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TRANSVERSAL CONFORMAL FIELDS OF FOLIATIONS

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1. Introduction

Let $(M, g_M, \mathcal{F})$ be a closed, oriented, connected Riemannian manifold of dimension $p+q$ with a transversally oriented foliation $\mathcal{F}$ of codimension $q \geq 2$ and a bundle-like metric g_M with respect to $\mathcal{F}$. Let Q be the normal bundle of $\mathcal{F}$ and $\pi : \Gamma(TM) \longrightarrow \Gamma(Q)$ the natural projection. We denote by D the transversal Riemannian connection of $\mathcal{F}$. Let $V(\mathcal{F})$ denote the set of infinitesimal automorphisms of $\mathcal{F}$ and $\bar{V}(\mathcal{F}) = \{ \nu \in \Gamma(Q) \mid \nu = \pi(Y),\ Y \in V(\mathcal{F}) \}$, the set of transversal infinitesimal automorphisms of $\mathcal{F}$.

Throughout this paper, we also use the following notation:

τ : the tension field of $\mathcal{F}$,

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$div_D\, \nu$: the transversal divergence of ν ,

$grad_D\, f$: the transversal gradient of a function f ,

ρ_D : the transversal Ricci operator,

Δ_D : the Laplacian acting on $\Omega^0(M,Q) = \Gamma(Q)$,

$\theta(Y)$: the transversal Lie derivative operator for $Y \in V(\mathcal{F})$,

$A_D(\nu) = \theta(Y) - D_Y \quad (\nu = \pi(Y) \in \bar{V}(\mathcal{F}))$,

$B_D(\nu) = A_D(\nu) + {}^tA_D(\nu) + \frac{2}{q}(div_D\nu)\cdot I$

(I : the identity map of $\Gamma(Q)$) .

In the case where $\mathcal{F}$ is a harmonic foliation ($\tau = 0$), geometric tranversal fields such as transversal Killing, transversal affine (projective, conformal) fields have been studied by Kamber, Tondeur, Molino and others ([1, 2, 3, 5]). For example, a transversal infinitesimal automorphism ν of $\mathcal{F}$ is a transversal Killing field of $\mathcal{F}$ if and only if ν satisfies $\Delta_D\, \nu = \rho_D(\nu)$ and $div_D\, \nu = 0$ ([1, 2, 5]). On the other hand, in the case where $\mathcal{F}$ is not a harmonic foliation, Nishikawa and Yorozu[4] give a necessary and sufficient condition for a transversal infinitesimal automorphism of $\mathcal{F}$ to be a transversal Killing field of $\mathcal{F}$.

A transversal infinitesimal automorphism $\nu = \pi(Y) \in \bar{V}(\mathcal{F})$ is called a transversal conformal field of $\mathcal{F}$ if ν satisfies $\theta(Y)g_Q = 2f\cdot g_Q$, where f is a function on M . The purpose of this paper is to find a necessary and sufficient condition for a transversal infinitesimal automorphism of $\mathcal{F}$ to be a transversal conformal field of $\mathcal{F}$, without assuming the harmonicity of $\mathcal{F}$. We prove the following theorem.

Theorem. Let $(M, g_M, \mathcal{F})$ be a closed, oriented, connected Riemannian manifold of dimension $p+q$ with a transversally oriented foliation $\mathcal{F}$ of codimension $q \geq 2$ and a bundle-like metric g_M with respect to $\mathcal{F}$. Let ν be a transversal infinitesimal automorphism of $\mathcal{F}$. Then ν is a transversal conformal field of $\mathcal{F}$ if and only if ν satisfies

$$\text{(i)} \quad \Delta_D \nu = D_{\sigma(\tau)} \nu + \rho_D(\nu) + (1 - \frac{2}{q}) \cdot \operatorname{grad}_D \operatorname{div}_D \nu$$

and

$$\text{(ii)} \quad \int_M g_Q(B_D(\nu)\nu, \tau) \, dM = 0 .$$

We shall be in C^∞-category. We use the following convention on the range of indices: $1 \leq i, j, \cdots \leq p$ and $p+1 \leq a, b, \cdots \leq p+q$. The authors would like to thank the referee for kind suggestion.

2. Preliminaries

Let $(M, g_M, \mathcal{F})$ be a closed, oriented, connected Riemannian manifold of dimension $p+q$ with a transversally oriented foliation $\mathcal{F}$ of codimension $q \geq 2$ and a bundle-like metric g_M with respect to $\mathcal{F}$. Let E and $Q = TM/E$ be the tangent bundle and the normal bundle of $\mathcal{F}$, respectively. The metric g_M gives a splitting σ of the exact sequence

$$0 \longrightarrow \Gamma(E) \longrightarrow \Gamma(TM) \underset{\sigma}{\overset{\pi}{\rightleftarrows}} \Gamma(Q) \longrightarrow 0$$

with $\sigma(\Gamma(Q)) = \Gamma(E^{\perp})$, where $E^{\perp}$ denotes the orthogonal complement bundle of E in TM with respect to g_M . Then g_M induces a metric g_Q on Q defined by $g_Q(\nu, \mu) = g_M(\sigma(\nu), \sigma(\mu))$ for $\nu, \mu \in \Gamma(Q)$. In a flat chart $U(x^i, x^a)$ with respect to $\mathcal{F}$, a local frame $\{X_i, X_a\} = \{\partial/\partial x^i, \partial/\partial x^a - \Sigma_j A^j{}_a \partial/\partial x^j\}$ is called the basic adapted frame to $\mathcal{F}$. Here $A^j{}_a$ are functions on U with $g_M(X_i, X_a) = 0$. We notice that $\{X_i\}$ spans $\Gamma(E|_U)$ and $\{X_a\}$ spans $\Gamma(E^{\perp}|_U)$. We put

$$g_{ij} = g_M(X_i, X_j) , \quad g_{ab} = g_M(X_a, X_b) ,$$

$$(g^{ij}) = (g_{ij})^{-1} , \quad (g^{ab}) = (g_{ab})^{-1} \quad ([5]).$$

A connection D on Q is defined by

$$D_X \nu = \pi([X, Y_\nu]) \qquad \text{if } X \in \Gamma(E) ,$$

$$D_X \nu = \pi(\nabla_X Y_\nu) \qquad \text{if } X \in \Gamma(E^{\perp}) ,$$

where $Y_\nu = \sigma(\nu)$, and ∇ denotes the Levi-Civita connection with respect to g_M . Since D is torsionfree and metrical with respect to g_Q ([1]), we call D the transversal Riemannian connection of $\mathcal{F}$. The curvature R_D of D is defined by

$$R_D(X,Y)\nu = D_X D_Y \nu - D_Y D_X \nu - D_{[X,Y]} \nu$$

for all $X, Y \in \Gamma(TM)$ and $\nu \in \Gamma(Q)$. We notice that $i(X)R_D = 0$, where $i(X)$ denotes the interior product with respect to $X \in \Gamma(E)$. The transversal Ricci operator $\rho_D : \Gamma(Q) \longrightarrow \Gamma(Q)$ is given by

$$\rho_D(\nu) = \Sigma_{a,b}\, g^{ab}\, R_D(\sigma(\nu), X_a)\pi(X_b) \quad ,$$

and let $\mathrm{Ric}_D(\nu) = g_Q(\rho_D(\nu), \nu)$.

We denote by $V(\mathcal{F})$ the set of all infinitesimal automorphisms of $\mathcal{F}$, that is,

$$V(\mathcal{F}) = \{\, Y \in \Gamma(TM) \mid [X,\ Y] \in \Gamma(E) \quad \text{for all} \quad X \in \Gamma(E) \,\} \quad .$$

A transversal infinitesimal automorphism ν of $\mathcal{F}$ is an element of the set

$$\bar{V}(\mathcal{F}) = \{\, \nu \in \Gamma(Q) \mid \nu = \pi(Y) \ ,\ Y \in V(\mathcal{F}) \,\} \quad .$$

The transversal Lie derivative operator $\theta(Y) : \Gamma(Q) \longrightarrow \Gamma(Q)$ for $Y \in V(\mathcal{F})$ is defined by

$$\theta(Y)\mu = \pi(\ [Y,\ Z_\mu]\ )$$

for all $\mu \in \Gamma(Q)$ with $\sigma(\mu) = Z_\mu$. For $\nu = \pi(Y) \in \bar{V}(\mathcal{F})$, we define an operator $A_D(\nu) : \Gamma(Q) \longrightarrow \Gamma(Q)$ by

$$A_D(\nu)\mu = \theta(Y)\mu - D_Y\, \mu$$

for all $\mu \in \Gamma(Q)$ ([1]). We notice that the definition of $A_D(\nu)$ is independent of the choice of Y with $\pi(Y) = \nu$ ([1]).

Definition. A transversal infinitesimal automorphism $\nu = \pi(Y) \in \bar{V}(\mathcal{F})$ is called a <u>transversal conformal field of</u> $\mathcal{F}$ if ν satisfies $\theta(Y)g_Q = 2f\cdot g_Q$, where f is a function on M .

The tension field τ of $\mathcal{F}$ is given by

$$\tau = \Sigma_{i,j} g^{ij} \pi(\nabla_{X_i} X_j) .$$

If $\tau = 0$, then $\mathcal{F}$ is called harmonic([1]). The transversal divergence $\mathrm{div}_D \nu$ of $\nu \in \Gamma(Q)$ is given by

$$\mathrm{div}_D \nu = \Sigma_{a,b} g^{ab} g_Q(D_{X_a} \nu, \pi(X_b)) ,$$

and the transversal gradient $\mathrm{grad}_D f$ of a function f on M is given by

$$\mathrm{grad}_D f = \Sigma_{a,b} g^{ab} X_a(f) \cdot \pi(X_b)$$

([5]).

Let $\Omega^r(M,Q)$ be the set of all Q-valued r-forms on M. We notice that $\Omega^0(M,Q) \cong \Gamma(Q)$. The global inner product $\langle\langle \ , \ \rangle\rangle$ on $\Omega^r(M,Q)$ is defined by

$$\langle\langle \xi, \eta \rangle\rangle = \int_M g_Q(\xi \wedge *\eta)$$

for all $\xi, \eta \in \Omega^r(M,Q)$ ([1]). Let $d_D : \Omega^r(M,Q) \longrightarrow \Omega^{r+1}(M,Q)$ be the exterior differential operator, and δ_D the adjoint operator of d_D with respect to $\langle\langle \ , \ \rangle\rangle$ ([1]). The Laplacian Δ_D acting on $\Omega^r(M,Q)$ is defined by

$$\Delta_D = \delta_D d_D + d_D \delta_D .$$

Then we have

Green's Theorem([6, 7]). For all $\nu \in \bar{V}(\mathcal{F})$, it holds that

$$\int_M \mathrm{div}_D\, \nu \quad dM = \langle\langle\, \nu,\ \tau\, \rangle\rangle \quad .$$

Proposition 1([7]). For all $\nu,\ \mu \in \bar{V}(\mathcal{F})$, it holds that

$$\langle\langle\, \Delta_D\, \nu,\ \mu\, \rangle\rangle = \langle\langle\, D\, \nu,\ D\, \mu\, \rangle\rangle \quad .$$

Proposition 2([7]). For all $\nu \in \bar{V}(\mathcal{F})$, it holds that

(i) $\mathrm{Ric}_D(\nu) + \mathrm{Tr}\, A_D(\nu)A_D(\nu) - (\, \mathrm{div}_D\nu\,)^2$

$$+ \mathrm{div}_D(\, A_D(\nu)\nu\,) + \mathrm{div}_D(\, (\mathrm{div}_D\nu)\nu\,) = 0 \quad ,$$

(ii) $\mathrm{Tr}\, A_D(\nu)A_D(\nu) = -\, \mathrm{Tr}\, {}^tA_D(\nu)A_D(\nu) + \frac{1}{2}\, \mathrm{Tr}\, (A_D(\nu) + A_D(\nu))^2$,

where $\mathrm{Tr}\, C$ denotes the trace of an operator $C : \Gamma(Q) \longrightarrow \Gamma(Q)$ with respect to g_Q , and ${}^tA_D(\nu)$ denotes the transposed operator of $A_D(\nu)$ with respect to g_Q .

Proposition 3([5]). A transversal conformal field ν of $\mathcal{F}$ satisfies

$$\Delta_D\, \nu = D_{\sigma(\tau)}\nu + \rho_D(\nu) + (\, 1 - \frac{2}{q}\,)\ \mathrm{grad}_D\, \mathrm{div}_D\nu \quad .$$

Let $B_D(\nu) : \Gamma(Q) \longrightarrow \Gamma(Q)$ ($\nu \in \bar{V}(\mathcal{F})$) be an operator defined by

$$B_D(\nu) = A_D(\nu) + {}^tA_D(\nu) + \frac{2}{q}\, (\mathrm{div}_D\nu)\cdot I \quad ,$$

where I denotes the identity map of $\Gamma(Q)$. Note that the operator $B_D(\nu)$ is symmetric.

Proposition 4([5]). A transversal infinitesimal automorphism ν of $\mathcal{F}$ is a transversal conformal field of $\mathcal{F}$ if and only if $B_D(\nu) = 0$.

3. Proof of Theorem

By Propositions 3 and 4, it is immediate that a transversal conformal field of $\mathcal{F}$ satisfies the conditions (i) and (ii). Conversely, we assume that a transversal infinitesimal automorphism ν of $\mathcal{F}$ satisfies the conditions (i) and (ii). We first note that, by Proposition 2, we have

$$\int_M \{ \mathrm{Ric}_D(\nu) - \mathrm{Tr}\ {}^tA_D(\nu)A_D(\nu) + \frac{1}{2}\mathrm{Tr}(A_D(\nu) + {}^tA_D(\nu))^2$$

$$- (\mathrm{div}_D\nu)^2 + \mathrm{div}_D(A_D(\nu)\nu)^2 + \mathrm{div}_D((\mathrm{div}_D\nu)\nu) \}dM = 0 .$$

By the condition (i) and $\mathrm{Ric}_D(\nu) = g_Q(\rho_D(\nu), \nu)$, we have

$$\int_M \mathrm{Ric}_D(\nu)\, dM = \langle\langle \Delta_D \nu, \nu \rangle\rangle - \langle\langle D_{\sigma(\tau)}\nu, \nu \rangle\rangle$$

$$- (1 - \frac{2}{q})\langle\langle \mathrm{grad}_D\ \mathrm{div}_D\nu, \nu \rangle\rangle .$$

By direct calculation, we have for $\nu \in \Gamma(Q)$

$$g_Q(\mathrm{grad}_D\ \mathrm{div}_D\nu, \nu) = \mathrm{div}_D((\mathrm{div}_D\nu)\nu) - (\mathrm{div}_D\nu)^2 .$$

Thus we have

$$(1) \quad \int_M \mathrm{Ric}_D(\nu)\, dM = \langle\langle \Delta_D \nu, \nu \rangle\rangle - \langle\langle D_{\sigma(\tau)}\nu, \nu \rangle\rangle$$

$$- (1 - \frac{2}{q}) \int_M \mathrm{div}_D((\mathrm{div}_D\nu)\nu)\, dM$$

$$+ (1 - \frac{2}{q}) \int_M (\mathrm{div}_D\nu)^2\, dM .$$

By direct calculation, we also have for $\nu \in \Gamma(Q)$

$$\mathrm{Tr}\ {}^tA_D(\nu)A_D(\nu) = \Sigma_{a,b}\ g^{ab}\ g_Q(\ D_{X_a}\nu,\ D_{X_b}\nu\ )\ ,$$

which implies

$$(2) \qquad \int_M \mathrm{Tr}\ {}^tA_D(\nu)A_D(\nu)\ dM = \langle\langle\ D\nu,\ D\nu\ \rangle\rangle \quad .$$

We have, by Green's Theorem,

$$\int_M \mathrm{div}_D(\ A_D(\nu)\nu\ )\ dM = \langle\langle\ A_D(\nu)\nu,\ \tau\ \rangle\rangle\ ,$$

$$\int_M \mathrm{div}_D(\ (\mathrm{div}_D\nu)\nu\ )\ dM = \langle\langle\ (\mathrm{div}_D\nu)\nu,\ \tau\ \rangle\rangle \quad .$$

On the other hand, we have

$$-\ g_Q(\ D_{\sigma(\tau)}\nu,\ \nu\ ) = g_q(\ {}^tA_D(\nu)\nu,\ \tau\ ) \quad .$$

Thus we have

$$(3) \quad \langle\langle\ A_D(\nu)\nu,\ \tau\ \rangle\rangle - \langle\langle\ D_{\sigma(\tau)}\nu,\ \nu\ \rangle\rangle$$

$$= \langle\langle\ (\ A_D(\nu) + {}^tA_D(\nu)\ )\nu,\ \tau\ \rangle\rangle \quad .$$

By direct calculation, we have

$$\mathrm{Tr}(\ A_D(\nu) + {}^tA_D(\nu)\ ) = -\ 2\ \mathrm{div}_D\nu \quad ,$$

which implies

$$(4) \quad \mathrm{Tr}\ (\ B_D(\nu)\ )^2 = \mathrm{Tr}\ (\ A_D(\nu) + {}^tA_D(\nu)\ )^2 + \frac{4}{q}\ (\mathrm{div}_D\nu)^2$$

$$+ \frac{4}{q}\ (\ \mathrm{div}_D\nu\ )\cdot\mathrm{Tr}(\ A_D(\nu) + {}^tA_D(\nu)\ )$$

$$= \mathrm{Tr}\ (\ A_D(\nu) + {}^tA_D(\nu)\ )^2 - \frac{4}{q}\ (\mathrm{div}_D\nu)^2 \quad .$$

By Proposition 1, (1), (2), (3) and (4), we have

$$0 = \int_M \{\ \mathrm{Ric}_D(\nu) - \mathrm{Tr}\ {}^tA_D(\nu)A_D(\nu) + \frac{1}{2}\ \mathrm{Tr}\ (\ A_D(\nu) + {}^tA_D(\nu)\ )^2$$

$$- (\mathrm{div}_D \nu)^2 + \mathrm{div}_D(A_D(\nu)\nu) + \mathrm{div}_D((\mathrm{div}_D\nu)\nu) \} dM$$

$$= \langle\langle \Delta_D \nu, \nu \rangle\rangle - \langle\langle D_{\sigma(\tau)}\nu, \nu \rangle\rangle$$

$$- (1 - \frac{2}{q}) \int_M \mathrm{div}_D((\mathrm{div}_D\nu)\nu)\, dM + (1 - \frac{2}{q}) \int_M (\mathrm{div}_D\nu)^2\, dM$$

$$- \langle\langle D\nu, D\nu \rangle\rangle + \frac{1}{2} \int_M \mathrm{Tr}\, (A_D(\nu) + {}^tA_D(\nu))^2\, dM$$

$$- \int_M (\mathrm{div}_D\nu)^2\, dM + \langle\langle A_D(\nu)\nu, \tau \rangle\rangle + \int_M \mathrm{div}_D((\mathrm{div}_D\nu)\nu)\, dM$$

$$= \langle\langle (A_D(\nu) + {}^tA_D(\nu))\nu, \tau \rangle\rangle + \frac{2}{q} \int_M \mathrm{div}_D((\mathrm{div}_D\nu)\nu)\, dM$$

$$+ \frac{1}{2}\{ \int_M \mathrm{Tr}\, (A_D(\nu) + {}^tA_D(\nu))^2\, dM - \frac{4}{q} \int_M (\mathrm{div}_D\nu)^2\, dM \}$$

$$= \langle\langle (A_D(\nu) + {}^tA_D(\nu))\nu, \tau \rangle\rangle + \frac{2}{q} \langle\langle (\mathrm{div}_D\nu)\nu, \tau \rangle\rangle$$

$$+ \frac{1}{2} \int_M \mathrm{Tr}\, (B_D(\nu))^2\, dM$$

$$= \langle\langle B_D(\nu)\nu, \tau \rangle\rangle + \frac{1}{2} \int_M \mathrm{Tr}\, (B_D(\nu))^2\, dM \quad .$$

By (ii) in Theorem, we have

$$(5) \qquad \int_M \mathrm{Tr}\, (B_D(\nu))^2\, dM = 0 \quad .$$

Since the operator $B_D(\nu)$ is symmetric, (5) implies that $B_D(\nu) = 0$. Therefore, by Proposition 4, ν is a transversal conformal field of $\mathcal{F}$.

4. Remark

On a non-compact foliated Riemannian manifold $(M, g_M, \mathcal{F})$, there exists a transversal infinitesimal automorphism of $\mathcal{F}$

satisfying the conditions (i) and (ii) in Theorem, but which is not a transversal conformal field of $\mathcal{F}$. For example, we have the following.

Let $M = R^1 \times R^3$ be a product manifold with a metric

$$f^2 \cdot (dx^1)^2 + \Sigma_{a=2}^{4} (dx^a)^2 ,$$

where (x^1, x^2, x^3, x^4) is a coordinate system of M , (x^1) and (x^2, x^3, x^4) being coordinate sytems of R^1 and R^3 respectively, and $f = f(x^2, x^3, x^4) = \exp(x^3 - x^4)$. The family $\{ R^1 \times \{t\} \}_{t \in R^3}$ defines a foliation $\mathcal{F}$ on M for which the metric is a bundle-like metric. We consider a vector field Y on M defined by

$$Y = x^2\, \partial/\partial x^2 + \partial/\partial x^3 + \partial/\partial x^4 .$$

Then Y is an infinitesimal automorphism of $\mathcal{F}$ so that Y induces a transversal infinitesimal automorphism $\nu = \pi(Y)$ of $\mathcal{F}$, that is,

$$\nu = x^2\, \pi(\partial/\partial x^2) + \pi(\partial/\partial x^3) + \pi(\partial/\partial x^4) .$$

The tension field τ of $\mathcal{F}$ is given by

$$\tau = - \pi(\partial/\partial x^3) + \pi(\partial/\partial x^4) .$$

Then we have

$$D_{\sigma(\tau)}\nu = 0 , \qquad D_{\sigma(\nu)}\nu = x^2\, \pi(\partial/\partial x^2) ,$$

$$g_Q(\nu, \tau) = 0 , \qquad \mathrm{div}_D\, \nu = 1 ,$$

$$g_Q((A_D(\nu) + {}^tA_D(\nu))\nu, \tau)$$

$$= - g_Q(D_{\sigma(\nu)}\nu, \tau) - g_Q(D_{\sigma(\tau)}\nu, \nu)$$

$$= 0 \quad .$$

Thus we have

(6) $$g_Q(\ B_D(\nu)\nu,\ \tau\ )$$
$$= g_Q(\ (\ A_D(\nu) + {}^tA_D(\nu) + (\ 1 - \frac{2}{3}\ )(\mathrm{div}_D \nu)\cdot I\ )\nu,\ \tau\ )$$
$$= 0 \quad .$$

On the other hand, we have

$$\rho_D(\nu) = 0 \quad ,$$
$$D_{\sigma(\tau)}\nu + \rho_D(\nu) + (\ 1 - \frac{2}{3}\ )\ \mathrm{grad}_D\ \mathrm{div}_D\ \nu = 0 \quad ,$$
$$\Delta_D\ \nu = 0 \quad .$$

Thus we have

(7) $$\Delta_D\ \nu = D_{\sigma(\tau)}\nu + \rho_D(\nu) + (\ 1 - \frac{2}{3}\ )\ \mathrm{grad}_D\ \mathrm{div}_D\ \nu \quad .$$

By (6) and (7), ν satisfies (i) and (ii) in Theorem. But we have

$$(\ \theta(Y)g_Q\ )(\ \pi(\partial/\partial x^2),\ \pi(\partial/\partial x^2)\ ) = 2 \quad ,$$
$$(\ \theta(Y)g_Q\ )(\ \pi(\partial/\partial x^3),\ \pi(\partial/\partial x^3)\ ) = 0 \quad .$$

Hence $\nu = \pi(Y)$ is not a transversal conformal field of $\mathcal{F}$.

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