell_2, \ell_2)$ defined by $g(\omega)(\alpha_n) = (r_n(\omega)\alpha_n)$ for all $(\alpha_n) \in \ell_2$. The function g is not essentially separably valued, since if ω_1 and ω_2 are different numbers in $[0, 1]$ there exists a Rademacher function r_n with $|r_n(\omega_1) - r_n(\omega_2)| = 2$ and hence, $\|g(\omega_1) - g(\omega_2)\|_{L(\ell_2, \ell_2)} \geqq 2$. Thus, g is not measurable. Define an operator $T: L_1(\mu, \ell_2) \to \ell_2$ by

$$T(f) = \int_{[0,1]} fg d\mu$$

and note that T is weakly compact. If T were representable by a kernel, then that kernel would be equal to g a.e. and so g would be measurable, which is a contradiction. However, we can use Proposition 4 to obtain a representation theorem for weakly compact operators by imposing further conditions on X^*.

THEOREM 5. *Let X^* be a separable Schur space. Then there is an isometric isomorphism between the space of weakly compact operators $W(L_1(\mu, X), Y)$ and the subspace of $L_\infty(\mu, W(X, Y))$ consisting of those functions whose essential range is in the uniformly weakly compact operators. In fact, T in $W(L_1(\mu, X, Y))$ and g in $L_\infty(\mu, W(X, Y))$ are in correspondence if, and only if,*

$$T(f) = \int_\Omega fg d\mu$$

for all f in $L_1(\mu, X)$.

Proof. Let T be in $W(L_1(\mu, X), Y)$. By the Factorization Lemma [2, p. 314], there is a reflexive space R and operators $S: L_1(\mu, X) \to R$ and $J: R \to Y$ such that $T = JS$. Suppose S is representable by a measurable kernel $h: \Omega \to L(X, R)$. Then T is representable by the measurable kernel $g: \Omega \to L(X, Y)$ given by $g(\omega)x = Jh(\omega)x$ for all x in X and ω in Ω. Hence, without loss of generality, we may assume that Y is reflexive.

Let $G: \Sigma \to L(X, Y)$ be the representing measure of T i.e.,

(i) $G(E)x = T(x\chi_E)$ for all x in X and E in Σ

(ii) $T(f) = \int_\Omega f dG$ for all f in $L_1(\mu, X)$ and

(iii) $$\|T\| = \sup_{\mu E > 0} \frac{\|G(E)\|}{\mu E}.$$

An appeal to [10, p. 345] produces a bounded function $g: \Omega \to L(X, Y)$ such that

(1) $g(\cdot)x: \Omega \to Y$ is Bochner integrable for all x in X and

(2) $G(E)x = \int_E g(\omega)x d\mu(\omega)$ for all x in X and E in Σ.

It follows quickly from the density of simple functions in $L_1(\mu, X)$ that

$$T(f) = \int_\Omega g f d\mu$$

for all f in $L_1(\mu, X)$. Consider, for each y^* in Y^*, the functions $y^*g(\cdot): \Omega \to X^*$. Since these functions are separably valued and weak* measurable, they are measurable by [4, p. 42]. Now $L(X, Y) = K(X, Y)$, since X^* is a Schur space and Y is reflexive. Consequently, Proposition 4 now produces a bounded measurable $h: \Omega \to K(X, Y)$

such that, for each y^* in Y^*, $y^*g(\cdot) = y^*h(\cdot)\mu$-a.e. Thus, for all y^* in Y^* and f in $L_1(\mu, X)$ we have that

$$\begin{aligned}\langle y^*, Tf\rangle &= \int_\Omega \langle y^*, g(\omega)f(\omega)\rangle d\mu(\omega)\\ &= \int_\Omega \langle y^*, h(\omega)f(\omega)\rangle d\mu\\ &= y^*\Big(\int_\Omega hfd\mu\Big)\end{aligned}$$

and so

$$T(f) = \int_\Omega hfd\mu\ .$$

It follows easily that

$$h(\omega)x \subseteqq \overline{T\{f\colon f \text{ in } L_1(\mu, X), \|f\|_1 \leqq 1\}}$$

for almost all ω in Ω and all x in X with $\|x\| \leqq 1$. Hence, the essential range of h consists of uniformly weakly compact operators.

The converse is proved in the same way as in Theorem 2 so we omit the proof.

Our final result follows from Theorem 5 in the same way that Corollary 3 follows from Theorem 2 so the proof is omitted.

COROLLARY 6. *Let X^* be a separable Schur space and let $G\colon \Sigma \to K(X; Y)$ be a μ-continuous vector measure of bounded variation. If, for each E_1 in Σ with $\mu E_1 > 0$, there exists an E_2 in Σ with $E_2 \subseteqq E_1$ and $\mu(E_2) > 0$ such that*

$$\left\{\frac{G(E)x}{\mu E}\colon x \text{ in } X, E \text{ in } \Sigma, E \subseteqq E_2, \mu E > 0, \|x\| \leqq 1\right\}$$

is relatively weakly compact, then there exists a Bochner integrable $g\colon \Omega \to K(X, Y)$ such that

$$G(E) = \int_E gd\mu$$

for each E in Σ.

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