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THE ISOMETRIES OF $H^\infty(E)$

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Let E be a uniformly convex and uniformly smooth complex Banach space. We prove that every onto isometry T on $H^\infty(E)$ is of the form

$$(TF)(z) = \mathcal{T}(F(t(z))) \qquad (F \in H^\infty(E), |z| < 1),$$

where $\mathcal{T}$ is an isometry from E onto E and t is a conformal map of the unit disc onto itself.

1. Introduction. Let H^∞ denote the set of all bounded analytic functions in the open unit disc with the norm $\|f\|_\infty = \sup_{|z|<1} |f(z)|$. Since H^∞ is a semi-simple commutative Banach algebra, the Gelfand transform $(f \to \hat{f})$ is an isometry from H^∞ onto a subalgebra $\widetilde{M}$ of $C(Y)$ where Y is the maximal ideal space of H^∞. One can show **[L-R-W]**:

To every extreme point L of the unit ball $(H^\infty)^*$ there corresponds a complex number α of absolute value 1 and a point $y \in Y$ (indeed, y is an element in the Choquet boundary of Y) such that

$$Lf = \alpha\hat{f}(y) \qquad (f \in H^\infty).$$

Using this result, K. deLeeuw, W. Rudin and J. Wermer (**[L-R-W]**; also see **[N]**) proved that every linear isometry T of H^∞ onto H^∞ is of the form

$$(Tf)(z) = \alpha f(t(z)) \qquad (f \in H^\infty, |z| < 1),$$

where α is a complex number of absolute value 1 and t is a conformal mapping of the unit disc onto itself. If E is a complex Banach space, then $H^\infty(E)$ denotes the set of all E-valued bounded analytic functions defined on the open unit disk Δ. We will show that there is a linear isometry from $H^\infty(E)$ onto a subspace $\widetilde{M}$ of $C((Y, \text{weak}^* \text{ topology}) \times (U, \text{norm topology}))$ where U is the unit ball of E^*. M. Cambern **[C1]** proved that: If E is a finite dimensional complex Hilbert space, then

to every extreme point L of the unit ball of $(H^\infty)^*$ there corresponds a point y in the Choquet boundary $B \subset Y$ of H^∞ and a point e^* in *the unit sphere of* E^* *such that*

$$L(F) = \widehat{F_{e^*}}(y) \qquad (F \in H^\infty(E))$$

where $F_{e^*}(z) = \langle F(z), e^* \rangle$. Using this result, he proved that if E is a finite dimensional Hilbert space, then every isometry T of $H^\infty(E)$ onto $H^\infty(E)$ is of the form

$$(TF)(z) = \mathscr{T}(F(t(z))) \qquad (F \in H^\infty(E), |z| < 1),$$

where $\mathscr{T}$ is an isometry from E onto E, and t is a conformal map of the unit disc onto itself. (In **[C2]**, he also proved that if E is a finite dimensional complex Banach space which does not split, then the conclusion of the above result is still true.)

In §2, we study the extreme points of the unit ball of $H^\infty(E)^*$. We prove that if E is uniformly convex and uniformly smooth, then for each point $y \in B$ and each point e^* in the unit sphere of E^*,

$$L_{y, e^*}(F) = \widehat{F_{e^*}}(y)$$

is an extreme point of the unit ball of $H^\infty(E)^*$. Moreover, if $\{L_{y_d, e_d^*}\}$ converges to a nonzero element in the weak* topology, then $\{y_d\}$ converges (in the weak* topology). In §3, we use these results to show if T is an isometry from $H^\infty(E)$ onto $H^\infty(E)$, then

(i) there exist a complex number α of absolute value 1 and a conformal mapping t of the unit disc onto itself such that for any $h \in H^\infty$ and any $F \in H^\infty(E)$ $T(h \cdot F) = \alpha \cdot h \circ t \cdot T(F)$,

(ii) T maps the set of all constant functions onto itself.

Hence, there exists an isometry $\mathscr{T}$ from E onto E such that

$$(TF)(z) = \mathscr{T}(F(t(z))) \qquad (F \in H^\infty(E), |z| < 1).$$

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2. Extreme points in $H^\infty(E)^*$**.** Let E be a complex Banach space and let X be the Hausdorff space

$$(Y, \text{ weak* topology}) \times (U, \text{ norm topology})$$

(where Y is the maximal ideal space of H^∞ and U is the closed unit ball of E^*). For each $F \in H^\infty(E)$, let $\widetilde{F}$ be a function on X which is defined by

$$\widetilde{F}(y, e^*) = \langle F_{e^*}, y \rangle.$$

(Note: $F_{e^*} \in H^\infty$ is defined by $F_{e^*}(z) = \langle F(z), e^* \rangle$.)

Lemma 1. *For each $F \in H^\infty(E)$, $\widetilde{F}$ is a continuous function on X. Moreover, the mapping $S\colon F \to \widetilde{F}$ is a linear isometry from $H^\infty(E)$ into $C(X)$.*

Proof. Suppose that $\{(y_d, e_d^*)\}$ converges to (y, e^*). Then

$$\begin{aligned}
&|\widetilde{F}(y, e^*) - \widetilde{F}(y_d, e_d^*)| \\
&\quad \le |\widetilde{F}(y_d, e^*) - \widetilde{F}(y_d, e_d^*)| + |\widetilde{F}(y, e^*) - \widetilde{F}(y_d, e^*)| \\
&\quad = |\langle \widehat{F}_{e^*-e_d^*}, y_d\rangle| + |\langle \widehat{F}_{e^*}, y - y_d\rangle| \\
&\quad \le \|e^* - e_d^*\| \cdot \|F\| + |\langle \widehat{F}_{e^*}, y - y_d\rangle|.
\end{aligned}$$

Since e_d^* converges to e^* in norm and y_d converges to y in the weak* topology, $\widetilde{F}(y_d, e_d^*)$ converges to $F(y, e^*)$. It is known that $\|F\| = \sup_{e^* \in U} \|F_{e^*}\| = \|\widetilde{F}\|$, and so the mapping $F \to \widetilde{F}$ is a linear isometry. □

Remark 1. If h is an element of H^∞ and F is an element of $H^\infty(E)$, then $h \cdot F \in H^\infty(E)$ and

$$\widetilde{h \cdot F}(y, e^*) = \langle h \cdot F_{e^*}, y\rangle = \hat{h}(y) \cdot \langle F_{e^*}, y\rangle \tag{1}$$

(since y is a maximal ideal).

Remark 2. For each $(y, e^*) \in X$, let L_{y,e^*} be a linear function on $H^\infty(E)$ which is defined by $L_{y,e^*}(F) = \widetilde{F}(y, e^*)$. It is known that the weak* closed convex hull of the Choquet boundary B (for H^∞) contains Y. By the proof of Lemma V.8.6 and Lemma V.8.5 **[D-S]**, every extreme point in the unit ball of $H^\infty(E)$ is in the weak* closure of the set $\{L_{y,e^*} : y \in B \text{ and } e^* \in U\}$.

Lemma 2. *Suppose that $\{(y_d, e_d^*)\}$ is a net in $B \times U$. If $\{L_{y_d, e_d^*}\}$ converges to a nonzero element in the weak* topology, then $\{y_d\}$ converges (in the weak* topology).*

Proof. Since B is compact in the weak* topology, y_d has a limit point, say y. We claim that $\{y_d\}$ converges to y in the weak* topology. If this is not true, then there is a weak* neighborhood V of y such that for any d_0

$$\begin{gathered}
(V \times U) \cap \{L_{y_d, e_d^*} : d > d_o\} \ne \varnothing \quad \text{and} \\
((Y \setminus V) \times U) \cap \{L_{y_d, e_d^*} : d > d_0\} \ne \varnothing.
\end{gathered}$$

Since $y \in B$ and $\text{w}^*\text{-}\lim L_{y_d, e_d^*} \ne 0$, there exist $\widetilde{F} \in M$ and $h \in H^\infty$

such that

(i) $\lim \widetilde{F}(y_d, e_d^*)$ converges to a positive number,

(ii) $1 = \|h\| = \hat{h}(y)$, and $|\hat{h}| < \frac{1}{2}$ on $Y \setminus V$.

This implies that $\langle h \cdot F, L_{y_d, e_d^*}\rangle = \hat{h}(y_d) \cdot \langle F, L_{y_d, e_d^*}\rangle$ does not converge. We get a contradiction since $\{L_{y_d, e_d^*}\}$ converges in the weak* topology. □

Let e^* be an element in U. We say e^* is a w*-*strongly exposed point* (in U) if there exists a unit vector $e \in E$ such that $\{\{f^* \in U: \langle e, f^*\rangle > \alpha\}: \alpha > 0\}$ is a neighborhood base for e^* in U in the norm topology. (We also say e is w* strongly exposed at f^*.) It is known that if E is uniformly smooth, then E^* is uniformly convex and every point in the unit sphere is a w*-strongly exposed point in the unit ball.

LEMMA 3. *If e^* is a* w*-*strongly exposed point in U and $y \in B$, then L_{y, e^*} is an extreme point of the unit ball of $H^\infty(E)^*$.*

Proof. Since e^* is a w*-strongly exposed point, there exists a unit vector $e \in E$ such that

$$\forall \varepsilon > 0\ \exists \delta > 0\ (f^* \in U \text{ and } \langle e, f^*\rangle > 1 - \delta) \Rightarrow \|f^* - e^*\| < \varepsilon.$$

Suppose that $\sum_{i=1}^{n_d} a_{i,d} = 1$, $a_{i,d} \geq 0$, and w*-$\lim \sum_{i=1}^{n_d} a_{i,d} L_{y_{i,d}, e^*_{i,d}} = L_{y, e^*}$. For any $\varepsilon > 0$ and any w*-neighborhood V of y, let $A_{\varepsilon, V, d}$ denote the set

$$\{i: \|e^* - e^*_{i,d}\| > \varepsilon \text{ or } y_{i,d} \notin V\}.$$

We claim that $\lim \sum_{i \in A_{\varepsilon, V, d}} a_{i,d} = 0$. Since $y \in B$, there is an $h \in H^\infty$ such that $\langle h, y\rangle = 1 = \|h\|$ and $|\hat{h}(y)| < 1 - \delta$ on $Y \setminus V$.

$$\begin{aligned} 1 = \langle e \cdot h, L_{y, e^*}\rangle &= \lim \sum_{i=1}^{n_d} \langle e \cdot h, a_{i,d} L_{y_{i,d}, e^*_{i,d}}\rangle \\ &= \lim \left(\sum_{i \in A_{\varepsilon, V, d}} \langle e \cdot h, a_{i,d} L_{y_{i,d}, e^*_{i,d}}\rangle + \sum_{i \notin A_{\varepsilon, V, d}} \langle e \cdot h, a_{i,d} L_{y_{i,d}, e^*_{i,d}}\rangle \right) \\ &\leq \limsup \left((1 - \delta) \sum_{i \in A_{\varepsilon, V, d}} a_{i,d} + \sum_{i \notin A_{\varepsilon, V, d}} a_{i,d} \right). \end{aligned}$$

So we must have

$$\lim \sum_{i \in A_{\varepsilon, V, d}} a_{i,d} = 0.$$

Let F be any function in $H^\infty(E)$. Then for any $\varepsilon > 0$ there exists a neighborhood V of y such that if $y' \in V$ then $|\langle F_{e^*}, y\rangle - \langle F_{e^*}, y'\rangle| < \varepsilon$. So if $\sum_{i\in B_d} a_{i,d} = \frac{1}{2}$, then

$$\begin{aligned}
&\left|2\left\langle F, \sum_{i\in B_d} a_{i,d} L_{y_{i,d}, e^*_{i,d}}\right\rangle - \langle F, L_{y,e^*}\rangle\right| \\
&\quad\leq \left|\left\langle F, \sum_{i\in B_d\setminus A_{\varepsilon,V,d}} 2\cdot a_{i,d}(L_{y_{i,d}, e^*_{i,d}} - L_{y_{i,d}, e^*} + L_{y_{i,d}, e^*} - L_{y,e^*})\right\rangle\right| \\
&\qquad + \sum_{i\in B_d\cap A_{\varepsilon,V,d}} 2\cdot a_{i,d}\cdot \|F\| \\
&\quad\leq 2\varepsilon\|F\| + \sum_{i\in B_d\cap A_{\varepsilon,V,d}} 2\cdot a_{i,d}\cdot\|F\|.
\end{aligned}$$

This implies $2\cdot\sum_{i\in B_d} a_{i,d} L_{y_{i,d}, e^*_{i,d}}$ converges to L_{y,e^*} in the w* topology. So L_{y,e^*} is an extreme point of the unit ball of $H^\infty(E)^*$. □

3. The isometries. Let T denote a fixed isometry of $H^\infty(E)$ onto itself. Then T^* is an isometry on $H^\infty(E)$ and T^* maps the extreme points of the unit ball of $H^\infty(E)^*$ onto itself.

LEMMA 4 (*See Lemma* 2.1 **[C1]**). *Suppose E^* is strictly convex. Let e_1^*, e_2^* be two* w*-*strongly exposed points in U and x be any element in B. Suppose that* w*-$\lim L_{y_d, f_d^*} = T^*(L_{x,e_1^*})$ *and* w*-$\lim L_{z_d, g_d^*} = T^*(L_{x,e_2^*})$. *If* w*-$\lim z_d = y'$ *and* w*-$\lim y_d = y$, *then* $y = y'$.

Proof. If $y \neq y'$, then there exist two disjoint neighborhoods V_1, V_2 of y and y'. Since y and y' are in B, for any $0 < \varepsilon < \frac{1}{2}$ there exist h_1 and h_2 such that $1 = \|h_1\| = \hat{h}_1(y) = \hat{h}_2(y') = \|h_2\|$, $|\hat{h}_1| < \varepsilon$ on $Y\setminus V_1$, and $|\hat{h}_2| < \varepsilon$ on $Y\setminus V_2$. If $\|\widetilde{F}_1\| = 1$ (resp. $\|\widetilde{F}_2\| = 1$) and $\lim \widetilde{F}_1(y_d, f_d^*) > 1-\varepsilon$ (resp. $\lim \widetilde{F}_2(z_d, g_d^*) > 1-\varepsilon$), then

(i) $\lim \hat{h}_1(y_d)\cdot\widetilde{F}(y_d, f_d^*) > 1-\varepsilon$ and $\lim \hat{h}_2(z_d)\cdot\widetilde{F}(z_d, g_d^*) > 1-\varepsilon$,

(ii) $\|\hat{h}_1\cdot\widetilde{F}_1 + \hat{h}_2\cdot\widetilde{F}_2\| < 1+\varepsilon$.

Since ε is arbitrary, we have

$$2 = \|\text{w}^*\text{-}\lim(L_{y_d, f_d^*} + L_{z_d, g_d^*})\| = \|L_{x,e_1^*} + L_{x,e_2^*}\| = \|e_1^* + e_2^*\|.$$

This contradicts the fact E^* is strictly convex. Hence, $y = y'$. □

REMARK 3. Clearly, the conclusion of the above lemma is still true if E^* does not contain a two dimensional l_1 space. In **[C2]**, M.

Cambern showed that if E is a finite dimensional complex Banach space which does not split, then the conclusion of the above lemma is true. (Note: we say a Banach space E splits if it is the direct sum of two nonzero subspaces E_1 and E_2 with sup norm.) But we do not know whether it is still true if E is an infinite dimensional Banach space.

Suppose that E^* is strictly convex. For any $y \in B$ and any w*-strongly exposed point $e^* \in U$ if $\text{w*-}\lim L_{z_d, g_d^*} = T^*(L_{y,e^*})$, then we define

$$\varphi(y) = \text{w*-}\lim z_d. \tag{2}$$

By Lemma 4, $\varphi(y)$ is independent of the choice of the w*-strongly exposed point in U.

LEMMA 5. *φ is a continuous function.*

Proof. Let e^* be any w*-strongly exposed point in U. If y_d converges to y, then L_{y_d,e^*} converges to L_{y,e^*} in the weak* topology. Since T^* is continuous with respect to w* topology, $T^*(L_{y_d,e^*})$ converges to $T^*(L_{y,e^*})$ in the weak* topology. If $\varphi(y_d)$ does not converge to $\varphi(y)$, then there is an $h \in H^\infty$ such that $\hat{h}(\varphi(y_d))$ does not converge to $\hat{h}(\varphi(y)) = 1$. Let F be any unit vector in $H^\infty(E)$ such that $\langle F, T^*(L_{y,e^*})\rangle = 1$. Then

$$\begin{aligned} 1 = \hat{h}(\varphi(y))\langle F, T^*(L_{y,e^*})\rangle &= \langle h\cdot F, T^*(L_{y,e^*})\rangle \qquad \text{(by (2))} \\ &= \lim\langle h\cdot F, T^*(L_{y_d,e^*})\rangle = \lim \hat{h}(\varphi(y_d))\langle F, T^*(L_{y_d,e^*})\rangle. \end{aligned}$$

But $\{\langle F, T^*(L_{y_d,e^*})\rangle\}$ converges to $\langle F, T^*(L_{y,e^*})\rangle = 1$, so this is impossible. Therefore, φ is a continuous function. □

REMARK 4. Similarly, one can show that if

(i) $\text{w*-}\lim y_d = y$,

(ii) $\{e_d^*\}$ is a net of w*-strongly exposed points such that $\{L_{y_d,e_d^*}\}$ converges to a nonzero element in w*-topology,

(iii) $\text{w*-}\lim L_{z_d,f_d^*} = T(\text{w*-}\lim L_{y_d,e_d^*})$,

then $\text{w*-}\lim z_d = \varphi(y)$. Hence, φ is one-to-one and onto.

LEMMA 6. *Suppose that E is uniformly smooth. There are a conformal map t of the unit disc onto itself and a complex number α*

of absolute value 1 *such that for each* $h \in H^\infty$ *and* $F \in H^\infty(E)$, $T(h \cdot F) = h \circ t \cdot T(F)$. *(So* $T^{-1}(h \cdot F) = h \circ t^{-1} \cdot T^{-1}(F)$.*)*

Proof. Since E is uniformly smooth, E^* is uniformly convex (so every point in the unit sphere is a w*-strongly exposed point in the unit ball). For any $h \in H^\infty$ and $F \in H^\infty(E)$,

$$\begin{aligned}\langle T(h \cdot F), L_{y,e^*}\rangle &= \langle h \cdot F, T^*(L_{y,e^*})\rangle = \hat{h}(\varphi(y)) \cdot \langle F, T^*(L_{y,e^*})\rangle \\ &= \hat{h}(\varphi(y)) \cdot \langle T(F), L_{y,e^*}\rangle \in \widehat{H^\infty}.\end{aligned}$$

Note. T is an onto mapping. There exists $F \in H^\infty(E)$ such that for any $y \in B$, $\langle T(F), L_{y,e^*}\rangle = 1$. This implies $\hat{h} \circ \varphi = \hat{\overline{h}}$ for some $\overline{h} \in H^\infty$. One can easily verify that

(i) $T(h \cdot F) = \overline{h} \cdot T(F)$ and $T^{-1}(\overline{h} \cdot F) = h \cdot T^{-1}(F)$,

(ii) $h \to \overline{h}$ is a linear isometry from H^∞ onto H^∞.

By the deLeeuw-Rudin-Wermer theorem, there exist a conformal t of the unit disc onto itself and a complex number α of absolute value 1 such that for each $h \in H^\infty$

$$\overline{h} = \alpha \cdot h \circ t.$$

Let h be the constant 1 function, and it is easy to see that $\alpha = 1$. So $T(h \cdot F) = h \circ t \cdot T(F)$. □

REMARK 5. Suppose that E^* is strictly convex. The above proof shows that if e^* is a w*-strongly exposed point (in U) and y is a point in B, then

$$\langle T(h \cdot F), L_{y,e^*}\rangle = \hat{h}(\varphi(y)) \cdot \langle T(F), L_{y,e^*}\rangle.$$

If w* cl(co$\{e^* : e^*$ is a w*-strongly exposed in $U\}) = U$, then for any $e^* \in U$ and any $y \in B$,

$$\langle T(h \cdot F), L_{y,e^*}\rangle = \hat{h}(\varphi(y)) \cdot \langle T(F), L_{y,e^*}\rangle.$$

So the conclusion of Lemma 6 is still true if E^* is strictly convex and E^* has RNP (**[B]** Theorem 4.2.13).

LEMMA 7. *Suppose that* E^* *is uniformly convex and* E *is complex strictly convex. If* $F \in H^\infty(E)$ *is a constant function, then* $T(F)$ *is a constant function.*

Proof. For any analytic E-valued constant function $F = e \cdot \mathbf{1}_\Delta$, let

$$T(F) = T(F)(0) \cdot \mathbf{1}_\Delta + z \cdot G.$$

Then

$$F = T^{-1}(T(F)(0) \cdot \mathbf{1}_\Delta + z \cdot G)$$
$$= T^{-1}(T(F)(0) \cdot \mathbf{1}_\Delta) + t^{-1} \cdot T^{-1}(G).$$

This implies

$$\|e\| = \|T^{-1}(T(F)(0) \cdot \mathbf{1}_\Delta)(t(0)) + t^{-1}(t(0)) \cdot T^{-1}(G)(t(0))\|$$
$$= \|T^{-1}(T(F)(0) \cdot \mathbf{1}_\Delta)(t(0))\| \leq \|T(F)(0)\|.$$

But $\|T(F)\| = \|e\|$. By Maximum Modulus Theorem **[T-W]**, $T(F)$ is a constant function (note: e is a complex strictly extreme point). □

THEOREM. *Suppose that E is a uniformly smooth and uniformly convex Banach space. Let T be any linear isometry of $H^\infty(E)$ onto $H^\infty(E)$. Then there are an isometry $\mathscr{T}$ from E onto E and a conformal map t of the unit disc onto itself such that*

$$(TF)(z) = \mathscr{T}(F(t(z))) \qquad (F \in H^\infty(E),\, |z| < 1).$$

Proof. Since T maps the set of all constant functions onto itself, there is an isometry $\mathscr{T}$ from E such that $T(e \cdot \mathbf{1}_\Delta) = \mathscr{T}(e) \cdot \mathbf{1}_\Delta$. For any unit vector $e \in E$, let f^* be a unit vector in E^* such that $\langle \mathscr{T} e, f^* \rangle = 1$. So e is w*-strongly exposed at $\mathscr{T}^*(f^*)$. Hence, if w*-$\lim L_{y_d, f_d^*} = T(L_{y, f^*})$, then f_d^* converges to $\mathscr{T}^*(f^*)$ in norm. And we have

$$T^*(L_{y, f^*}) = L_{\varphi(y), \mathscr{T}^*(f^*)}.$$

By Choquet's theorem, for each $|z| < 1$ there is a probability measure μ_z such that $\mu_z(B) = 1$ and $f(z) = \int_B \hat{f}(y)\, d\mu_z(y)$ for any $f \in H^\infty$. By the proof of Lemma 6,

$$h \circ t(z) = \overline{h}(z) = \int_B \hat{h}(\varphi(y))\, d\mu_z(y),$$

and so

$$\langle T(F(z)), f^* \rangle = \int_B \widehat{TF_{f^*}}(y)\, d\mu_z(y) = \int_B \langle TF, L_{y, f^*} \rangle\, d\mu_z(y)$$
$$= \int_B \langle F, L_{\varphi(y), \mathscr{T}^*(f^*)} \rangle\, d\mu_z(y) = \int_B \widehat{F_{\mathscr{T}^*(f^*)}}(\varphi(y))\, d\mu_z(y)$$
$$= F_{\mathscr{T}^*(f^*)}(t(z)) = \langle F(t(z)), \mathscr{T}^*(f^*) \rangle$$
$$= \langle \mathscr{T}(F(t(z))), f^* \rangle.$$

This implies that $T(f(z)) = \mathscr{T}(F(t(z)))$. □

REMARK 6. The conclusion of the above theorem is still true if E satisfies the following conditions:

(i) E^* does not contain two dimensional complex l_1 space,

(ii) the unit ball of E^* is the w*-closed convex hull of its w*-strongly exposed points,

(iii) the unit ball of E is the closed convex hull of its complex extreme points.

It is known that if E splits, then the conclusion of the theorem does not hold. But we do not know whether the converse is true or not.

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