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CORRECTION TO
CHARACTERISTIC CLASSES WITH VALUES IN COMPLEX COBORDISM

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In the proof of Theorem 7 it is asserted that there exists $\alpha \in U^*(B_G \times M)$ such that

$$\alpha q(\tau(M)) = w^{m'}, \quad r_0^*(\alpha) = 0 .$$

The proof of this assertion is not correct. Under a further assumption that $U_*(M)$ *is projective over* $U_*(pt)$, this is proved correctly as follows.

It follows from Lemma 2 and (7.1) that $i^*(id \underset{G}{\times} f^k)^* \Delta' = 0$ for $i^*\colon U^*(E_G \underset{G}{\times} M^k) \to U^*(E_G \underset{G}{\times} (M^k - M))$ induced by the inclusion. Therefore there exists $\alpha \in U^{i-2m(k-1)}(B_G \times M)$ such that

$$(id \underset{G}{\times} f^k)^* \Delta' = j^* \phi_{\nu_1}(\alpha) ,$$

where $\phi_{\nu_1}\colon U^*(B_G \times M) \cong U^*(E_G \underset{G}{\times} (M^k, M^k - M))$ is the Thom isomorphism, and $j^*\colon U^*(E_G \underset{G}{\times} (M^k, M^k - M)) \to U^*(E_G \underset{G}{\times} M^k)$ is induced by the inclusion. It is easily seen that the diagram

$$\begin{tikzcd} U^{i-2m(k-1)}(M) \arrow[d, "d_!"] & U^{i-2m(k-1)}(B_G \times M) \arrow[l, "r_0^*"'] \arrow[r, "\cdot e(\nu_1)"] \arrow[d, "j^* \circ \phi_{\nu_1}"] & U^i(B_G \times M) \\ U^i(M^k) & U^i(E_G \underset{G}{\times} M^k) \arrow[l, "r^*"] \arrow[ur, "(id \underset{G}{\times} d)^*"'] & \end{tikzcd}$$

is commutative, where r and r_0 are the inclusions, and $d_!$ is the Gysin homomorphism induced by the diagonal map $d\colon M \to M^k$. Consequently we have

$$(8.1) \qquad (id \underset{G}{\times} d)^* (id \underset{G}{\times} f^k)^* \Delta' = \alpha \cdot e(\nu_1) ,$$

$$(8.2) \qquad r^* (id \underset{G}{\times} f^k)^* \Delta' = d_! r_0^*(\alpha) .$$

It follows from (6.1), (6.3) and (8.1) that

$$\alpha q(\tau(M)) = (id \times f)^* q(\tau(M')) ,$$

and from (8.2) that

$$d_! r_0^*(\alpha) = (f^k)^* r'^*(\Delta') ,$$

where $r' \colon M'^k \to E_G \underset{G}{\times} M'^k$ is the inclusion. Since f is null-homotopic, these imply that

$$\alpha q(\tau(M)) = w^{m'} , \quad d_! r_0^*(\alpha) = 0 .$$

Thus it suffices to prove that $d_!$ is injective. Since $U_*(M)$ is projective over $U_*(pt)$, it holds that

$$U^*(M) \cong \mathrm{Hom}_{U_*(pt)}(U_*(M), U_*(pt))$$

(see [2]). Therefore, if $b \in U_*(M)$ is not zero then there exists $\beta \in U^*(M)$ such that $\langle \beta, b \rangle \neq 0$. Since $\langle \beta \times 1 \times \cdots \times 1, d_* b \rangle = \langle \beta, b \rangle$, it follows that d_* and hence $d_!$ is injective.