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NONEXISTENCE RESULTS FOR CLASSES OF ELLIPTIC SYSTEMS

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Abstract. We consider the system

$$
\begin{aligned}
-\Delta u &= \lambda f(u,v);\ x \in \Omega\\
-\Delta v &= \lambda g(u,v);\ x \in \Omega\\
u &= 0 = v;\ x \in \partial\Omega,
\end{aligned}
$$

where Ω is a ball in $R^N, N \geq 1$ and $\partial\Omega$ is its boundary, λ is a positive parameter, and f and g are smooth functions that are negative at the origin (semipositone system) and satisfy certain linear growth conditions at infinity. We establish nonexistence of positive solutions when λ is large. Our proofs depend on energy analysis and comparison methods.

1. Introduction

Consider the system

$$
\begin{aligned}
-\Delta u &= \lambda f(u,v);\ x \in \Omega\\
-\Delta v &= \lambda g(u,v);\ x \in \Omega\\
u &= 0 = v; x \in \partial\Omega,
\end{aligned}
\tag{1.1}
$$

where Ω is a ball in R^N, $\partial\Omega$ is its boundary, λ is a positive parameter, and f,g satisfy the following:

(H1) $f,g : [0,\infty) \times [0,\infty) \to R$ are C^1 functions such that $f(0,0) < 0$, $g(0,0) < 0$, and $f_u > 0$, $f_v > 0$, $g_u > 0$, and $g_v > 0$ for all $u > 0$ and $v > 0$.

(H2) there exist positive numbers K_i and M_i, $i = 1,2$ such that $f(u,v) \geq K_1 v - M_1$ and $g(u,v) \geq K_2 u - M_2$ for all $u > 0$ and $v > 0$.

(H3) there exist positive numbers β_1, β_2, γ_1, and γ_2 such that $f(0,\beta_1) = g(0,\beta_2) = f(\gamma_1,0) = g(\gamma_2,0) = 0$.

Then we establish

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Theorem 1.1. *Let* (H1), (H2) *and* (H3) *hold. Then there exists a positive number* λ^* *such that the system* (1.1) *has no positive solution for* $\lambda > \lambda^*$.

A simple example satisfying the hypotheses (H1)–(H3) is $f(u,v) = (u+1)^{p_1} + v^{q_1} - \epsilon_1, g(u,v) = u^{p_2} + (v+1)^{q_2} - \epsilon_2$, where $p_1 > 0, q_1 \geq 1, p_2 \geq 1, q_2 > 0, \epsilon_1 > 1$ and $\epsilon_2 > 1$. We will establish some preliminary results in Section 2 and prove Theorem 1.1 in Section 3. Note that when $f(0,0)$ and $g(0,0)$ are both non-negative or when $f(0,0) < 0$ and $g(0,0) \geq 0$ (or $f(0,0) \geq 0$ and $g(0,0) < 0$), the corresponding non-existence result is easier to establish (see Appendices A and B). Further, when either $f(u,v) = \widetilde{f}(v)$ or $g(u,v) = \widetilde{g}(u)$, the non-existence result is again easier to establish (see Remark 3.1). Also see [11] for a recent non-existence result in the case when $f(u,v) = \widetilde{f}(v)$ and $g(u,v) = \widetilde{g}(u)$. The purpose of this paper is to extend the results in [11] for the case when f and g depend on u as well as v. Due to this strong coupling, the extensions are challenging and non-trivial. Note that it is well known in the literature that the study of positive solutions to such semipositone problems is mathematically very challenging (see [5, 15]).

When Ω is a ball and $N > 1$ by [9] all nonnegative solutions are positive componentwise. Hence, by [18] solutions are radially symmetric and decreasing. The proofs of our main results rely heavily on this property.

For an existence result for positive solutions for classes of superlinear nonlinearities and λ small, see [14]. Also see [12] for a similar existence result for a class of p-Laplacian systems in an annulus and [8] for a recent survey on semipositone systems. In the single equation case, see [1, 4, 6] for nonexistence results and [1–4, 7, 10, 13, 16, 17] for existence results.

2. Some preliminary results

Without loss of generality we assume that Ω is the unit ball in R^N. Let (u,v) be a positive solution of (1.1). Then u and v are radial, decreasing and satisfy

$$\begin{aligned} -(r^{(N-1)}u')' &= \lambda\, r^{N-1} f(u,v);\ 0<r<1 \\ -(r^{(N-1)}v')' &= \lambda\, r^{N-1} g(u,v);\ 0<r<1 \\ u'(0) = 0 = v'(0),&\quad u(1) = 0 = v(1). \end{aligned} \tag{2.1}$$

We first recall two lemmas from [11].

Lemma 2.1. *Assume that there exists* $z \geq 0\, (z \not\equiv 0)$ *on* $\bar{I}$ *where* $I = (a,b)$ *and a constant* σ *such that*

$$-(r^{N-1}z')' \geq \sigma r^{N-1} z;\ r \in I. \tag{2.2}$$

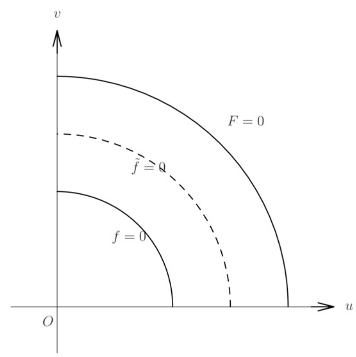

Figure 2.1

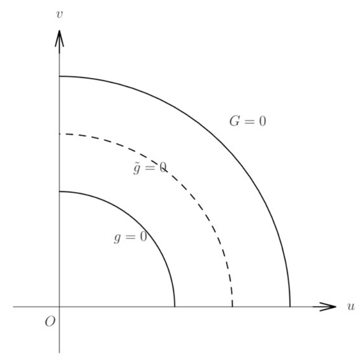

Figure 2.2

Let $\lambda_1 = \lambda_1(I) > 0$ *denote the principal eigenvalue of*

$$\begin{aligned} -(r^{N-1}\phi^{'})' &= \lambda r^{N-1}\phi; \qquad r \in (a,b) \\ \phi(a) &= 0 = \phi(b), \end{aligned} \tag{2.3}$$

where $0 < a < b \leq 1$. *Then* $\sigma \leq \lambda_1(I)$.

Proof. See Theorem B in the Appendix section in [11]. □

Lemma 2.2. *There exists a positive constant* C *such that for* λ *large,*

$$u(\tfrac{1}{4}) + v(\tfrac{1}{4}) \leq C.$$

Proof. Due to our assumption (H2) the proof is exactly as in Lemma 2.1 in [11]. □

We next state and prove a crucial lemma for the strongly coupled system (2.1). For any function h define $D_h := \{(u,v) : u > 0,\, v > 0,\, h(u,v) < 0\}$ and $H(u,v) := \int_0^u h(s,v)ds + \int_0^v h(u,t)dt$. Let $\widetilde{f}(u,v) = \frac{f(u,v)+F(u,v)}{2}$ and $\widetilde{g}(u,v) = \frac{g(u,v)+G(u,v)}{2}$.

Lemma 2.3. *For* λ *sufficiently large,* $(u(3/4), v(3/4)) \in \overline{D}_f \bigcup \overline{D}_g$.

Proof. From (H1) and (H3), we know that $D_f \subset\subset D_{\widetilde{f}} \subset\subset D_F$ and $D_g \subset\subset D_{\widetilde{g}} \subset\subset D_G$. (See Figure 2.1 and Figure 2.2.)

First, we claim that for λ large, $(u(1/2), v(1/2)) \in D_{\widetilde{f}} \bigcap D_{\widetilde{g}}$. Suppose $(u(1/2), v(1/2)) \notin D_{\widetilde{f}}$; then $\exists\, (\rho_1, \rho_2) \in$ curve $\widetilde{f} = 0$ (see Figure 2.3) such that $\rho_1 \leq u(1/2)$ and $\rho_2 \leq v(1/2)$.

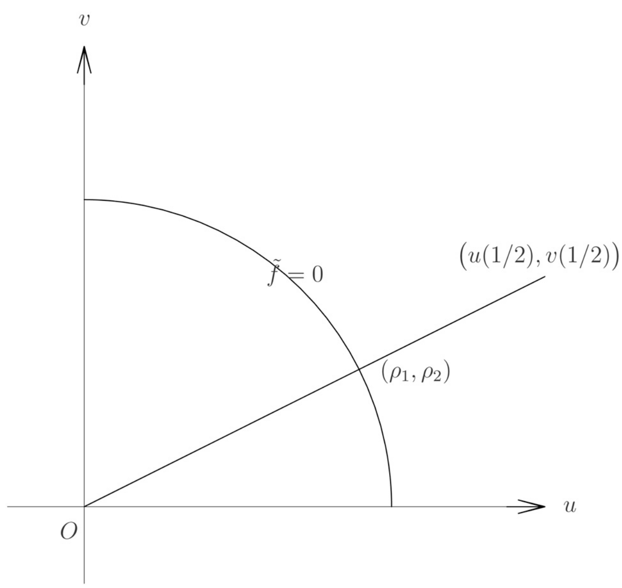

Figure 2.3

Then on $J_1 := (\frac{1}{4}, \frac{1}{2})$,

$$-(r^{N-1}u^{'})^{'} = \lambda\, r^{N-1}f(u,v) \geq \lambda r^{N-1}f(\rho_1,\rho_2)$$
$$\geq \frac{\lambda f(\rho_1,\rho_2)}{C} r^{N-1}u \geq \frac{\lambda f(\rho_1^*,\rho_2^*)}{C} r^{N-1}u,$$

where C is a constant as in Lemma 2.2, and $(\rho_1^*, \rho_2^*) \in$ curve $\widetilde{f} = 0$ such that $f(\rho_1^*, \rho_2^*) := \min\{f(u,v) : u \geq 0, v \geq 0, \text{with } \widetilde{f}(u,v) = 0\}$. Then by Lemma 2.1

$$\frac{\lambda f(\rho_1^*,\rho_2^*)}{C} \leq \lambda_1(J_1).$$

But in the above inequality, $f(\rho_1^*, \rho_2^*)$ and C are fixed positive constants; hence, it cannot hold for λ large. A similar contradiction can be reached for the case when $(u(1/2), v(1/2)) \notin D_{\widetilde{g}}$. Hence, for λ large, $(u(1/2), v(1/2)) \in D_{\widetilde{f}} \bigcap D_{\widetilde{g}}$.

Now suppose $(u(3/4), v(3/4)) \notin \overline{D}_f \bigcup \overline{D}_g$. Then by the mean value theorem, there exist $C_1, C_2 \in (1/2, 3/4)$ such that $|u^{'}(C_2)| \leq k$ and $|v^{'}(C_1)| \leq k$, where k is a constant independent of λ. Also $-(r^{N-1}u^{'})^{'} \geq 0$ on $[1/2, 3/4)$, and hence

$$-r^{N-1}u^{'}(r) \leq -C_2^{N-1}\, u^{'}(C_2) \text{ on } [1/2, C_2).$$

Thus,

$$|u^{'}(r)| \le \frac{C_2^{N-1}}{r^{N-1}}|u^{'}(C_2)| \le \frac{1}{(\frac{1}{2})^{N-1}}(\frac{3}{4})^{N-1}k = (3/2)^{N-1}k \quad \text{on } [1/2, C_2).$$

Similarly, $|v^{'}(r)| \le (\frac{3}{2})^{N-1}k$ on $[\frac{1}{2}, C_1)$. Hence, there exists $r_0 \in [\frac{1}{2}, \frac{3}{4})$ such that

$$|u^{'}(r_0)| \le \tilde{C}, \qquad |v^{'}(r_0)| \le \tilde{C},$$

where $\tilde{C}$ is a constant independent of λ. Now define

$$E(r) = \frac{1}{2}(u^{'})^2 + \frac{1}{2}(v^{'})^2 + u^{'}(r)v^{'}(r) + \lambda F(u(r), v(r)) + \lambda G(u(r), v(r)).$$

Then

$$\begin{aligned} E^{'}(r) &= u^{'}u^{''} + v^{'}v^{''} + u^{''}v^{'} + u^{'}v^{''} + \lambda f(u,v)u^{'} + \lambda f(u,v)v^{'} \\ &\quad + \lambda \int_0^{u(r)} f_2^{'}(s, v(r))v^{'}ds + \lambda \int_0^{v(r)} f_1^{'}(u(r), t)u^{'}dt \\ &\quad + \lambda g(u,v)u^{'} + \lambda g(u,v)v^{'} + \lambda \int_0^{u(r)} g_2^{'}(s, v(r))v^{'}ds \\ &\quad + \lambda \int_0^{v(r)} g_1^{'}(u(r), t)u^{'}dt \\ &\le -\frac{N-1}{r}(u^{'})^2 - \frac{N-1}{r}(v^{'})^2 - \frac{2(N-1)}{r}u^{'}v^{'} \\ &= -\frac{N-1}{r}(u^{'} + v^{'})^2 \le 0, \end{aligned}$$

$E(1) = \frac{1}{2}(u^{'})^2 + \frac{1}{2}(v^{'})^2 + u^{'}(1)v^{'}(1) \ge 0$. So $E \ge 0$ on [0,1]. But

$$\begin{aligned} E(r_0) &\le 2\widetilde{C}^2 + \lambda F(u(r_0), v(r_0)) + \lambda G(u(r_0), v(r_0)) \\ &\le 2\widetilde{C}^2 + \lambda F(\delta_1, \delta_2) + \lambda G(\delta_1^*, \delta_2^*), \end{aligned}$$

where

$$F(\delta_1, \delta_2) = \max\{F(u,v) : (u,v) \in \overline{D_{\widetilde{f}} - D_f}\},$$
$$G(\delta_1^*, \delta_2^*) = \max\{G(u,v) : (u,v) \in \overline{D_{\widetilde{g}} - D_g}\}.$$

Here, $F(\delta_1, \delta_2)$ and $G(\delta_1^*, \delta_2^*)$ are negative constants. Thus $E(r_0) < 0$ for λ sufficiently large, a contradiction. Hence Lemma 2.3 is proven. □

3. Proof Of Theorem 1.1

Proof. Let λ be large such that Lemma 2.2 and Lemma 2.3 hold. Then there are three cases we need to consider.

Case 1: $(u(3/4), v(3/4)) \in \overline{D}_f \bigcap \overline{D}_g$. In this case

$$-(r^{N-1}u')' = \lambda r^{N-1} f(u,v) \le 0 \quad \text{on } (\tfrac{3}{4},1), \qquad u(\tfrac{3}{4}) \le C,\ u(1) = 0.$$

Thus by comparison arguments $u(r) \le w(r)$, where w is the solution of

$$-(r^{N-1}w')' = 0 \quad \text{on } (\tfrac{3}{4},1), \quad w(\tfrac{3}{4}) = C,\ w(1) = 0.$$

But $w(r) = \frac{C}{\int_{3/4}^1 s^{1-N}ds} \int_r^1 s^{1-N} ds$ decreases from C to 0 on $[3/4, 1]$. Thus for a fixed point $(\beta_1, \beta_2) \in D_f \bigcap D_g \,\exists\, r_1 \in (3/4, 1)$ (which is independent of λ) such that $w(r_1) \le \beta_1$, so $u(r_1) \le \beta_1$.

In the same way

$$-(r^{N-1}v')' = \lambda r^{N-1} g(u,v) \le 0 \quad \text{on } (\tfrac{3}{4},1), v(\tfrac{3}{4}) \le C,\ v(1) = 0.$$

So $\exists\, r_2 \in (3/4, 1)$ (which is independent of λ) such that $v(r_2) \le \beta_2$. Let $r_3 = \max(r_1, r_2)$ (r_3 is independent of λ). Then $u \le \beta_1$ and $v \le \beta_2$ on $(r_3, 1)$.

Now on $J_2 = (r_3, 1)$

$$\begin{aligned} -(r^{N-1}(\beta_1 - u)')' &= -\lambda r^{N-1} f(u,v) \ge -\lambda r^{N-1} f(\beta_1, \beta_2) \\ &\ge \frac{\lambda[-f(\beta_1,\beta_2)]}{\beta_1} r^{N-1}(\beta_1 - u). \end{aligned}$$

Then by Lemma 2.1

$$\frac{\lambda[-f(\beta_1,\beta_2)]}{\beta_1} \le \lambda_1(J_2).$$

Here $-f(\beta_1, \beta_2)$ and β_1 are two fixed positive numbers, and hence we have a contradiction for λ large.

Case 2: $(u(3/4), v(3/4)) \in \overline{D}_f - (\overline{D}_f \bigcap \overline{D}_g)$, and $v(3/4) < y_0$, where $y_0 = \max\{y : (x,y) \in \overline{D}_f \bigcap \overline{D}_g\}$. In this case we can choose a β_1 small enough such that $(\beta_1, v(3/4)) \in \overline{D}_f \bigcap \overline{D}_g$ (see Figure 3.1).

However,

$$-(r^{N-1}u')' = \lambda r^{N-1} f(u,v) \le 0 \quad \text{on } (\tfrac{3}{4},1), \quad u(\tfrac{3}{4}) \le C,\ u(1) = 0.$$

Just as before, $u(r_1) \le w(r_1) \le \beta_1$. Then $(u(r_1), v(r_1)) \in \overline{D}_f \bigcap \overline{D}_g$. And as before we can get a contradiction for λ large.

Similarly, if $(u(3/4), v(3/4)) \in \overline{D}_g - (\overline{D}_f \bigcap \overline{D}_g)$, and $u(3/4) \le x_0$, where $x_0 = \max\{x : (x,y) \in \overline{D}_f \bigcap \overline{D}_g\}$, we can also get a contradiction for λ large.

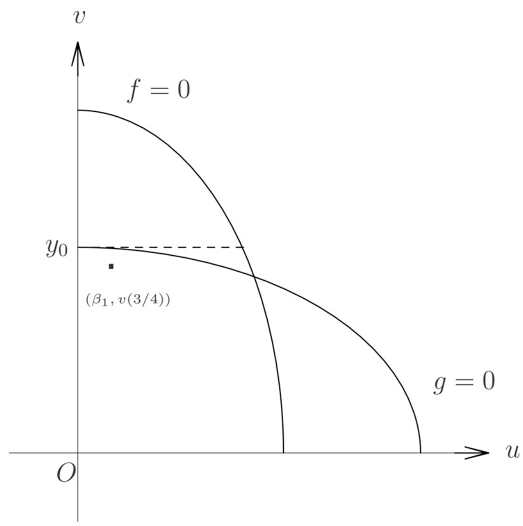

Figure 3.1

Case 3: $(u(3/4), v(3/4)) \in \{(x,y) : (x,y) \in \overline{D}_f - (\overline{D}_f \bigcap \overline{D}_g)$, and $y \geq y_0\} = D^*$ (say), where $y_0 = \max\{y : (x,y) \in \overline{D}_f \bigcap \overline{D}_g\}$. Let the curve $h = 0$ be such that it divides the region D^* into two regions D_1^* andD_2^*, as shown in Figure 3.2. Here the curve $h = 0$ is chosen so that D_1^* is strictly inside the region $\overline{D}_f$ and D_2^* is strictly outside the region $\overline{D}_g$.

If $(u(3/4), v(3/4)) \in D_1^*$, then $\exists\,(\rho_1, \rho_2) \in$ curve $h = 0$ such that $u(3/4) \leq \rho_1, v(3/4) \leq \rho_2$. Then on $J_3 := (\frac{3}{4}, 1)$

$$\begin{aligned}-(r^{N-1}(C-u)')' &= -\lambda\, r^{N-1} f(u,v) \geq -\lambda\, r^{N-1} f(\rho_1, \rho_2)\\ &\geq -\lambda\, r^{N-1} f(\rho_1^*, \rho_2^*) \geq \frac{\lambda[-f(\rho_1^*, \rho_2^*)]}{C} r^{N-1}(C-u).\end{aligned}$$

Here C is a constant as in Lemma 2.2, and $(\rho_1^*, \rho_2^*) \in$ curve $h = 0$ is such that $f(\rho_1^*, \rho_2^*) = \max\{f(u,v) : u \geq 0,\, v \geq 0, \text{with}\, h(u,v) = 0\}$. Then by Lemma 2.1,

$$\frac{\lambda[-f(\rho_1^*, \rho_2^*)]}{C} \leq \lambda_1(J_3)$$

Here $-f(\rho_1^*, \rho_2^*)$ and C are fixed positive constants. Hence, we obtain a contradiction for λ large.

If $(u(3/4), v(3/4)) \in D_2^*$, using the same method as in the proof of the claim in Lemma 2.3, we can get a contradiction for λ large.

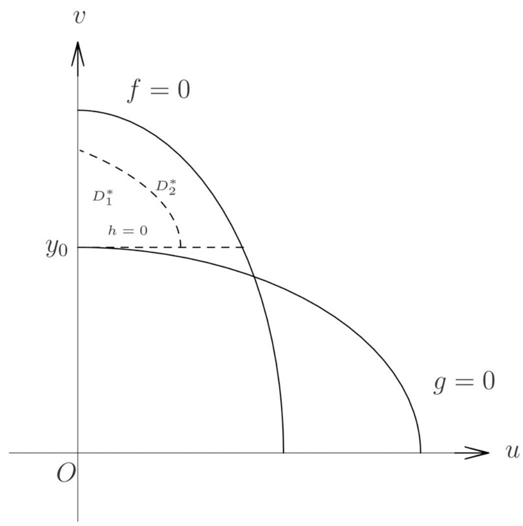

Figure 3.2

Similarly, if $(u(3/4), v(3/4)) \in \overline{D}_g - (\overline{D}_f \bigcap \overline{D}_g)$, and $u(3/4) \geq x_0$, where $x_0 = \max\{x : (x, y) \in \overline{D}_f \bigcap \overline{D}_g\}$. we can also get a contradiction for λ large.

Therefore for λ large, (1.1) has no positive solution for all the cases. Thus, Theorem 1.1 is proven. □

Remark 3.1. Consider the system

$$\begin{aligned} -\Delta u &= \lambda f(u, v); \quad x \in \Omega \\ -\Delta v &= \lambda g(u); \quad x \in \Omega \\ u &= 0 = v; \quad x \in \partial\Omega, \end{aligned}$$

where $f(u, v)$ satisfies conditions in (H1)–(H3) and $g : [0, \infty) \to R$ is a C^1 function such that $g'(u) > 0$ for $u > 0$, $g(0) < 0$, and $\exists K_2 > 0$ and $M_2 > 0$ such that $g(u) \geq K_2 u - M_2$, for all $u \geq 0$. Again this system has no positive solution for λ large. Here the proof is made easier by considering the following two situations (see Figure 3.3 and Figure 3.4):

Remark 3.2. Our non-existence result also easily extends to the multi-parameter system

$$\begin{aligned} -\Delta u &= \lambda f(u, v); \; x \in \Omega \\ -\Delta v &= \mu g(u, v); \; x \in \Omega \end{aligned}$$

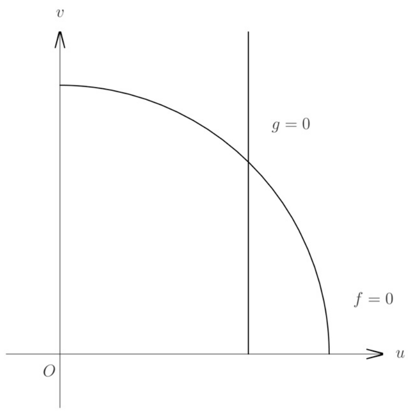

Figure 3.3

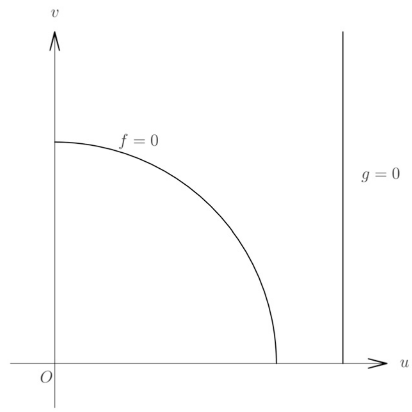

Figure 3.4

$$u = 0 = v;\ x \in \partial\Omega,$$

when f and g satisfy the hypotheses (H1)–(H3). In particular, there exists no positive solution when both λ and μ are large.

APPENDIX A

Consider the system

$$\begin{aligned} -\Delta u &= \lambda f(u,v);\ x \in \Omega \\ -\Delta v &= \lambda g(u,v);\ x \in \Omega \\ u &= 0 = v; x \in \partial\Omega, \end{aligned} \tag{A.1}$$

where Ω is a smooth bounded region in R^N, $\partial\Omega$ is its boundary and λ is a positive parameter. Let $f, g : [0,\infty) \times [0,\infty) \to R$ be continuous and assume that there exist $\sigma_1 > 0$, $\sigma_2 > 0$, $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$f(u,v) \geq \sigma_1 v + \varepsilon_1,\ \forall u, v \in [0,\infty) \tag{A.2}$$

and

$$g(u,v) \geq \sigma_2 u + \varepsilon_2, \forall u, v \in [0,\infty). \tag{A.3}$$

Then we prove the following:

Theorem A. *Let* (A.2)–(A.3) *hold. Then the system* (A.1) *has no positive solution if* $\lambda > \frac{\lambda_1}{\sqrt{\sigma_1\sigma_2}}$ *where* λ_1 *is the first eigenvalue of the* $-\Delta$ *with Dirichlet boundary conditions.*

Proof. Multiplying the first equation in (A.1) by a positive eigenfunction, say ϕ corresponding to λ_1, and using (A.2) we obtain

$$-\int_\Omega \Delta u\,\phi\,dx \geq \int_\Omega \lambda(\sigma_1 v + \varepsilon_1)\,\phi\,dx.$$

That is,

$$\int_\Omega u\,\lambda_1\,\phi\,dx \geq \int_\Omega \lambda(\sigma_1 v + \varepsilon_1)\,\phi\,dx. \tag{A.4}$$

Similarly, using the second equation in (A.1) and (A.3) we obtain

$$\int_\Omega v\lambda_1\phi\,dx \geq \int_\Omega \lambda(\sigma_2 u + \varepsilon_2)\,\phi\,dx. \tag{A.5}$$

Combining (A.4) and (A.5) we obtain

$$\int_\Omega [\lambda_1 - \lambda^2 \frac{\sigma_1\sigma_2}{\lambda_1}]\,v\phi\,dx \geq \int_\Omega \lambda\,[\lambda \frac{\sigma_2\varepsilon_1}{\lambda_1} + \epsilon_2]\phi\,dx. \tag{A.6}$$

Inequality (A.6) clearly leads to a contradiction if $\lambda > \frac{\lambda_1}{\sqrt{\sigma_1\sigma_2}}$. Hence the result. □

APPENDIX B

Consider the above system (A.1), but with f and g satisfying the following conditions:

$(B.1)$ $f, g : [0,\infty)\times[0,\infty) \to R,$ continuous,
$f_u > 0,\ f_v > 0,\ g_u > 0,\ g_v > 0, f(0,0) < 0,\ g(0,0) \geq 0;$

$(B.2)$ $\exists K_1 > 0, K_2 > 0, M > 0$ such that
$f(u,v) \geq K_1 v - M, g(u,v) \geq K_2 u,$ for all $u \geq 0,\ v \geq 0.$

Then we prove

Theorem B. *Let* $(B.1)$ *and* $(B.2)$ *hold. Then the system* (A.1) *has no positive solution for* λ *large.*

Proof. It is easy to see that Lemma 2.2 holds in this case as well.

Let $D_f := \{(u,v) : u > 0, v > 0, f(u,v) < 0\}$. Then we first claim that for fixed $(\beta_1,\beta_2) \in D_f$, $u(1/2) < \beta_1$ and $v(1/2) < \beta_2$ for λ sufficiently large (see Figure A.1).

Suppose $u(1/2) \geq \beta_1$; then on $J_1 := (1/4, 1/2)$,

$$-(r^{N-1}v')' = \lambda\,r^{N-1}g(u,v) \geq \lambda\,r^{N-1}g(\beta_1,0) \geq \frac{\lambda g(\beta_1,0)}{C} r^{N-1}v.$$

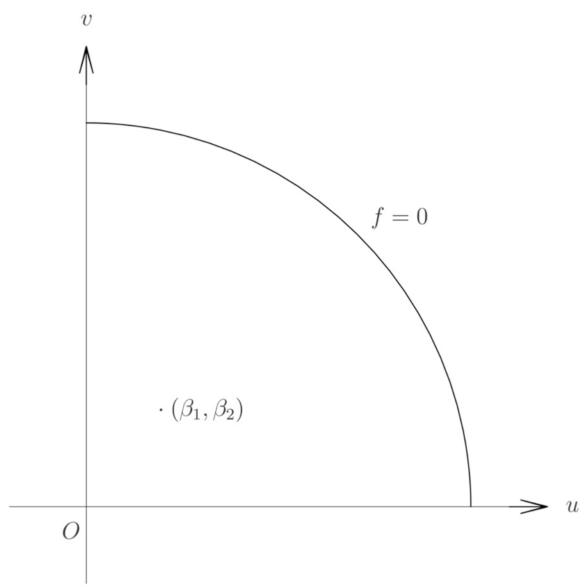

Figure A.1

Here, $g(\beta_1, 0)$ is a fixed number, and C is a constant as in Lemma 2.2. Then by Lemma 2.1

$$\frac{\lambda g(\beta_1, 0)}{C} \leq \lambda_1(J_1),$$

which is a contradiction for λ large. In the same way, we can show that $v(1/2) \geq \beta_2$ is also impossible for λ large. Thus we have proven the claim.

Now for λ sufficiently large, on $J_2 = (1/2, 1)$

$$\begin{aligned} -(r^{N-1}(\beta_1 - u)')' &= -\lambda r^{N-1} f(u, v) \geq -\lambda r^{N-1} f(\beta_1, \beta_2) \\ &\geq \frac{\lambda[-f(\beta_1, \beta_2)]}{\beta_1} r^{N-1}(\beta_1 - u). \end{aligned}$$

Then by Lemma 2.1,

$$\frac{\lambda[-f(\beta_1, \beta_2)]}{\beta_1} \leq \lambda_1(J_2),$$

which is a contradiction for λ large. Hence Theorem B is proven. □

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