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DOUBLE TANGENT BALL EMBEDDINGS OF CURVES IN E^3

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An arc or curve J in E^3 has congruent double tangent balls if there exists a positive number δ such that for each $p \in J$, there are two three-dimensional balls B and B', each with radius δ, such that $\{p\} = B \cap B' = (B \cup B') \cap J$. Such an arc or simple closed curve is shown to be tamely embedded in E^3. An example is given to show that the "uniform" radii are required for this conclusion and to show the necessity of having two tangent balls at each point rather than just one. The proof applies as well to show that any subset of E^3 having these congruent double tangent balls must locally lie on a tame 2-sphere.

1. Introduction. Questions about tameness of a 2-sphere in E^3 when the sphere has double tangent balls apparently originated with R. H. Bing **[1]**. Bothe **[2]** and I **[7]** independently showed that such a 2-sphere is tamely embedded in E^3, and, more recently Daverman, Wright and I **[4]**, **[8]** showed the existence of wild $(n-1)$-spheres in E^n having double tangent balls for each $n > 3$.

Let J be a subset of E^3, and let $p \in J$. Then J is said to have δ *double tangent balls* at p provided there exist a positive number δ and two 3-dimensional balls B and B' each of radius δ such that $B \cap B' = \{p\} = (B \cup B') \cap J$. If there exists a positive δ such that J has these δ double tangent balls at each of its points, then J is said to have *congruent double tangent balls*. When J is a 2-sphere one may also require that the interiors of the double tangent balls lie in different components of $E^3 - J$, as was done in the previous studies **[2]**, **[3]**, **[4]**, **[6]**, **[7]**, **[8]**. However this paper concentrates on curves in E^3 where no such restriction can be imposed because the curves are not assumed to be subsets of spheres. It should also be noted that the global uniformity (congruence) of the tangent balls over J was not part of Bing's question, although Griffith **[6]** did answer the question for 2-spheres in E^3 with this extra hypothesis. In fact, an $(n-1)$-sphere in E^n is tame when it has congruent double tangent balls on opposite sides of the sphere at each of its points **[4]**.

It is natural to wonder whether an arc J in E^3 is tame with the weaker hypothesis that it have double (but not necessarily congruent) tangent balls or that it have congruent (but not necessarily double) balls tangent to J at each of its points. After all, either of these two weaker conditions implies the tameness of a 2-sphere in E^3 (**[2]**, **[7]**, **[5]**), provided the balls are in the appropriate complementary domains of the sphere. Wild arcs in E^3 having double tangent balls at each of their points are easy to construct from known examples. Such an arc F can be constructed to also satisfy the condition that a $\delta > 0$ exists such that, for each $p \in F$, there is a ball B_p of radius δ intersecting F precisely at p. It follows that neither hypothesis can be removed from the tameness theorem. The example F is easy to construct provided one knows how to adjust the Fox-Artin arc A (see Figure 4 of [3]) so that it lies in a three page book B (the product of triod and an interval). Persinger [9, p. 171] describes this adjustment and attributes it to Posey [10]. First one obtains a regular projection of A into the xy-plane with all the double points lying on the x-axis, and then each of the overcrossings is lifted into the xz-coordinate plane. From A, F is obtained as $h(A)$ where h is a space homeomorphism taking B onto a 3-page book with cylindrical pages (Figure 1 shows $h(B)$).

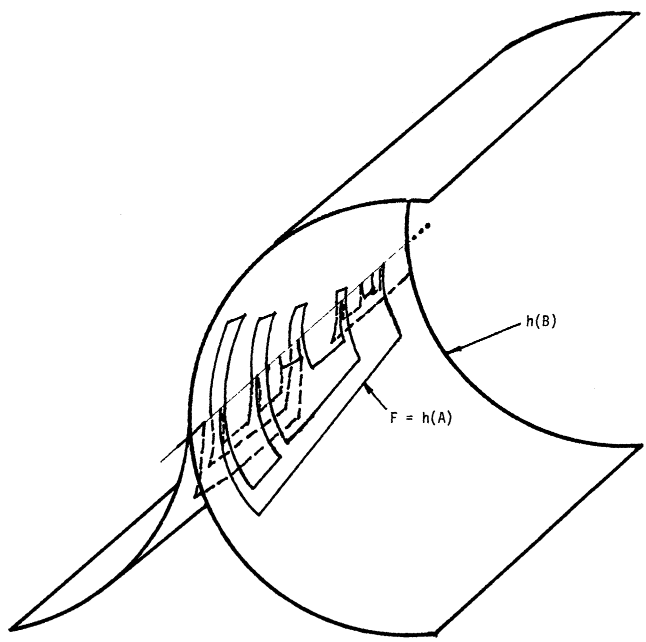

FIGURE 1

2. Tameness of curves in E^3 with congruent double tangent balls. The focus of this paper is a proof that a simple closed curve J embedded in E^3 so that it has congruent double tangent balls is tamely embedded. Let δ be a positive number smaller than the hypothesized common radius of the congruent double tangent balls to J. For each $p \in J$, define $\mathfrak{B}_p$ to be the set such that a ball B belongs to $\mathfrak{B}_p$ if and only if B has radius δ and there exists a ball B' distinct from B such that B' also has radius δ, $p \in B \cap B'$, and $(\operatorname{Int} B \cup \operatorname{Int} B') \cap J = \varnothing$. Define $\mathfrak{B} = \{\mathfrak{B}_p \mid p \in J\}$, and notice that both $\bigcup \mathfrak{B}_p$ and $\bigcup(\bigcup \mathfrak{B})$ are nonempty closed point sets.

For each $p \in J$, let D_p be the 2-sphere of radius δ centered at p, and let C_p be the set of centers of all balls in $\mathfrak{B}_p$. Then C_p is a closed proper subset of D_p; the next lemma describes more precisely the location of C_p in D_p.

LEMMA 2.1. *If J is a simple closed curve in E^3, $p \in J$, and J has congruent double tangent balls at p, then C_p lies in an equator of the sphere D_p.*

Proof. Let $\{p_i\}$ be a sequence of points of J converging to p, and, for each i, choose r_i as the point of D_p on the ray from p through p_i. Let r be a point to which some subsequence of $\{r_i\}$ converges, and define H_p as the closed hemisphere of D_p farthest from r. If there were a point n of $C_p - H_p$, the ball B centered at n and tangent to J at p would contain some point p_i in its interior, which would contradict the condition that $J \cap \operatorname{Int} B = \varnothing$. Thus $C_p \subset H_p$, and, from the fact that C_p contains its reflection in p, it is clear that C_p lies in the great circle $\operatorname{Bd} H_p$.

The points of J are divided into two classes according to whether C_p actually is an equator of D_p or merely a proper subset of whatever equators contain it. Let

$$E = \{p \in J \mid \text{there exists an equator } E_p \text{ of } D_p \text{ containing } C_p \text{ such that } E_p \neq C_p\} \text{ and}$$

$$F = \{p \in J \mid C_p \text{ is an equator of } D_p\}.$$

From Lemma 2.1, $J = E \cup F$.

LEMMA 2.2. *If J is a simple closed curve embedded in E^3 so as to have congruent double tangent balls and p belongs to the set E defined above, then some neighborhood of p in J lies on the boundary of a tame 3-cell. Thus J is locally tame at each point of E.*

Proof. Since C_p is closed it follows from the definition of E that there is an equator E_p of D_p containing C_p and an arc A in E_p such that A contains no pair of antipodal points of D_p and, for each tangent ball pair in $\mathfrak{B}_p$, one of the two centers lies in A. These properties of A insure that the intersection of all balls from $\mathfrak{B}_p$, whose centers lie in A, is a convex 3-cell X. Let x be a point in the interior of X, and let B^* be the union of all balls from $\bigcup \mathfrak{B}$ that contain x. Since $\bigcup(\bigcup \mathfrak{B})$ is closed it follows that $\operatorname{Bd} B^*$ contains an open subset U of J with $p \in U$. Furthermore, the radial map at x from $\operatorname{Bd} B^*$ to a round 2-sphere centered at x is a homeomorphism, so near p J lies on the boundary of the 3-cell B^*. Since this radial homeomorphism extends to a neighborhood of Bd B^*, Bd B^* is tamely embedded.

THEOREM 2.3. *If J is a simple closed curve embedded in E^3 so as to have congruent double tangent balls, then J is tamely embedded in E^3.*

Proof. It is convenient to scale the measurements so that the uniform radius δ of the tangent balls is 2. From Lemma 2.2, J is locally tame except possibly at points in the closed set F. Let $p \in F$, and impose a coordinate system on E^3 so that p is the origin and E_p, the equator of centers of tangent balls at p, is the horizontal circle $x^2 + y^2 = 4$, $z = 0$. For each t in the vertical interval $[-2, 2]$, the horizontal plane P_t, defined by $z = t$, intersects $\bigcup \mathfrak{B}_p$. Define G as $\{(x, y, z) \mid x^2 + y^2 \leq 4 \text{ and } |z| < 1\} - \bigcup \mathfrak{B}_p$ so that G is the union of two congruent, open, trumpet-shaped 3-cells each with p in its closure. For each t such that $0 < |t| < 1$, let G_t denote the open circular 2-cell $G \cap P_t$, let $G_0 = \{p\}$, and let A be an arc in J such that p lies in its interior and $A \subset G \cup \{p\}$. Then J is locally tame at p if and only if A is locally tame at p.

The strategy is much like that in [5]; A is "unwound" with a space homeomorphism h so that the orthogonal projection of $h(A)$ into the yz-plane has no multiple points. A straight line L_t in P_t is called a *projective line* if no line in P_t parallel to L_t contains two points of A. Through a sequence of alphabetically named lemmas I shall identify suitable restrictions on a vertical interval $[-u, u]$ and on A to insure the existence of a family $\{L_t \mid -u \leq t \leq u\}$ of projective lines whose directions are continuous except at a certain closed 0-dimensional subset of $[-u, u]$. This family will provide a means for rotating various levels to obtain the desired homeomorphism h.

LEMMA A. *There exists $u_1 \in (0, 1]$ such that, for any ball B in $\bigcup \mathfrak{B}$ containing a point of $A \cap G_t$, $|t| < u_1$, the circular 2-cell $B \cap P_t$ has radius at least* 1.

Since balls from $\mathfrak{B}_p$ intersect P_0 in circular 2-cells of radius 2, Lemma A follows easily from the fact that $\bigcup(\bigcup \mathfrak{B})$ is closed.

If $|t| < u_1$, $q \in G_t \cap A$, and B and B' are tangent balls from $\mathfrak{B}_q$, it follows from Lemma A that $B \cap P_t$ and $B' \cap P_t$ contain unique tangent circular disks D and D', respectively, of radius 1 such that $D \cap D' = \{q\}$. The line through the centers of D and D' is called a *normal line* at q. There may be infinitely many of these normal lines at the point q but they all lie in P_t and are in a one-to-one correspondence with the set of such tangent unit disks coming from tangent balls of $\mathfrak{B}_q$.

If L and L' are two intersecting lines in E^3, $\theta(L, L')$ denotes the radian measure of the small angle between L and L'. If L and L' are coplanar and do not intersect, $\theta(L, L') = 0$.

LEMMA B. *There exists $u_2 \in (0, u_1)$ such that if $|t| < u_2$ and P_t contains normal lines N_x and N_y at two distinct points x and y of G_t, then $\theta(N_x, N_y) < \pi/36$.*

Proof. Choose u_2 such that $0 < u_2 < u_1$ and such that if c is any point of $P_t(|t| < u_2)$ at a distance 1 or more from some point of G_t, then G_t subtends an angle less than $\pi/36$ at c. Let $|t| < u_2$, let x and y be two points of $G_t \cap A$, and let N_x and N_y be normal lines in P_t at x and y, respectively. Suppose $\theta(N_x, N_y) \geq \pi/36$, and let c_y and c_y' be the centers of the two tangent unit disks at y relative to which N_y is defined. Let c_x and c_x' be the analogous two centers on N_x. The definition of u_2 insures $\sphericalangle\, yc_yx$ and $\sphericalangle\, yc_y'x$ have measures less than $\pi/36$, so it follows from the above supposition that N_x intersects N_y at a point b between c_y' and c_y on N_y. Thus $d(b, y) < 1$, and, analogously, $d(b, x) < 1$. Assume for convenience that $d(b, x) \leq d(b, y)$ so that x lies in the interior of the disk D of radius $d(b, y)$ centered at b. But D lies in one of the unit disks centered at c_y and c_y' both of which lie in tangent balls from $\bigcup \mathfrak{B}$. This yields the contradiction that a ball from $\bigcup \mathfrak{B}$ contains the point x of J in its interior.

LEMMA C. *If $|t| \leq u_2$, $q \in G_t \cap A$, N_q is a normal line at q in P_t, and L is a line in P_t such that $\theta(N_q, L) \leq \pi/3$, then L is a projective line.*

Proof. The conclusion is clear if $G_t \cap A = \{q\}$; otherwise let L' be a line in P_t parallel to L such that L' contains a point x of $G_t \cap (A - \{q\})$, and let N_x be a normal line at x in P_t. By Lemma B, $\theta(N_x, N_q) < \pi/36$, and from the hypotheses, $\theta(N_x, L') < \pi/36 + \pi/3 = 13\pi/36$. The normal N_x contains the centers c and c' of the two unit disks D and D' used to define N_x. Because of the condition on $\theta(N_x, L')$ above, the chord $L' \cap D$ subtends an angle at c of measure larger than $\pi - 2(13\pi/36) > \pi/36$. Then by the definition of u_2 in the first line of the proof of Lemma B, $L' \cap G_t$ lies in $D \cup D'$. Therefore $L' \cap A = \{x\}$, and L is projective.

Each point q of $G_t \cap F$, where $t \in [-u_2, u_2]$, has the property that each horizontal line through q is a normal line at q. As a consequence there exists a circular horizonal neighborhood Q of q lying entirely in the union of the corresponding unit tangent disks; that is $Q \cap A = \{q\}$. More generally such a neighborhood exists when there are two distinct normal lines through a point q, the maximum radius of the neighborhood depends on the angle between the two normals. Define $S = \{q \in A \mid$ there exist two normal lines N and N' at q such that $\theta(N, N') \geq \pi/36\}$. Elementary trigonometry reveals that, for each $q \in S$, there is a circular horizontal neighborhood Q_q of q lying entirely in the union of four balls in $\mathcal{B}_q$ such that Q_q is centered at q and has radius $r_0 = 2\sin\pi/72$. Define u_3 to be small enough that $u_3 < u_2$ and the diameter of G_t is less than r_0 when $|t| < u_3$. Then $G_t \subset Q_q$ if $q \in S$ and $|t| < u_3$. Define T to be $\{t \in [-u_3, u_3] \mid G_t \cap S \neq \varnothing\}$, and notice that $G_t \cap A$ is a singleton set if $t \in T$. Furthermore, if $t \in ([-u_3, u_3] - T)$ and N and N' are any two normals at a point q of $A \cap G_t$, then $\theta(N, N') < \pi/36$.

A family $\{L_w \mid w \in (r, s)\}$ of lines L_w in P_w is said to be *continuous* provided $\{L_{t_i}\}$ converges to L_t whenever $\{t_i\}$ converges to t in (r, s).

LEMMA D. *If* $|t| < u_3$, $t \notin T$, *and* $P_t \cap A \neq \varnothing$, *then there exists an open interval* (r, s) *containing* t, *a continuous family* $\{L_w \mid w \in (r, s)$ *and* $P_w \cap A \neq \varnothing\}$ *of projective lines, and a corresponding family* $\{N_w\}$ *of normal lines such that, whenever* $w \in (r, s)$ *and* $P_w \cap A \neq \varnothing$, $L_w \cup N_w \subset P_w$ *and* $\theta(L_w, N_w) < \pi/12$.

Proof. Let q be a point of $A \cap P_t$. Suppose there exists a sequence $\{t_i\}$ converging to t such that each P_{t_i} contains a normal line N_i such that $\theta(N_t, \omega(N_i)) \geq \pi/12$ where ω denotes the vertical projection of E^3 onto P_t and N_t is any normal line at q in P_t. Then the limiting set of $\{N_i\}$ in P_t contains a normal line N such that $\theta(N, N_t) \geq \pi/12$. If N is not a normal line at q, then Lemma B is contradicted. On the other hand $t \notin T$ so $q \notin S$ and it follows that N and N_t cannot both be normal at q. Since no such sequence $\{t_i\}$ exists there must exist an interval (r, s) containing t and a family of normal lines $\{N_w \mid w \in (r, s)$ and $P_w \cap A \neq \varnothing\}$ such that $N_w \subset P_w$ and $\theta(\omega(N_w), N_t) < \pi/12$ for each appropriate w in (r, s). For each $w \in (r, s)$ define $L_w = \omega^{-1}(N_t) \cap P_w$, and notice that $\theta(L_w, N_w) < \pi/12$. From Lemma C it is clear that each L_w is a projective line, and the lemma follows.

LEMMA E. *There exist an arc* A *in* J *and a homeomorphism* h *of* E^3 *onto itself such that* $p \in \operatorname{Int} A$ *and the orthogonal projection of* $h(A)$ *into the yz-plane is injective*; *hence* J *is locally tame at* p.

Proof. It is beneficial to consider first the simplified situation where the point p of S is not a limit point of the closed set S. In this situation choose u_4 such that $0 < u_4 < u_3$ and $[-u_4, u_4] \cap T = \{0\}$; and let A be an arc in J such that $p \in \operatorname{Int} A$ and $A \cap G_t = \varnothing$ if $|t| > u_4$. For convenience in writing, assume A intersects the upper "trumpet" of G. Mark a decreasing sequence $\{s_i\}$ of points in $(0, u_4]$ converging to 0 such that each of the intervals $[s_{i+1}, s_i]$ lies in the interior of an open interval (r, s)

of the type guaranteed by Lemma D. At a point s_i where two such open intervals overlap Lemma D provides two projective lines L_{s_i} and L'_{s_i} and two normal lines N_{s_i} and N'_{s_i} such that $\theta(L_{s_i}, N_{s_i}) < \pi/12$ and $\theta(L'_{s_i}, N'_{s_i}) < \pi/12$. By Lemma B (or by the choice of u_3 if N_{s_i} and N'_{s_i} are normals at the same point), $\theta(N_{s_i}, N'_{s_i}) < \pi/36$; therefore, $\theta(L_{s_i}, L'_{s_i}) < \pi/36 + \pi/12 + \pi/12 = 7\pi/36$. The point is that any line L in P_{s_i} within $7\pi/36$ of L_{s_i} is known to be a projective line by Lemma C because $\theta(L, N_{s_i}) \leq \theta(L, L_{s_i}) + \theta(L_{s_i}, N_{s_i}) < 7\pi/36 + \pi/12 < \pi/3$. Thus one can rotate L'_{s_i} through projectives to L_{s_i}. In order to obtain a continuous family of projective lines over $(s_{i+1}, s_i) \cup (s_i, s_{i-1})$ one accomplishes this rotation through a small vertical interval containing s_i.

A continuous family $\{L_w \mid 0 < |w| < u_4\}$ of projective lines can be constructed using this procedure at each point of overlap and repeating it for $[-u_4, 0]$ if A pierces P_0 at p. It is convenient to assume each L_w intersects the z-axis. The homeomorphism h is constructed to take each horizontal plane onto itself, to be fixed on the z-axis, and to be fixed outside $\cup\{G_t \mid 0 < |t| < u_4\}$. Specifically, h isometrically rotates the various concentric circular sections of G_t to bring the segments $G_t \cap L_t$ into the vertical xz-plane. With sufficient controls to be described later, h is a space homeomorphism such that the orthogonal projection of $h(A)$ into the yz-plane is injective. This completes the special case.

The general case is much the same except the scheme above is applied to each of countably infinitely many intervals rather than to just two. Let T' be the union of all the endpoints of the nondegenerate components of T together with the points of T lying in degenerate components of T. Then T' is a closed 0-dimensional subset of $[-u_3, u_3]$. It is convenient to assume u_3 and $-u_3$ belong to T' because then each component of $[-u_3, u_3] - T'$ is an open interval. Name A as an arc in J such that $p \in \operatorname{Int} A$ and $G_t \cap A = \varnothing$ if $|t| > u_3$, and let V be a component of $[-u_3, u_3] - T'$ with endpoints r and s in T' named so that $r < s$. Since $A \in G_t$ is a point for each t in T, it follows that if $V \subset T$ there is a vertical plane P such that each $P \cap P_t$, $t \in V$, is a projective line. If $V \not\subset T$, Lemma D can be used just as in the special case to construct a continuous family $\{L_t\}$ of projective lines over V.

Let $A(V)$ denote the compact set $\cup\{A \cap G_t \mid r \leq t \leq s\}$, $A(V) \cap P_r = \{x\}$, and $A(V) \cap P_s = \{y\}$. There is a 3-cell $C(V)$ such that $C(V) \cap P_r = \{x\}$, $C(V) \cap P_s = \{y\}$, $A(V) \subset C(V)$, $\operatorname{diam} C(V) < 3 \operatorname{diam} A(V)$, and, for each $t \in V$, the horizontal section $C(V) \cap P_t$ of $C(V)$ is a circular disk containing $A(V) \cap P_t$ in its interior. Let G'_t denote the interior of the disk $C(V) \cap P_t$, for t in V. It is also important that $C(V)$ be constructed so that the 2-sphere $\operatorname{Bd} C(V)$ lies in $\{x, y\} \cup \{\cup\{\operatorname{Bd} G'_t \mid t \in V\})$ because this insures that $A(V) - \{x, y\}$ lies in the interior of $C(V)$ and that the circles $\{\operatorname{Bd} G'_t \mid t \in V\}$ and the degenerate circles $\{x\}$

and $\{y\}$ together form a closed continuous collection. To provide room to match two homeomorphisms later, let $\{G_t'' \mid t \in V\}$ be a collection of open disks arising as horizontal sections of a second 3-cell $C'(V)$ where $C'(V) \subset \{x, y\} \cup \operatorname{Int} C(V)$, $C'(V)$ is constructed to satisfy the conditions on $C(V)$, and G_t'' is concentric with G_t'. Then $A \cap P_t \subset G_t' \subset$ closure $G_t'' \subset G_t'$ for $t \in V$. For each t in V, let L_t' be the line through the center of the open disk G_t' parallel to the projective line L_t. A homeomorphism h_V, fixed on $(E^3 - \cup\{G_t' \mid t \in V\}) \cup \{x, y\} \cup$ {the centers of the disks G_t'} and taking each P_t onto itself, that rotates the open segments $\{G_t'' \cap L_t' \mid t \in V\}$ into planes parallel to the xz-plane, is obtained using the annular regions $\{G_t' - G_t'' \mid t \in V\}$ to "feather out" the rotation to the identity outside $\cup\{G_t' \mid t \in V\}$.

The desired space homeomorphism h is defined by specifying that it agree with h_V on $\cup\{P_t \mid t \in V\}$, for each component V of $[-u_3, u_3] - T'$, and that it be the identity elsewhere. The construction of $C(V)$ insures that a sequence $\{h_{V_i}\}$ of homeomorphisms, where $\{V_i\}$ is a sequence of components of $[-u_3, u_3] - T'$ converging to a point t_0 in T', must converge to the identity on P_{t_0}. As in the special case the orthogonal projection of $h(A)$ into the yz-plane is injective because of the realignment of the projective segments $\{G_t'' \cap L_t'\}$ parallel to the vertical xz-plane, and the proof is complete.

3. Generalizations to arbitrary sets in E^3.

The proof given for Theorem 2.3 actually establishes a much stronger result. For example, an arc embedded in E^3 that has congruent double tangent balls must be tamely embedded. Moreover, any subset J of E^3 that has congruent double tangent balls must locally lie on a tame 2-sphere in E^3 in the sense that each point of J lies in an open subset of J that lies in a tame 2-sphere. The set J need not be closed because its closure will have congruent double tangent balls if it does.

THEOREM 3.1. *If J is a subset of E^3 such that J has congruent double tangent balls, then J locally lies on a tame 2-sphere in E^3.*

COROLLARY 3.2. *If U is an open subset of a 2-sphere Σ in E^3 such that Σ has congruent double tangent balls over U, then Σ is locally tame at each point of U.*

The hypothesis of Corollary 3.2 does not specify that the double tangent balls at a point p of U lie on the opposite sides of Σ. In this sense Corollary 3.2 generlizes Griffith's theorem **[6]**; however, the seeming generality is illusory because my hypotheses actually imply that the tangent balls lie on opposite sides of Σ, as can be verified by the reader. I have not verified the higher dimensional analogue of Corollary 3.2 nor do I have an answer to the following question for $n > 3$.

Question 3.3. Is an $(n-2)$-sphere in E^n flat if it has congruent double tangent balls?

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