HITOSHI KUMANO-GO

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1935–1982

Hitoshi Kumano-go was born on October 4, 1935 in Arita, Wakayama Prefecture, Japan. After finishing Taikyû Senior High School in 1954, he entered Osaka University and graduated in 1958. After spending four years at the graduate school as a student of Professor Mitio Nagumo, he was appointed an assistant at Osaka University in 1962. In September 1963 he received his doctor's degree. He was promoted to assistant professor in 1964, to associate professor in 1967, and to full professor in 1971. He spent the academic years 1967–1969 as a Visiting Member at the Courant Institute of Mathematical Sciences of New York University. He was a member of the editorial committee of the mathematical journals "Communications in Partial Differential Equations" since the first edition and "Funkcialaj Ekvacioj" since 1976. During the period 1974–1977 he was an editor of "Osaka Journal of Mathematics". In May 1981 he entered Osaka University Hospital where he was found to be afflicted with a brain tumor. Hitoshi Kumano-go died on August 24, 1982 at the age of 46.

H. Kumano-go was mainly interested in partial differential equations. He is especially noted for his study of pseudo-differential operators as well as Fourier integral operators. The two years he spent at the Courant Institute was for him a time of great mathematical growth and maturity. In the extremely active, scientific atmosphere of the Courant Institute he made researches in pseudo-differential operators with Professors Kurt O. Friedrichs, Peter D. Lax, Louis Nirenberg, etc., making a significant contribution to the foundation and development of that theory. After returning to Japan he continued his research on pseudo-differential operators. His study covered a wide variety of topics on these operators such as the boundedness, complex powers, indices as Fredholm operators, etc. In addition, he entered into the study of Fourier integral operators. With the aid of his profound investigation on the products of Fourier integral operators he manifested various important facts on hyperbolic systems through the construction of the fundamental solutions of these systems. The outcomes of these extensive researches are contained in his book "Gibibun Sayôso" written in Japanese and in its English edition "Pseudo-Differential Operators" which is an outstanding contribution to the field of pseudo-differential operators in view of its uniqueness and the wealth of material. His other book "Henbibun Hôteishiki (Partial Differential Equations)" written in Japanese is an excellent textbook on partial differential equa-

tions with some introductory exposition of pseudo-differential operators.

A brief survey of H. Kumano-go's papers on mathematics is as follows.

Continuing the study of his respected teacher, Professor Mitio Nagumo, H. Kumano-go investigated the singular perturbation of second order partial differential equations in [1]*.

Papers [2], [3], [5], [7], [9] are concerned with the local and global uniqueness of the solutions of the Cauchy problem for partial differential equations. In this series of papers and [8] H. Kumano-go made an important contribution to the theory of singular integrals of Calderón-Zygmund type introducing operators with general symbols such as symbols of mixed homogeneity. With the aid of this extension of singular intergals H. Kumano-go proved uniqueness theorems for a wide class of differential equations. An essential role is played by the extensions of the inequality established by A.P. Calderón in his proof of the uniqueness in the Cauchy problem, and this type of inequality is applied to the problem of existence and propagation of regularity in [4]. The propagation of regularity was also discussed in papers [14], [15] in a different form.

Papers [6] and [13] are concerned with the non uniqueness in the Cauchy problem. In paper [6] H. Kumano-go constructed a function which satisfies the wave equation with vanishing inhomogeneous term and has a support coinciding with the complement of an open cylinder parallel to the time axis. Paper [10] is related to similar problems.

Papers [11] and [12] are concerned with the global uniqueness in the Cauchy problem and the construction of null solutions. A differential operator is characterized by the smallest function class of Gevrey to which null solutions can belong.

In paper [18] the authors gave another proof of Boas on analytic functions with an application to the integration problem of completely integrable systems with singularities.

In paper [16] H. Kumano-go developed Friedrichs' theory of pseudo-differential operators with a general basic weight function $\lambda(\xi)$ in place of $(1+|\xi|^2)^{1/2}$. This generality of basic weight functions is important in applications. With the aid of these results he proved some uniqueness theorems in the Cauchy problem generalizing previous results by A.P. Calderón, S. Mizohata and himself.

In papers [17], [20], [25] H. Kumano-go studied pseudo-differential operators with symbols in Hörmander's class $S^m_{\rho,\delta}$. He showed that the set of operators with symbols in $\bigcup_{-\infty<m<\infty} S^m_{\rho,\delta}$ makes an algebra with respect to sum, product and adjoint. The symbols of the product and adjoint are given in an exact

*) Numbers in brackets refer to the bibliography.

form without relying upon asymptotic expansions. He extended Hörmander's theorem on L^2-boundedness and Lax-Nirenberg's theorem on a sharp form of Gårding's inequality to the case when m and s are arbitrary real numbers. A number of important theorems such as pseudo-local property and the invariance under coordinate transformations were also established. These results were extended to the case of general basic weight functions in [23].

In paper [19] H. Kumano-go constructed a pseudo-differential operator with a symbol belonging to $S^0_{\rho,1}$ for $0<\rho<1$ which cannot be extended as a bounded operator in L^2. This result is a negative answer to a problem proposed by L. Nirenberg.

In paper [27] H. Kumano-go defined a new type of oscillatory integrals, and investigated pseudo-differential operators in the set of mildly increasing functions. As an application he gave a simple proof of Beals-Fefferman's inequality on some partition of unity which was used in their improvement of Nirenberg-Treve's theorem on local solvability.

Papers [21] and [22] are concerned with the L^p theory of pseudo-differential operators. Among others the authors extended Kagan's result on L^p boundedness of pseudo-differential operators. Papers [24] and [28] are concerned with the complex powers P_z of a system P of pseudo-differential operators. The method is to construct directly the symbol of the complex power by Dunford integrals for the symbols of parametrices for $P-\zeta I$. In [28] it is shown that an operator with complex powers and a slowly varying symbol is an operator of Fredholm type with vanishing index. Paper [26] is the announcement of this last result.

In paper [29] the authors defined oscillatory integrals for a new class of functions and studied pseudo-differential operators of class $S^m_{\lambda,\rho,\delta}$ with a basic weight function $\lambda(x,\xi)$ dependent also on x. As an application some theorem on hypoellipticity was established. With the aid of the oscillatory integrals developed in this paper H. Kumano-go introduced in [30] a class of multiple symbols and defined the associated pseudo-differential operators. Some powerful estimates on the associated single symbols were established. As an application the Calderón-Vaillancourt theorem on the boundedness of pseudo-differential operators was proved by symbol calculus alone without recourse to Cotlar's lemma on almost orthogonal operators in a Hilbert space.

Paper [32] is concerned with the calculus of pseudo-differential operators associated with difference approximation. The authors extended the stability theorems for the Friedrichs schemes obtained previously by M. Yamaguchi, T. Nogi and R. Vaillancourt.

In paper [33] H. Kumano-go considered a factorizable pseudo-differential operator. He investigated the Levi condition through that of its factors, and constructed the fundamental solution represented by a Fourier integral operator.

Paper [35] is concerned with pseudo-differential operators with symbols differentiable only a finite number of times with respect to the variable x. Various fundamental theorems on pseudo-differential operators are extended to the case of such symbols. The main tool is what should be called the Friedrichs part of the symbol with respect to the variable x.

In the series of papers [31], [34], [36], [37], [38], [40] H. Kumano-go investigated with some of his colleagues the product of Fourier integral operators, and the results were applied to the construction of the fundamental solution of a first order hyperbolic system and the study of the wave front sets of solutions. In the first paper [31] of the series H. Kumano-go developed a global calculus of Fourier integral operators, and applied the result to the construction of the fundamental solution of a single hyperbolic pseudo-differential equation of first order. Paper [34] is devoted to the same problem for regularly hyperbolic systems. In paper [36] the authors obtained an explicit representation of multi-products of phase functions together with precise estimates. Using this result the authors obtained in paper [37] a theorem on the representation of the multi-products of Fourier integral operators modulo a smoothing operator. This result was applied to the construction of the fundamental solution of a hyperbolic system with diagonal principal part and the study of the wave front sets of solutions. The subsequent paper [38] is devoted to the same problem for hyperbolic systems with some singularity in lower order terms at the initial time together with an application to single hyperbolic equations of arbitrary order. In paper [40] it was shown that some interesting phenomena can occur concerning the propagation of singularity of the solutions of hyperbolic equations according to the choice of initial values.

In paper [39] the authors investigated Fourier integral operators containing a parameter and constructed the fundamental solution of the equation of Schrödinger type.

H. Kumano-go's contributions were not limited to his own personal mathematical achievements. He also trained many young mathematicians of high ability. He was well known for the kindly interest which he took in the careers of young mathematicians. He was always ready to help, encourage and advise them. He looked through all their manuscripts, helping to improve and develop new ideas.

With his death Japanese mathematics lost an eminent scholar and a brilliant teacher.

Hiroki TANABE

List of publications of Hitoshi Kumano-go

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