## On Wendt's Theorem of Knots, II

## By Shin'ichi KINOSHITA

1. Recently R. H. Fox introduced in his paper [1] an operation  $\tau$  called a single twist. Using this operation  $\tau$ , we introduce now a numerical knot<sup>1</sup> invariant  $\overline{s}(k)$  defined as the minimal number of  $\tau^n$  which change the given knot k to the trivial one, where the natural number n is not fixed. By definition of  $\overline{s}(k)$  and  $s(k)^{2}$ 

 $\bar{s}(k) \leq \bar{s}(k) \leq s(k)$ .

Then the purpose of this note is to prove

 $(*) \qquad e_g \leq (g-1)\bar{s}(k),$ 

where  $e_g$  is the minimal number of essential generators of the 1-dimensional homology group of the *g*-fold cyclic covering space of *S*, branched along *k*. From the above inequality (\*) it follows that

$$e_{g} \leq (g-1)\overline{s}(k)$$
,  $e_{g} \leq (g-1)s(k)$ ,

where the former is proved in [2] and the latter is due to H. Wendt [3].

2. Now we prove our inequality (\*). Let k be a knot. Suppose that k is deformed into k' by  $\tau^{n}$ . Then we are only to prove that

$$e_g(k') \leq e_g(k) + (g-1) \, .$$

Let F(S-k) be the fundamental group of S-k. By [1] we may assume that

$$F(S-k) = (a, b, A, B, x_1, x_2, \cdots;$$
  

$$a = A, b = B, r_1 = 1, r_2 = 1, \cdots),$$
  

$$F(S-k') = (a, b, A, B, x_1, x_2, \cdots;$$
  

$$a = A, b = A^n B, r_1 = 1, r_2 = 1, \cdots).$$

The 1-dimensional homology groups of S-k and S-k' are infinite cyclic. We denote by t a generator of either group. Then abelianization of F(S-k) or F(S-k') maps A into  $t^q$  and B into 1, where q is an integer. By usual methods the presentations of F(S-k) and F(S-k') can be transformed to the following one:

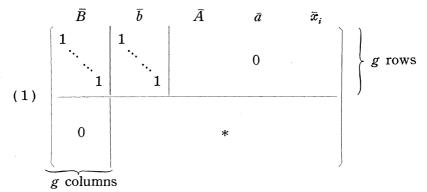
2)  $\bar{s}(k)$  and s(k) are defined in [2].

<sup>1)</sup> A knot is a polygonal simple closed curve in the 3-sphere S.

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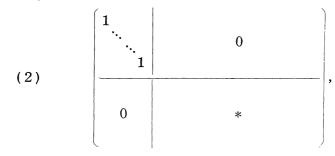
$$F(S-k) = (t, \ \bar{a}, \ \bar{b}, \ \bar{A}, \ \bar{B}, \ \bar{x}_1, \ \bar{x}_2, \cdots :$$
  
$$\bar{a} = \bar{A}, \ \bar{b} = \bar{B}, \ \bar{r}_1 = 1, \ \bar{r}_2 = 1, \cdots, \ tf_1^{-1} = 1),$$
  
$$F(S-k') = (t, \ \bar{a}, \ \bar{b}, \ \bar{A}, \ \bar{B}, \ \bar{x}_1, \ \bar{x}_2, \cdots :$$
  
$$\bar{a} = \bar{A}, \ t^{nq}\bar{b} = (t^q\bar{A})^n\bar{B}, \ \bar{r}_1 = 1, \ \bar{r}_2 = 1, \cdots, \ tf_2^{-1} = 1),$$

Furthermore we may suppose that  $f_1 = f_2$ . Then the 1-dimensional homology group of the *g*-fold cyclic covering space of *S*, branched along *k*, is given by the matrix:

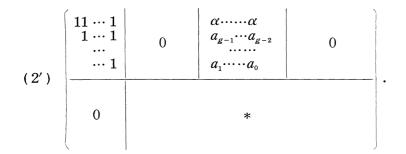


and that of S, branched along k', is given by the following one:

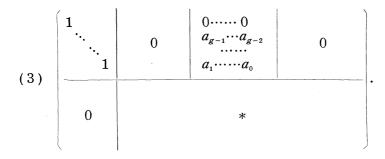
Putting  $\sum_{i=0}^{q-1} a_i = \alpha$ , we can transform (1) and (1') to the following one, respectively:



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(2') is equivalent to



From (2) and (3) it is easy to see that

$$e_g(k') \leq e_g(k) + (g-1).$$

Thus our proof is complete.

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## References

[1] R. H. Fox: Congruence classes of knots, Osaka Math. J. 10, 37-41 (1958).
[2] S. Kinoshita: On Wendt's theorem of knots, Osaka Math. J. 9, 61-66 (1957).
[3] H. Wendt: Die gordische Auflösung von Knoten, Math. Z. 42, 680-696 (1937).