HOMEOMORPHISMS OF 3-MANIFOLDS AND TOPOLOGICAL ENTROPY

Dedicated to Professor Itiro Tamura on his 60th birthday

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1. Introduction

The topological entropy \( h(f) \) of a self-map \( f \) of a metric space is a measure of its dynamical complexity (for the definition of topological entropy see section 2 below). In \([T]\) Thurston has shown that any homeomorphism \( f \) of a compact hyperbolic surface is isotopic to \( \varphi \) which is either periodic, pseudo-Anosov, or reducible (see also \([F-L-P]\), \([H-T]\), \([M]\)). We call \( \varphi \) Thurston’s canonical form of \( f \). In section 2 we show that \( h(\varphi) \leq h(f) \) i.e. \( \varphi \) attains the minimal entropy in its isotopy class. Hence from the dynamical viewpoint Thurston’s canonical form plays an important role (\([H]\), \([K]\), \([Smi]\)).

In this paper, we find a similar canonical form of a homeomorphism of a class of geometric 3-manifolds (for the definition and fundamental properties of geometric 3-manifolds see \([Sc]\)). We note that every self-homeomorphism of a hyperbolic 3-manifold is homotopic to a periodic one (\([Mo]\)). In the following, we consider homeomorphisms of an \( H^2 \times R, \text{SL}_2(R), \) or Nil 3-manifold \( M \).

Then our main result is:

Theorem 2. Let \( f \) be a homeomorphism of an \( H^2 \times R, \text{SL}_2(R), \) or Nil 3-manifold \( M \). Then \( f \) is homotopic to \( \varphi \) such that either:

(i) \( \varphi \) is of type periodic,
(ii) \( \varphi \) is of type pseudo-Anosov, or
(iii) there is a system \( \Sigma \) of tori in \( M \) such that \( \varphi \) is reducible by \( \Sigma \). There is a \( \varphi \)-invariant regular neighborhood \( \eta(\Sigma) \) of \( \Sigma \) such that each \( \varphi \)-component of \( M - \text{Int} \eta(\Sigma) \) satisfies (i) or (ii). Each component \( \eta(T_j) \) of \( \eta(\Sigma) \) is mapped to itself by some positive iterate \( \varphi^n \) of \( \varphi \) and \( \varphi^n|_{\eta(T_j)} \) is a twist homeomorphism.

For the definitions of the terms which appear in Theorem 2, see section 4 below. We note that if \( M \) is sufficiently large, then \( \varphi \) is isotopic to \( f \) (\([Wa]\)).

In section 5 we show that the above \( \varphi \) attains the minimal entropy in its homotopy class, and \( h(\varphi) \) is positive if and only if \( \varphi \) contains a component of type pseudo-Anosov.
2. Preliminaries

Let \( f: X \rightarrow X \) be a continuous map of a metric space \((X, d)\). In this section we recall the definition of the topological entropy \( h(f) \) of \( f \) in [Bo], and show that Thurston’s canonical form of a surface homeomorphism attains the minimal entropy in its homotopy class.

Let \( K (\subset X) \) be compact, \( \varepsilon > 0 \) a positive number, and \( n \) a positive integer. We say that \( E(\subset K) \) is \((n, \varepsilon)\)-separated, if \( x, y \in E, x \neq y \), then there is \( 0 \leq i < n \) such that \( d(f^i(x), f^i(y)) \geq \varepsilon \). Let \( s_K(n, \varepsilon) \) be the maximal cardinality of an \((n, \varepsilon)\)-separated set in \( K \). We say that \( E(\subset K) \) is \((n, \varepsilon)\)-spanning for \( K \), if \( x \in K \), then there is a \( y \in E \) such that \( d(f^i(x), f^i(y)) < \varepsilon \) for all \( i \) with \( 0 \leq i < n \). Let \( r_K(n, \varepsilon) \) be the minimal cardinality of an \((n, \varepsilon)\)-spanning set in \( K \). Let \( \delta_K(\varepsilon) = \limsup_{\varepsilon \rightarrow 0} \frac{1}{n} \log(s_K(n, \varepsilon)) \), and \( \rho_K(\varepsilon) = \limsup_{\varepsilon \rightarrow 0} \frac{1}{n} \log(r_K(n, \varepsilon)) \). Then it can be shown that \( \lim_{\varepsilon \rightarrow 0} \delta_K(\varepsilon) = \lim_{\varepsilon \rightarrow 0} \rho_K(\varepsilon) \), and we denote this value by \( h_K(f) \). Finally, we put \( h(f) = \sup \{ h_K(f) | K : \text{compact} \subset X \} \).

Theorem A. Let \( X, Y \) be compact metric spaces, \( p: X \rightarrow Y, f: X \rightarrow X, g: Y \rightarrow Y \) continuous maps such that \( f \circ p = p \circ g \). Then \( h(g) \leq h(f) \leq h(g) + \sup \{ h_{p \circ f}(g); y \in Y \} \).

Let \( F \) be a compact hyperbolic surface. A measured foliation \((\mathcal{F}, \mu)\) on \( F \) is a pair of a singular foliation on \( F \) and a transverse invariant measure \( \mu \) of \( \mathcal{F} \).
may have a finite number of singularities \( a_1, \ldots, a_n \), where

\( a_i \) is an \( r_i \)-pronged saddle with \( r_i \geq 3 \). If \( M \) has boundary, then each boundary component is a leaf of the foliation and has at least one singularity. A self-homeomorphism \( f: F \to F \) is pseudo-Anosov if there is a pair of mutually transverse measured foliations \((\mathcal{L}^s, \mu^s), (\mathcal{L}^u, \mu^u)\) and a number \( \lambda > 1 \) such that \( f \) preserves two foliations \( \mathcal{L}^s, \mathcal{L}^u \) and \( f_\ast(\mu^s) = 1/\lambda \cdot \mu^s, f_\ast(\mu^u) = \lambda \cdot \mu^u \). Then \( \lambda \) is called the expanding factor of \( f \). \( f \) is reducible by \( \Gamma \) if \( \Gamma \) is a system of mutually disjoint and non-parallel loops, each of which is non-contractible, non-peripheral and \( f(\Gamma) = \Gamma \). \( f \) is periodic if there is a positive integer \( n \) such that \( f^n = \text{id}_F \). Let \( A \) be an annulus, \( g: A \to A \) a homeomorphism, \( \overline{A} = [-1, 1] \times \mathbb{R} \) the universal cover of \( A \) where the covering translations are generated by \((x, y) \to (x, y+1)\). \( g \) is a twist homeomorphism if there is a lift \( \tilde{g}: \tilde{A} \to \tilde{A} \) of \( g \) such that \( \tilde{g}(x, y) = (\pm x, h(x, y)) \) for some map \( h \).

Then the precise statement of Thurston's result is:

**Theorem B** (Thurston [T1]). If \( f \) is a self-homeomorphism of a compact hyperbolic surface \( F \) then \( f \) is isotopic to \( \phi \) such that either:

(i) \( \phi \) is periodic,

(ii) \( \phi \) is pseudo-Anosov, or

(iii) there is a system of simple loops \( \Gamma \) on \( F \) such that \( \phi \) is reducible by \( \Gamma \).

There is a \( \phi \)-invariant regular neighborhood \( \eta(\Gamma) \). Each \( \phi \)-component of \( F - \text{Int} \eta(\Gamma) \) satisfies (i) or (ii). Each component, \( A_i \), of \( \eta(\Gamma) \) is mapped to itself by some positive iterate \( \phi^m \) of \( \phi \), and \( \phi^m \mid_{A_i} \) is a twist homeomorphism.

Then we can show:

**Proposition 2.1.** Thurston's canonical form \( \phi \) attains the minimal entropy in its homotopy class. Moreover \( h(\phi) > 0 \) if and only if \( \phi \) contains a pseudo-Anosov component.

Proof. If \( \phi \) is periodic then \( h(\phi) = 0 \) for \( h(\phi^*) = n \cdot h(\phi) \). If \( \phi \) is pseudo-Anosov then by [F-L-P] Exposé 10, \( h(\phi) > 0 \) and it attains the minimal entropy in its homotopy class. Suppose that \( \phi \) is reducible. Let \( A_1, \ldots, A_m \) be the components of \( \eta(\Gamma) \) and \( F_1, \ldots, F_n \) be the closures of the components of \( F - \bigcup A_i \). Let \( l \) be a positive integer such that \( \phi^l(A_i) = A_i \) and \( \phi^l(F_j) = F_j \). By the definition of topological entropy we see that \( h(\phi^l) = \max \{ h(\phi^l \mid_{A_i}), h(\phi^l \mid_{F_j}) \} \). It is easy to show that for each homeomorphism \( g \) of a circle we have \( h(g) = 0 \). By Theorem A we see that \( h(\phi^l \mid_{A_i}) = 0 \) for each \( i \). Hence \( h(\phi^l) = \max \{ h(\phi^l \mid_{F_j}) \} \).

If \( \phi^l \mid_{F_j} \) is periodic, then \( h(\phi^l) = 0 \). Hence \( h(\phi^l) = 0 \).

If \( \phi^l \) contains a pseudo-Anosov component, then \( h(\phi^l) = \log \lambda \), where \( \lambda \) is the expanding factor of some pseudo-Anosov component \( \phi^l \mid_{F_j} \). Since \( \chi(F_i) \)
<0, there is an essential, non-peripheral (not necessarily simple) loop \( \alpha \) on \( F_i \). Then by [F-L-P] \( \lim_{n \to \infty} \frac{1}{n} \log L(\varphi_i^n(\alpha)) = \lambda_i \), where \( L(\alpha) \) denotes the infimum of the length of loops which are homotopic to \( \alpha \). By [F-L-P] if \( g \) is homotopic to \( \varphi' \) then \( h(g) \geq \log \lambda_i \), i.e. \( \varphi' \) attains the minimal entropy in its homotopy class. Since \( h(\varphi') = 1 \cdot h(\varphi) \) we see that \( \varphi \) attains the minimal entropy in its homotopy class.

This completes the proof of Proposition 2.1.

3. Homeomorphisms of 2-dimensional orbifolds

In this section, we give a classification theorem for homeomorphisms of 2-dimensional orbifolds. We assume that the reader is familiar with [T2, §13] or [Sc, §2].

By [Sc], [T2] every singularity on a 2-dimensional orbifold is either cone, reflector line, or corner reflector. Throughout this paper we consider orbifolds whose singularities are cones. Let \( O(O' \text{ resp.}) \) be a 2-dimensional orbifold with singularities \( x_1, \ldots, x_n \) (\( n \geq 1 \)) (\( x'_1, \ldots, x'_{n'} \) (\( n' \geq 1 \)) resp.), where the cone angle of \( x_i \) (\( x'_i \) resp.) is \( 2\pi/p_i \) (\( 2\pi/p'_i \) resp.).

Let \( f \) be a map from \( O \) to \( O' \). \( f \) is called \textit{O-homeomorphism} if \( f \) satisfies:

(i) \( f \) is a topological homeomorphism,
(ii) \( f(\text{sing}(O)) = \text{sing}(O') \), where \( \text{sing}(O) \) denotes the set of the singularities of \( O \),
(iii) if \( f(x_i) = x'_i \) then \( p_i = p'_i \).

Let \( f, f': O \to O' \) be \textit{O-homeomorphisms}. \( f \) and \( f' \) are \textit{O-isotopic} if there is a topological isotopy \( F_t: O \to O' \) (\( 0 \leq t \leq 1 \)) such that each \( F_t \) is an \textit{O-homeomorphism} and \( F_0 = f, F_1 = f' \).

The definition of a measured foliation \( (\mathcal{F}, \mu) \) on \( O \) is the same as the definition for surfaces in section 2 except the fact that if \( x \) is a singularity of \( O \) then \( \mathcal{F} \) may have 1-pronged saddle at \( x \). A self-\textit{O-homeomorphism} \( f: O \to O \) is \textit{pseudo-Anosov} if there are a pair of mutually transverse measured foliations \( (\mathcal{F}, \mu), (\mathcal{F}', \mu') \) and a number \( \lambda > 1 \) such that \( f \) preserves the two foliations, and \( f_*(\mu) = 1/\lambda \cdot \mu, f_*(\mu') = \lambda \cdot \mu' \).
Let $a_1, a_2 \subset O$ be simple loops which do not meet singularities of $O$. $a_1$ and $a_2$ are parallel if $a_1 \cup a_2$ bounds an annulus which does not contain singularities. $a_1$ is peripheral if $a_1$ is parallel to a component of $\partial O$. $a_1$ is essential if $a_1$ is not peripheral and $a_1$ does not bound a disk on $O$ which contains at most one singular point.

A self-$O$-homeomorphism $f: O \to O$ is reducible by $\Gamma$ if $\Gamma$ is a system of simple loops on $O$ each of which does not meet a singular point and is essential, which are mutually disjoint and non-parallel, and $f(\Gamma) = \Gamma$.

Then we have:

**Theorem 1.** Let $O$ be a compact 2-dimensional orbifold whose (possibly empty) singular points are all cone type and $f$ a self-$O$-homeomorphism of $O$. Then $f$ is $O$-isotopic to $\phi$ such that either:

(i) $\phi$ is periodic,
(ii) $\phi$ is pseudo-Anosov, or
(iii) there is a system $\Gamma$ of simple loops on $O$ such that $\phi$ is reducible by $\Gamma$.

There is a $\phi$-invariant regular neighborhood $\eta(\Gamma)$ of $\Gamma$ such that each $\phi$-component of $O - \text{Int} \eta(\Gamma)$ satisfies (i) or (ii). Each component, $A_j$, of $\eta(\Gamma)$ is mapped to itself by some positive iterate $\phi^m$ of $\phi$ and $\phi^m|_{A_j}$ is a twist homeomorphism.

**Proof.** First, suppose that $O$ contains no singularities i.e. $O$ is a surface. In case of $\chi(O) < 0$ Theorem 1 is just Theorem B in section 2. There are four distinct compact surfaces with Euler characteristic zero, say annulus, Möbius band, Klein bottle, and torus [Sc]. It is easy to see that every homeomorphism of an annulus or a Möbius band is homotopic to a periodic one, and then is isotopic to a periodic one [E]. By Lickorish [Li] every homeomorphism of a Klein bottle is isotopic to a periodic one. Let $O$ be a torus, $A$ a $2 \times 2$ matrix representing $f_*: (O)\pi_1 \to \pi_1(O)$ for a fixed basis of $\pi_1(O)$. Then $f$ is isotopic to a reducible, periodic, or (pseudo-)Anosov map according to whether $A$ is conjugate to $\begin{pmatrix} e & n \\ 0 & e \end{pmatrix}$ ($e = \pm 1, n \neq 0$), to $\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ with $|\lambda_1| = |\lambda_2| = 1$, or to $\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ with $|\lambda_1| \neq |\lambda_2|$. In case of $\chi(O) > 0$ Theorem 1 is just Theorem B in section 2. There are three distinct compact surfaces with positive Euler characteristic say sphere, disk, and projective plane. By Smale [Sma], [F-L-P, Expose 2] every homeomorphism of them is isotopic to a
periodic one.

We suppose that $O$ has singularities $x_1, \ldots, x_n$ ($n \geq 1$) which are cone type. Let $S$ be a surface obtained from $O - \{x_1, \ldots, x_n\}$ by adding a circle to each non-compact end. By moving $f$ by an $O$-isotopy we may suppose that $f|_{O - \{x_1, \ldots, x_n\}}$ extends to $f: S \to S$.

If $\chi(S) \geq 0$, then by the above $f$ is isotopic to a periodic map $\bar{f}$. Let $\varphi: O \to O$ be the projection of $\bar{f}$. Then $\varphi$ is $O$-isotopic to $f$, and periodic.

If $\chi(S) < 0$, then by Theorem B $f$ is isotopic to Thurston's canonical form $\bar{f}$. Let $\varphi: O \to O$ be the projection of $\bar{f}$. Then $\varphi$ is $O$-isotopic to $f$, and we easily see that $\varphi$ satisfies the conclusion (i), (ii), or (iii) of Theorem 1 according to $\varphi$ is periodic, pseudo-Anosov, or reducible.

This completes the proof of Theorem 1.

4. Homeomorphisms of $H^2 \times R$, $\widehat{SL_2(R)}$ and Nil-manifolds

Throughout this section let $M$ be a compact, orientable 3-manifold with an $H^2 \times R$, $\widehat{SL_2(R)}$, or Nil structure. In this section we prove Theorem 2 and investigate some properties of homeomorphisms of type periodic and pseudo-Anosov. By [Sc] $M$ admits a Seifert fibration $\rho: M \to O$ where $O$ is a good 2-dimensional orbifold whose (possibly empty) singularities are all cones and by moving $f$ by a homotopy we may suppose that $f$ is fiber preserving. We note that this deformation can be realized by an isotopy if $M$ is sufficiently large [Wa]. Then we have an $O$-homeomorphism $\psi: O \to O$ which satisfies:

$$
\begin{array}{c}
M \\
\downarrow \rho \\
O
\end{array}
\xrightarrow{f}
\begin{array}{c}
M \\
\downarrow \rho \\
O
\end{array}
\xrightarrow{\psi}

A fiber preserving self-homeomorphism $f$ is of type periodic if $\psi$ is periodic. $f$ is of type pseudo-Anosov if $\psi$ is pseudo-Anosov. Let $F$ be a 2-sided surface properly embedded in a 3-manifold $M'$. $F$ is incompressible if $i_*: \pi_1(F) \to \pi_1(M')$ is injective. Let $E$ be a subset of a Seifert fibered manifold $S$. $E$ is saturated if $E$ is a union of fibers of $S$. $f: M \to M$ is reducible by $\Sigma$ if $\Sigma$ is a system of mutually disjoint, non-parallel incompressible tori, and $f(\Sigma) = \Sigma$. Let $q: N \to A$ be a circle bundle over an annulus $A$, $g: N \to N$ a fiber preserving homeomorphism, $\varphi: A \to A$ a map induced from $g$. $g$ is a twist homeomorphism if $\varphi$ is a twist homeomorphism.

**Lemma 4.1.** Let $M$ be an $H^2 \times R$, $\widehat{SL_2(R)}$, or Nil-manifold with a Seifert fibration, $T$ an incompressible torus in $M$. Then $T$ is isotopic to a saturated torus.

**Proof.** Assume that $T$ is not isotopic to a saturated torus. By [Sc],
Seifert fibrations on $M$ are unique up to isotopy. Then by [J] Theorem VI. 34 we have either:

(i) $M$ is a torus bundle over a circle and $T$ is a fiber of the bundle, or
(ii) $M = M_1 \cup M_2$ where $M_1 \cap M_2 = \partial M_1 = \partial M_2 = T$, and $M_i$ ($i = 1, 2$) is a twisted $I$-bundle over the Klein bottle.

Assume that (i) holds. The monodromy of $M$ is represented by a $2 \times 2$ matrix $A \in SL_2(\mathbb{Z})$. By [Sc] a torus bundle over the circle admits either an $E^3$, Nil, or Sol structure, and $M$ admits a Nil structure if and only if $A$ is conjugate to $\begin{pmatrix} \epsilon & n \\ 0 & \epsilon \end{pmatrix}$ ($\epsilon = \pm 1$, $n \neq 0$). Then we easily see that there is a Seifert fibration on $M$ such that $T$ is saturated with respect to the fibration. This contradicts the assumption. Assume that (ii) holds. By [Sc] $M$ admits an $E^3$ or Sol structure and does not admit an $H^2 \times R$, $\widehat{SL_2(R)}$, or Nil structure, a contradiction. Hence $T$ is isotopic to a saturated torus.

Proof of Theorem 2. If $T$ is a non-peripheral, saturated, incompressible torus in $M$, then $p(T)$ is an essential loop on $O$. Conversely, if $a$ is an essential loop on $O$, then $p^{-1}(a)$ is a non-peripheral, saturated, incompressible torus in $M$. Moreover, if $a_1$, $a_2$ are mutually non-parallel, essential loops on $O$, then $p^{-1}(a_1)$, $p^{-1}(a_2)$ are mutually non-parallel incompressible tori. From these facts the proof of Theorem 2 follows immediately from Theorem 1.

Now, we investigate homeomorphisms of type periodic on $M$. Suppose that $f$ is of type periodic. Let $G$ be a subgroup of $\text{Out } \pi_1(M)$ generated by $f_*$. Let $\psi : \text{Out } \pi_1(M) \to \text{Out } \pi_1^{orb}(O)$ be a canonical homomorphism, where $\pi_1^{orb}(O)$ denotes the fundamental group of $O$ as an orbifold ([T; §13]). Then we have an exact sequence:

$$1 \to \ker \psi \to G \xrightarrow{\psi} \psi(G) \to 1.$$ 

$\psi(G)$ is a finite cyclic group. If $O$ is non-orientable, then by Kojima [Ko] $\ker \psi \mid_G$ is a finite group and $G$ itself a finite cyclic group. This fact together with [Zi] implies:

**Proposition 4.2.** If $O$ is non-orientable and $f : M \to M$ is a homeomorphism of type periodic, then $f$ is homotopic to a periodic one.

5. Topological entropy of homeomorphisms of $H^2 \times R$, $\widehat{SL_2(R)}$, or Nil-manifolds

In this section we see that the map $\varphi$ obtained in Theorem 2 attains the minimal entropy in its homotopy class. Throughout this section, $M$ denotes a compact, orientable $H^2 \times R$, $\widehat{SL_2(R)}$, or Nil manifold with Seifert fibration $M \to O$. 
Lemma 5.1. Let \( g: M \rightarrow M \) be a fiber preserving homeomorphism. Then \( h(g) = h(\psi) \), where \( \psi: O \rightarrow O \) is the homeomorphism induced from \( g \).

Proof. By the argument in the proof of Lemma 3.1 of [S-S] we see that \( h_C(g) = 0 \) for each fiber \( C \) of \( M \). Hence, by Theorem A we have \( h(g) = h(\psi) \).

By Lemma 5.1 we have:

Proposition 5.2. If \( f: M \rightarrow M \) is a homeomorphism of type periodic, then \( h(f) = 0 \).

For the homeomorphisms of type pseudo-Anosov, we have:

Proposition 5.3. If \( f: M \rightarrow M \) is a homeomorphism of type pseudo-Anosov, then \( h(f) > 0 \). Moreover, it attains the minimal entropy in its homotopy class.

Proof. By [Sc] \( M \) admits a finite covering \( p: \widetilde{M} \rightarrow M \) such that the Seifert bundle structure on \( M \) lifts to a circle bundle structure \( q: \widetilde{M} \rightarrow S \). We may suppose that some power of \( f, f^n \), lifts to a homeomorphism \( \tilde{f}: \widetilde{M} \rightarrow \widetilde{M} \). Let \( \tilde{\psi}: S \rightarrow S \) be a homeomorphism induced from \( \tilde{f} \). Then we have \( h(f) = 1/n \cdot h(f^n) \).

By Lemma 5.1 \( h(f^n) = h(\tilde{f}) = h(\tilde{\psi}) \).

Then we note that \( \tilde{\psi} \) is a pseudo-Anosov homeomorphism. Let \( \lambda > 1 \) be the expanding factor of \( f \). Then \( \lambda^* \) is the expanding factor of \( \tilde{\psi} \). By [F-L-P] \( h(\tilde{\psi}) = n \cdot \log \lambda > 0 \). Hence, we have \( h(f) = \log \lambda \). Since \( \chi(S) < 0 \), there is a loop \( l \) in \( \widetilde{M} \) such that \( q(l) (\subset S) \) is an essential loop of \( S \). We note that \( q(f(l)) = \tilde{\psi}(q(l)) (\subset S) \). Then by [Sc] we see that \( L(l') \geq L(q(l')) \) for each loop \( l' \) on \( \widetilde{M} \), where \( L(l') \) denotes the infimum of the length of loops which are homotopic to \( l' \). Since \( \lim \frac{1}{m} \cdot \log L(f^m(l)) = \lambda^* \), we see that \( \lim \frac{1}{m} \cdot \log L(f^m(l)) \geq \lambda^* \). Hence if \( f' \) is homotopic to \( f \) then \( h(f') \geq h(f) = n \cdot \log \lambda \). From this we see that \( f \) attains the minimal entropy in its homotopy class.

By using the argument as in the proof of Proposition 2.1 and by Lemma 5.1 we easily have:

Proposition 5.4. Let \( \varphi: M \rightarrow M \) be as in Theorem 2. Then \( \varphi \) attains the minimal entropy in its homotopy class and \( h(\varphi) \) is positive if and only if \( \varphi \) contains a component of type pseudo-Anosov.

References


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