



BOOK REVIEW

Lectures on Differential Geometry, by Iskander A. Taimanov, European Mathematical Society, Zürich, 2008, viii + 221pp., ISBN 978-3-03719-050-0.

In this book the author gives an introduction to the basics of differential geometry by keeping in mind the natural origin of many geometrical quantities, as well as the applications of differential geometry and its methods to other sciences.

The book is divided into three parts. The first part - Curves and Surfaces, is structured into Chapter 1 and Chapter 2.

In the first chapter the author gives: a definition of regular parameterized curve in the Euclidean space \mathbb{R}^n ; length and arclength parameter of such curve; the Frenet formulas and natural equations for a plane curve and a curve in three-dimensional space; the proof, that the natural equations determine the curve up to motion of the space; a definition of k -dimensional smooth submanifold in \mathbb{R}^{n+k} ; the proof, that orthogonal group $O(n)$ is an $\frac{n(n-1)}{2}$ - dimensional submanifold in \mathbb{R}^{n^2} .

The second chapter is devoted to the differential geometry of regular surfaces (two-dimensional smooth submanifold) in \mathbb{R}^3 . The chapter contains the following themes: metric on regular surfaces and curvature of a curve on a surface; derivational equations and Bonnet's theorem; the Gauss theorem; covariant derivative and geodesics; the Euler-Lagrange equations, the Gauss-Bonnet formula; minimal surfaces.

The second part - Riemannian geometry, is splitted into three chapters - 3, 4 and 5. Chapter 3 provides the foundations of: topological spaces; topological manifolds; smooth manifolds; submanifolds; smooth maps; tensors; the tangent bundle TM and the cotangent bundle T^*M of a smooth manifold M ; the action of maps on tensors, Lie derivative of a tensor field along a vector field. The theorem that the tangent bundle TM carries the structure of a smooth manifold such that the projection $\pi : TM \rightarrow M$ is a smooth map and every point $x \in M$ has a neighbourhood U such that the inverse image $\pi^{-1}(U)$ of U is diffeomorphic to the direct product $U \times \mathbb{R}^n$, $n = \dim M$, is proved. The theorem that every closed smooth

manifold can be embedded into Euclidean space of a sufficiently large dimension is proved also.

In Chapter 4 the foundations of the Riemannian manifolds are given. The metric tensor, affine connection and covariant derivative, Riemannian connections, the curvature tensor, the sectional curvature, the geodesics, the exponential map, the geodesic and semigeodesic coordinates are introduced. The theorem, that on each Riemannian manifold there is a unique symmetric connection compatible with the metric is proved. It is proved, that for a Riemannian manifold the geodesics are the natural analog of straight lines in the Euclidean space as the shortest curves.

In Chapter 5 the author gives short introductions to the hyperbolic geometry (the Lobachevskii plane), the Minkowski space and the geometrical principles of the special relativity theory.

The third part - Supplement chapters, is more advanced and assumes that the reader is familiar with the first two parts of the book. This part consists of four chapters - 6, 7, 8 and 9.

In Chapter 6 - Minimal surfaces and complex analysis, the author proves that each two-dimensional Riemannian manifold is conformally Euclidean, introduces the conformal parameter, gives the Weierstrass representation of a minimal compact surface without boundary.

In Chapter 7 - Elements of Lie Group Theory, the author treats the following Lie groups and their Lie algebras: the complete linear group $GL(n)$, the unimodular (or special linear) group $SL(n)$, the special orthogonal group $SO(n)$, the general linear group $SL(n, \mathbb{C})$, the unitary group $U(n)$, the special unitary group $SU(n)$. The geometry of the simplest linear groups - $SU(2)$, $SO(3)$, $SO(4)$ and the quaternions is given.

Chapter 8 - Elements of Representation Theory, contains: the basic notation of representation theory; representation of finite groups; representation of Lie groups. It is proved: that each unitary representation $\rho : G \rightarrow GL(V)$ decomposes into direct sum of irreducible representations; Schur's lemma for two irreducible representations; the theorem that if G is a finite group and $\rho : G \rightarrow GL(n, \mathbb{C})$ is a finite-dimensional representation, then ρ is unitary; the theorem that every finite group is linear; the theorem for characters of irreducible representations of a finite group; various assertions for representations of compact and noncompact Lie groups.

Chapter 9 - Elements of Poisson and Symplectic Geometry, is the largest chapter in the book. It contains the following themes: The Poisson bracket and Hamilton's equations; the Lagrangian formalism; examples of Poisson manifolds; Darboux's

theorem and Liouville's theorem; Hamilton's variational principle; reduction of the order of the system; Euler's equations; Integrable Hamiltonian systems.

A Poisson bracket on a smooth manifold M is bilinear product $f, g \rightarrow \{f, g\}$ defined on the space $C^\infty(M)$ of all smooth functions on M such that the space $C^\infty(M)$ is a Lie algebra with respect this form and the Leibniz identity $\{fg, h\} = f\{g, h\} + g\{f, h\}$ holds. In this case the manifold M is called a Poisson manifold. A manifold M is called symplectic if a nondegenerate closed two-form ω is given on M . It is proved that: if a Poisson bracket is given on M then an antisymmetric tensor of type $(2, 0)$ is given on M then an antisymmetric bilinear form on the space of functions on M is determined which satisfies the Leibniz identity; a nondegenerate two-form ω on M determines a Poisson bracket if and only if this form is closed. The author exposes symplectic and Poisson geometry in their close relation with mechanics and theory of integrable systems.

The bibliography of the book consist of 16 entries. There are also index pages. The book offers 62 problems which are given to complete the discussed theoretical material. There are also many examples.

The personal opinion of the reviewer is that the book is interesting for both the mathematicians and the physicists.

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