Commun. Math. Phys. 183, 205–252 (1997)

Communications in
Mathematical
Physics

$N=2$ Affine Superalgebras and Hamiltonian Reduction in $N=2$ Superspace

C. Ahn[1], E. Ivanov[2,], A. Sorin[2,***]**

[1] Dept. of Physics, Kyung Hee Univ., Seoul 130-701, Korea. cahn@photon.kyunghee.ac.kr
[2] Bogoliubov Laboratory of Theoretical Physics, JINR, 141980, Dubna, Moscow Region, Russia

Received: 15 September 1995 / Accepted: 20 July 1996

Abstract: We construct $N=2$ affine current algebras for the superalgebras $sl(n|n-1)^{(1)}$ in terms of $N=2$ supercurrents subjected to nonlinear constraints and discuss the general procedure of the hamiltonian reduction in $N=2$ superspace at the classical level. We consider in detail the simplest case of $N=2$ $sl(2|1)^{(1)}$ and show how $N=2$ superconformal algebra in $N=2$ superspace follows via the hamiltonian reduction. Applying the hamiltonian reduction to the case of $N=2$ $sl(3|2)^{(1)}$, we find two new extended $N=2$ superconformal algebras in a manifestly supersymmetric $N=2$ superfield form. Decoupling of four component currents of dimension 1/2 in them yields, respectively, $u(2|1)$ and $u(3)$ Knizhnik–Bershadsky superconformal algebras. We also discuss how the $N=2$ superfield formulations of $N=2$ W_3 and $N=2$ $W_3^{(2)}$ superconformal algebras come out in this framework, as well as some unusual extended $N=2$ superconformal algebras containing constrained $N=2$ stress tensor and/or spin 0 supercurrents.

1. Introduction

For the last several years important progress has been achieved in understanding the role of world-sheet superconformal symmetry and target space symmetry of nonlinear σ-models in the context of string theory and topological field theory [1–3]. The BRST structure of the bosonic string (W_n string) generates a topologically twisted $N=2$ superconformal algebra [4] ($N=2$ super-W_n algebra [5, 6]). In obtaining these results, heavy use of the hamiltonian reduction from WZNW models based on the superalgebra $sl(n|n-1)$ has been made. Futhermore, any superstring theory possesses $N=3$ twisted supersymmetry [5]. Recently, BRST structure has been systematically constructed for superstrings with N supersymmetries by the hamiltonian reduction of the affine extension of $osp(N+2|2)$ [7]. The $N=2$ analog for topological strings is the twisted $N=4$ $su(2)$ superconformal algebra (SCA) which has been obtained by the reduction of the affine extension of $sl(2|2)$ in [8].

** E-mail: eivanov@thsun1.jinr.dubna.su
*** E-mail: sorin@thsun1.jinr.dubna.su

As these and many other examples demonstrate, the hamiltonian reduction is a powerful method of deducing new conformal [9–11] and superconformal algebras and analysing the symmetry structure of the conformal field theory and string theory models. Since a natural arena for studying various superconformal symmetries and the related field theory models is provided by superspace, it is tempting to have convenient superspace generalizations of the hamiltonian reduction. The $N=1$ superspace version of this procedure in various aspects was discussed in Ref. [12]. On the other hand, a lot of interesting models (both in string theory and topological field theory) reveal $N=2$ superconformal symmetries, manifestly covariant formulations of which require $N=2$ superspace. Motivated by this, in the present paper we generalize the hamiltonian reduction procedure to $N=2$ superspace.

Let us recollect some well-known facts which are relevant to the problems we address in the present paper.

Knizhnik [13] and Bershadsky [14] have proposed SCAs with quadratic nonlinearity having as subalgebras $u(n)$ and $so(n)$ affine algebras. It has been shown later [15] that the nonlinear $so(3)$ and $so(4)$ Knizhnik–Bershadsky (KB) SCAs can be embedded as subalgebras in the usual linear $so(3)$ and $so(4)$ extended SCAs [16] after passing to some new basis for the currents of the latter (related to the standard one by an invertible nonlinear transformation). By construction, the usual $N=2$ and $N=4$ $su(2)$ SCAs [16] are the same as $u(1)$ and $u(2)$ KB SCAs, respectively.

Polyakov [17] has found that there exist two types of classical hamiltonian reductions for $sl(3)$: one yields the W_3 algebra while the other leads to $W_3^{(2)}$ which is a $u(1)$ "quasi" SCA in the sense that dimension 3/2 fields are bosonic ("wrong" statistics) and, besides, it reveals a quadratic nonlinearity in the $u(1)$ current in its operator product expansions (OPEs). Bershadsky [18] has further explained its structure in detail. In Ref. [19] new infinite families of nonlinear extended conformal algebras, $u(n)$ and $sp(2n)$ quasi SCAs, have been found. Independently it has been shown [11, 20] that $u(n)$ quasi SCAs can be constructed by the hamiltonian reductions of affine algebras $sl(n)^{(1)}$, based on non-principal embeddings of $sl(2)$ into $sl(n)$. A $N=2$ supersymmetric extension of $W_3^{(2)}$ containing both $W_3^{(2)}$ and $N=2$ SCA as genuine subalgebras have been constructed in [21, 22] by means of hamiltonian reduction of the affine $sl(3|2)^{(1)}$ (at the level of component currents). Recently, a formulation of this extended SCA in terms of constrained $N=2$ superfields has been presented [23].

It was demonstrated in [24–26] that new SCAs with quadratic nonlinearity, so-called $Z_2 \times Z_2$ graded SCAs, can be obtained by combining both fermionic and bosonic spin-3/2 currents in the same $osp(m|2n)$ or $u(m|n)$ supermultiplet. The $u(n)$ KB SCAs and the algebra $W_3^{(2)}$ can be identified with $Z_2 \times Z_2$ graded SCAs associated with the superalgebras $u(n|0)$and $u(0|1)$, respectively.[1]

By applying the classical hamiltonian reduction to the affine Lie superalgebra $sl(n|2)^{(1)}$ and putting the constraint on the currents valued in its bosonic $sl(2)$ part, in [27] the classical $u(n)$ KB SCAs has been recovered in a new setting. In [28], this analysis was promoted to $N=1$ superspace and a $N=1$ extension of $u(n)$ KB SCAs has been constructed (at the classical level). However, an attempt to incorporate $N=2$ supersymmetry has failed. As we will show, this happened just because nonlinear constraints on $N=2$ affine supercurrents have not been involved in the game.

[1] There exist other conventions for these superalgebras, see, e.g., Ref. [25].

As was already said, the aim of this paper is to develop the hamiltonian reduction at the classical level directly in $N=2$ superspace. In short, its main steps are: (i) construction of an $N=2$ affine current algebra for some superalgebra admitting a complex structure (we limit our consideration here to the superalgebras $sl(n|n-1)$); (ii) imposing appropriate constraints on the relevant superalgebra valued $N=2$ supercurrents; (iii) deducing $N=2$ extended superconformal algebras in the $N=2$ superfield formalism. We would like to especially emphasize that we are always dealing with the $N=2$ superfield approach in our scheme. To our knowledge, this was not done before. Another point to be mentioned is that our construction here is purely algebraic and does not resort to any specific field theory realization of $N=2$ affine current superalgebras, e.g. to their WZNW realizations. This is the difference from, e.g., Ref. [12] where an $N=1$ superspace version of the hamiltonian reduction was discussed in the WZNW context. Also, we will be mainly interested in such extended $N=2$ SCAs which include as a subalgebra the standard linear $N=2$ SCA, i.e., contain an $N=2$ superconformal stress tensor among their defining supercurrents.

The paper is organized as follows. In Sect. 2 we construct an $N=2$ $sl(n|n-1)^{(1)}$ current algebra in terms of $N=2$ supercurrents subjected to nonlinear constraints. In Sect. 3 we describe the general procedure of the hamiltonian reduction in $N=2$ superspace and in Sect. 4 we exemplify it by the simplest case of $N=2$ $sl(2|1)^{(1)}$ which gives rise to the standard $N=2$ SCA. In Sect. 5 we consider the case of $N=2$ $sl(3|2)^{(1)}$. We reproduce the previously known $N=2$ W_3 and $N=2$ $W_3^{(2)}$ SCAs in the $N=2$ superfield formulation and find two new $N=2$ extended SCAs. We explain how the factorization of the dimension 1/2 component currents in these superalgebras works. And finally in Sect. 6 we end with a few closing remarks. In the Appendices, we give notations for $sl(n|n-1)$ superalgebras, $u(m|n)$ SCA and a different realization of $sl(n|n-1)$.

2. $N=2$ Current Algebra for $sl(n|n-1)^{(1)}$

In [29] Hull and Spence have constructed an $N=2$ current algebra for the bosonic algebra g in terms of $N=2$ superfield currents satisfying *nonlinear* constraints. The only essential restriction on g is that it is even-dimensional and admits a complex structure. The quadratic terms appearing in the r.h.s. of superoperator product expansions (SOPEs) between the supercurrents are necessary for the consistency between these SOPEs and the aforementioned nonlinear constraints. The nonlinearity of the $N=2$ current algebra while it is written in terms of $N=2$ supercurrents is the price for manifest $N=2$ supersymmetry. When formulated via ordinary currents or $N=1$ supercurrents, the algebra can be put in a linear form (in an appropriate basis).

If g is an ordinary bosonic algebra, all the $N=2$ affine supercurrents are fermionic and we cannot put them to be constants. On the other hand, this kind of constraints imposed on bosonic (super)currents is of common use in the standard hamiltonian reduction scheme. We are going to generalize the latter to $N=2$ superspace, expecting such a generalization to allow us to deduce extended $N=2$ SCAs (both previously known and new) in a manifestly supersymmetric $N=2$ superfield fashion. To be able to impose the aforementioned constraints on the affine supercurrents, we need to have bosonic ones among them. A natural way to achieve this is to deal with $N=2$ affine extensions of *superalgebras*. So we are led to generalize the approach of Ref. [29] to the superalgebras admitting a complex structure. In this paper we confine our consideration to the superalgebras $sl(n|n-1)$.

Let g be a classical simple Lie superalgebra $g = g_0 \oplus g_1$, where g_0 is the bosonic subalgebra and g_1 is the fermionic subspace, with the generators t_A satisfying graded commutation relations $[t_A, t_B\} = F_{AB}{}^C t_C$. Let us introduce new structure constants, $f_{AB}{}^C = (-1)^{(d_A+1)d_B} F_{AB}{}^C$, where for $t_A \in g_\alpha, \alpha \in 0,1$ we used the grading $d_A = \alpha + 1$. Therefore, $f_{AB}{}^C$ are antisymmetric in the indices A, B when A, B correspond to bosonic generators and symmetric otherwise. It is convenient to choose a complex basis for g, so that its generators are labelled by a and $\bar{a}$, $a = 1, 2, \ldots, \frac{1}{2}\dim g = \frac{1}{2}((2n-1)^2 - 1)$. In this basis the complex structure associated with the second supersymmetry has eigenvalue $+i$ on the generators t_a and $-i$ on the conjugated ones $t_{\bar{a}} (= t_a^\dagger)$. The Killing metric $g_{a\bar{b}}$ is given by $Str(t_a t_{\bar{b}})$, $g_{a\bar{b}}$ being symmetric for the indices related to bosonic generators and antisymmetric otherwise. Any index can be raised and lowered with $g^{a\bar{b}}$ and $g_{a\bar{b}}$.

The affine superalgebra $\hat{g} = sl(n|n-1)^{(1)}$ we deal with in this paper has an equal number $2n(n-1)$ of fermionic and bosonic supercurrents. For example, in the fermionic g valued supercurrent in the fundamental representation $\mathcal{J} \equiv \mathcal{J}_A t_B g^{AB}$, top-left $n \times n$ and bottom-right $(n-1) \times (n-1)$ matrix elements are fermionic, so that $d_a, d_{\bar{a}} = 1$. Then the bosonic supercurrents are entries of the top-right $n \times (n-1)$ and bottom-left $(n-1) \times n$ blocks in the supercurrent matrix, so for them $d_a, d_{\bar{a}} = 2$. In the scheme of hamiltonian reduction which will be explained in the next section we impose non-zero constraints just on these supercurrents.

We refer the reader to Ref. [29] for details of how the $N=2$ current algebra can be formulated in $N=2$ superspace. The only new thing to be kept in mind in our case is that now there are extra *bosonic* supercurrents besides the fermionic ones. The presence of supercurrents with different statistics will play an important role in our construction. This property will manifest itself in the appearance of some extra (-1) factors in the r.h.s. of SOPEs defining the $N=2$ affine superalgebra.

With all these remarks taken into account, we summarize the $N=2$ affine current algebra corresponding to $sl(n|n-1)^{(1)}$ with the level k as the following set of SOPEs between $N=2$ superfield currents satisfying the appropriate nonlinear constraints:[2]

$$\mathcal{J}_a(Z_1)\mathcal{J}_b(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}} f_{ab}{}^c \mathcal{J}_c - \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}(-1)^{(d_a+1)d_{\bar{c}}}(-1)^{(d_b+1)d_{\bar{c}}} f_{a\bar{c}}{}^d f_b{}^{\bar{c}e} \mathcal{J}_d \mathcal{J}_e ,$$

$$\mathcal{J}_{\bar{a}}(Z_1)\mathcal{J}_{\bar{b}}(Z_2) = \frac{\theta_{12}}{z_{12}} f_{\bar{a}\bar{b}}{}^{\bar{c}} \mathcal{J}_{\bar{c}} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}(-1)^{(d_{\bar{a}}+1)d_c}(-1)^{(d_{\bar{b}}+1)d_c} f_{\bar{a}c}{}^{\bar{d}} f_{\bar{b}}{}^{c\bar{e}} \mathcal{J}_{\bar{d}} \mathcal{J}_{\bar{e}} ,$$

$$\mathcal{J}_a(Z_1)\mathcal{J}_{\bar{b}}(Z_2) = -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{1}{2}k g_{a\bar{b}} + \frac{1}{z_{12}} k g_{a\bar{b}} + \frac{\theta_{12}}{z_{12}} f_{a\bar{b}}{}^c \mathcal{J}_c - \frac{\bar{\theta}_{12}}{z_{12}} f_{a\bar{b}}{}^{\bar{c}} \mathcal{J}_{\bar{c}}$$

$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[f_{a\bar{b}}{}^c \bar{\mathcal{D}}\mathcal{J}_c - \frac{1}{k}(-1)^{(d_a+1)d_{\bar{c}}}(-1)^{(d_{\bar{b}}+1)d_{\bar{c}}} f_{a\bar{c}}{}^d f_{\bar{b}}{}^{c\bar{e}} \mathcal{J}_d \mathcal{J}_{\bar{e}} \right] , \qquad (2.1)$$

---

[2] By Z we denote the coordinates of $1D$ $N=2$ superspace, $Z = (z, \theta, \bar{\theta})$. From now on we do not write down explicitly the regular parts of SOPEs. All the supercurrents (currents) appearing in the r.h.s. of SOPEs (OPEs) are evaluated at the point Z_2 (z_2).

where

$$\theta_{12} = \theta_1 - \theta_2, \qquad \bar\theta_{12} = \bar\theta_1 - \bar\theta_2, \qquad z_{12} = z_1 - z_2 + \frac{1}{2}(\theta_1\bar\theta_2 + \bar\theta_1\theta_2), \tag{2.2}$$

and the constraints on the supercurrents read

$$\mathscr{D}\mathscr{J}_a - \frac{1}{2k}(-1)^{d_a} f_a{}^{bc}\mathscr{J}_b\mathscr{J}_c = 0, \qquad \bar{\mathscr{D}}\mathscr{J}_{\bar a} + \frac{1}{2k}(-1)^{d_{\bar a}} f_{\bar a}{}^{\bar b\bar c}\mathscr{J}_{\bar b}\mathscr{J}_{\bar c} = 0 \tag{2.3}$$

(the summation is assumed over repeated indices). Here, we work with complex fermionic covariant derivatives

$$\mathscr{D} = \frac{\partial}{\partial\theta} - \frac{1}{2}\bar\theta\partial, \qquad \bar{\mathscr{D}} = \frac{\partial}{\partial\bar\theta} - \frac{1}{2}\theta\partial$$

satisfying the algebra

$$\{\mathscr{D},\bar{\mathscr{D}}\} = -\partial\,(= -\partial_z)\,, \tag{2.4}$$

all other anticommutators are vanishing. If we restrict the indices in (2.1) and (2.3) to the fermionic supercurrents we reproduce the $N=2$ $sl(n)^{(1)} \oplus sl(n-1)^{(1)} \oplus u(1)^{(1)}$ affine current algebra [29]. We have checked that the whole $N=2$ current superalgebra (2.1) with the nonlinear constraints (2.3) satisfies the standard Z_2 graded Jacobi identities and that SOPEs of the l.h.s. of (2.3) with any affine supercurrent vanish on the shell of constraints (the presence of nonlinear terms in the r.h.s. of (2.1) is crucial for this). When we consider this superalgebra at the quantum level (to all orders in contractions between the supercurrents), then there appears an extra term, $\frac{1}{2}(-1)^{d_a+1} f_{a\bar c}{}^{d} f_{\bar b}^{\bar c\bar d}$ in $\frac{\theta_{12}\bar\theta_{12}}{z_{12}^2}$ in the r.h.s. of SOPE $\mathscr{J}_a(Z_1)\mathscr{J}_{\bar b}(Z_2)$. This is due to the fact that there exist additional contractions between the supercurrents at the quantum level. In the remainder of this paper we will deal with the classical relations (2.1) and (2.3).

Generalizing the well-known Sugawara construction to $N=2$ superspace yields the following formula for the improved $N=2$ stress tensor in terms of the affine supercurrents $\mathscr{J}_a, \mathscr{J}_{\bar a}$,

$$\mathscr{T}_{sug} = \frac{1}{k} g^{a\bar b}\mathscr{J}_a\mathscr{J}_{\bar b} + \alpha_i\bar{\mathscr{D}}\mathscr{H}_i + \alpha_{\bar j}\mathscr{D}\mathscr{H}_{\bar j}\,. \tag{2.5}$$

We denote by $\mathscr{H}_i, \mathscr{H}_{\bar i}$ $(i = 1,2,\ldots,n-1)$ the supercurrents associated with Cartan generators of $sl(n|n-1)$. The $N=2$ super stress tensor satisfies the following SOPE:

$$\mathscr{T}_{sug}(Z_1)\mathscr{T}_{sug}(Z_2) = \frac{c}{z_{12}^2} + \left[\frac{\theta_{12}\bar\theta_{12}}{z_{12}^2} - \frac{\theta_{12}}{z_{12}}\mathscr{D} + \frac{\bar\theta_{12}}{z_{12}}\bar{\mathscr{D}} + \frac{\theta_{12}\bar\theta_{12}}{z_{12}}\partial\right]\mathscr{T}_{sug} \tag{2.6}$$

with

$$c = -2k\,\alpha_i\alpha_{\bar j}g_{i\bar j}\,. \tag{2.7}$$

With respect to this $\mathscr{T}_{sug}$, the supercurrents $\mathscr{H}_i, \mathscr{H}_{\bar i}$ are quasi-superprimary superfields of the dimension 1/2 with $u(1)$ charge $+1,-1$, respectively. All other affine $N=2$

supercurrents are superprimary,

$$\mathcal{T}_{sug}(Z_1)\mathcal{J}_{a(\bar{a})}(Z_2) = \left[s_{a(\bar{a})}\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} - \frac{\theta_{12}}{z_{12}}\mathcal{D} + \frac{\bar{\theta}_{12}}{z_{12}}\bar{\mathcal{D}} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\partial + q_{a(\bar{a})}\frac{1}{z_{12}} \right] \mathcal{J}_{a(\bar{a})} \,. \tag{2.8}$$

Their dimensions (superspins) $s_{a(\bar{a})}$ and $u(1)$ charges $q_{a(\bar{a})}$ depend in a certain way on the parameters $\alpha_i, \alpha_{\bar{j}}$. The explicit formulas for them will be given later, for each specific example we will consider.

Our last remark in this section is that for the superalgebra $sl(n|n-1)$ (and seemingly for other superalgebras admitting a complex structure) the above $N=2$ extension actually coincides with the $N=1$ extension given in [12, 30]. In other words, the latter possesses a *hidden* $N=2$ supersymmetry which becomes manifest in terms of constrained $N=2$ supercurrents. Indeed, due to the fact that the generators of $sl(n|n-1)$ can be divided into the pairs of mutually conjugated ones, the relevant $N=1$ supercurrent for each pair is complex and its component content (two real spins $\frac{1}{2}$ and two real spins 1) is such that these components can be combined into a $N=2$ supermultiplet.[3] To explicitly demonstrate this, let us solve the constraints (2.3) via unconstrained $N=1$ supercurrents $J_a, J_{\bar{a}}$,

$$\mathcal{J}_a = J_a + \theta^1 \left[iDJ_a + (-1)^{d_a}\frac{1}{k} f_a{}^{bc} J_b J_c \right],$$

$$\mathcal{J}_{\bar{a}} = J_{\bar{a}} - \theta^1 \left[iDJ_{\bar{a}} + (-1)^{d_{\bar{a}}}\frac{1}{k} f_{\bar{a}}{}^{\bar{b}\bar{c}} J_{\bar{b}} J_{\bar{c}} \right] . \tag{2.9}$$

Here the $N=1$ supercurrents are defined on a real $N=1$ superspace $\widetilde{Z} = (z, \theta^2)$, $\theta^2 \equiv \frac{1}{2}(\theta + \bar{\theta})$, $D = \frac{\partial}{\partial\theta^2} - \theta^2\partial$ is a $N=1$ covariant fermionic derivative and $\theta^1 \equiv \frac{1}{2i}(\theta - \bar{\theta})$ is an extra fermionic coordinate. The SOPEs between the $N=1$ $sl(n|n-1)^{(1)}$ affine supercurrents $J_A(\widetilde{Z})$ [12, 30] are given by

$$J_A(\widetilde{Z}_1)J_B(\widetilde{Z}_2) = \frac{1}{\widetilde{z_{12}}}kg_{AB} + \frac{\widetilde{\theta_{12}}}{\widetilde{z_{12}}} f_{AB}{}^C J_C \,, \tag{2.10}$$

where

$$\widetilde{\theta_{12}} = \theta_1^2 - \theta_2^2, \qquad \widetilde{z_{12}} = z_1 - z_2 - \theta_1^2\theta_2^2 \,. \tag{2.11}$$

In a complex basis, the indices $A, B, \ldots$ can be divided into the two sets of the barred and unbarred indices, thus demonstrating that the number of $N=1$ supercurrents in the present case coincides with the number of $N=2$ ones (of course, these complex $N=1$ supercurrents are reducible, each containing two real $N=1$ supermultiplets). The superalgebra (2.10) is equivalent to the superalgebra (2.1) supplemented with the nonlinear constraints (2.3). The $N=1$ superfield formulation clearly demonstrates that the nonlinearities in the r.h.s. of Eqs. (2.1) are fake: they appear as the price for manifest $N=2$ supersymmetry. In what follows the $N=1$ formulation will be a useful guide of how to impose constraints on the relevant $N=2$ supercurrents

[3] Similar arguments for the case of the bosonic algebra g were given in [31].

corresponding to different embeddings of $sl(2|1)$ into $sl(n|n-1)$ and to extract those preserving $N=2$ supersymmetry from their general set.

In the next sections, we will discuss different hamiltonian reductions of the $N=2$ $sl(n|n-1)^{(1)}$ current algebra in $N=2$ superspace for the particular cases of $n=2,3$. But before that we will sketch the basic peculiarities of the $N=2$ superspace version of the hamiltonian reduction procedure.

3. Hamiltonian Reduction

To illustrate the basic idea of different reductions, we start by considering how we can obtain extended $N=2$ SCAs by imposing reduction constraints on the $N=2$ affine supercurrents which we defined in Sect. 2.

From now on we will deal with the matrix elements $\mathcal{J}_{mn}$ of the $sl(n|n-1)$ valued affine $N=2$ supercurrent (with the $sl(n|n-1)$ generators in the fundamental representation) rather than with its adjoint representation components labelled by indices $a, \bar{a}$. The explicit relation between them is given by

$$\mathcal{J}_{mn} = \begin{pmatrix} \mathcal{H}_{\bar{1}} & & & & & & \\ & \mathcal{H}_{\bar{2}} + \mathcal{H}_1 & & \text{unbarred indices} & & & \\ & & \cdot & & & & \\ & & & \mathcal{H}_{\overline{n-1}} + \mathcal{H}_{n-2} & & & \\ & & & & \mathcal{H}_{n-1} & & \\ & \text{barred indices} & & & & \mathcal{H}_{\bar{1}} + \mathcal{H}_1 & \\ & & & & & \cdot & \\ & & & & & & \mathcal{H}_{\overline{n-1}} + \mathcal{H}_{n-1} \end{pmatrix} \tag{3.1}$$

(see also Appendices A and B).

We will consider only linear reduction constraints as in [9–11]. Then we are led to equate some of $\mathcal{J}_{mn}$ (we denote the corresponding subset of indices by the symbol "hat") to constants

$$\Phi_{\hat{m}\hat{n}} \equiv \mathcal{J}_{\hat{m}\hat{n}} - c_{\hat{m}\hat{n}} = 0\,. \tag{3.2}$$

The entries of the constant supermatrix $c_{\hat{m}\hat{n}}$ can be either 0, which is possible both for bosonic and fermionic supercurrents, or 1, which is admissible only for bosonic supercurrents. In order to produce $N=2$ supersymmetric algebras these constraints should be invariant with respect to $N=2$ superconformal transformations generated by the improved $N=2$ stress tensor (2.5), which means that the constrained supercurrents with nonzero $c_{\hat{m}\hat{n}}$ should have zero spin and $u(1)$ charge.

In Ref. [30], W superalgebras which can be obtained by the reductions associated with different embeddings of $osp(1|2)$ into $sl(n|n-1)$ have been classified in $N=1$ superspace. Once we know the constraints in $N=1$ superspace, the relation (2.9) gives us constraints in $N=2$ superspace. Some of the constraints in $N=1$ superspace, being rewritten in $N=2$ superspace, explicitly break $N=2$ supersymmetry. Meanwhile, we wish to deal with only those reductions which preserve $N=2$ supersymmetry, because our eventual aim is to get extended SCAs containing $N=2$ SCA as a subalgebra. Only a subset of constraints in $N=1$ superspace preserves

$N=2$ supersymmetry, namely those which after substitution into (2.9) produce no explicit θ's in the r.h.s., i.e. lead to the $N=2$ constraints in the form (3.2). Thus, we can choose the appropriate subset of constraints in $N=1$ superspace and then extract the constraints in $N=2$ superspace from (2.9).

Then the first-class constraints, i.e. those which commute among themselves on the constraints shell, generate a gauge invariance. An infinitesimal gauge transformation of $\mathcal{J}_{kl}$ induced by $\Phi_{\hat{m}\hat{n}}$ with a gauge parameter $\Lambda_{\hat{m}\hat{n}}$ can easily be calculated,

$$\delta_\Lambda \mathcal{J}_{kl}(Z_2) = \frac{1}{2\pi i}\oint dZ_2 \Lambda_{\hat{m}\hat{n}}(Z_1)(\Phi_{\hat{m}\hat{n}}(Z_1)\mathcal{J}_{kl}(Z_2))\,|_{\{\Phi_{\hat{m}\hat{n}}=0\}}\,, \tag{3.3}$$

where the symbol $|_{\{\Phi_{\hat{m}\hat{n}}=0\}}$ means that after computing the SOPEs we should pass on the constraints shell by imposing the constraints (3.2) on the resulting expression and the gauge parameters $\Lambda_{\hat{m}\hat{n}}$ are general $N=2$ superfields which do not depend on $\mathcal{J}_{kl}$. It is clear that the variation of the l.h.s. of (2.3) vanishes identically because the SOPEs of (2.3) with any $\mathcal{J}_{kl}$, and, in particular, with $\Phi_{\hat{m}\hat{n}}$ are zero on the shell of (2.3) (see the discussion in the paragraph below (2.4)).

By definition, an extended $N=2$ SCA constructed by the hamiltonian reduction based on the constraints (3.2) is a superalgebra generated by gauge invariant differential – polynomial functionals of affine supercurrents $\mathcal{J}_{kl}$, including some $N=2$ stress tensor. It is possible to find these superalgebras by using Dirac construction. Let us recall its main steps.

At first, we should fix the gauge, which means that we are led to enlarge the original set of first-class constraints by adding the gauge-fixing conditions (standard gauge-fixing procedure), such that the total set of constraints becomes second-class. We denote this extended set of constraints by $\Psi_{\hat{m}\hat{n}}$. The number of constraints $\Psi_{\hat{m}\hat{n}}$ is exactly twice the number of $\Phi_{\hat{m}\hat{n}}$. For the remaining unconstrained supercurrents we will use in this section Greek indices, $\alpha, \beta, \ldots$. Clearly, once a gauge freedom with respect to the Λ transformations has been somehow fixed, the surviving supercurrents $\check{\mathcal{J}}_{\alpha\beta}$ are expressed as some gauge invariant differential functionals of the original affine supercurrents.

Secondly, we should construct Dirac brackets between these gauge invariant supercurrents. We generalize this procedure to the $N=2$ supersymmetric case and represent Dirac brackets in an equivalent form of SOPEs. The new rules for calculation of SOPEs of the gauge invariant supercurrents which we denote by brackets with a star, $(\mathcal{J}_{\alpha\beta}(Z_1)\mathcal{J}_{\gamma\sigma}(Z_2))^*$, can be defined in terms of original SOPEs of the affine supercurrents as follows (the supercurrents entering this star bracket are gauge invariant, but for brevity we omit the symbol ˇ above them):

$$\begin{aligned}(\mathcal{J}_{\alpha\beta}(Z_1)\mathcal{J}_{\gamma\sigma}(Z_2))^* &\equiv (\widetilde{\mathcal{J}_{\alpha\beta}}(Z_1)\widetilde{\mathcal{J}_{\gamma\sigma}}(Z_2))|_{\{\Psi_{\hat{m}\hat{n}}=0\}} \\ &- \left[\frac{1}{(2\pi i)^2}\oint\oint d\,Z_3 d\,Z_4 (\widetilde{\mathcal{J}_{\alpha\beta}}(Z_1)\Psi_{\hat{i}\hat{j}}(Z_3))\,\triangle^{\hat{i}\hat{j},\hat{k}\hat{l}}(Z_3,Z_4)(\Psi_{\hat{k}\hat{l}}(Z_4)\widetilde{\mathcal{J}_{\gamma\sigma}}(Z_2))\right]\Bigg|_{\{\Psi_{\hat{m}\hat{n}}=0\}}\,,\end{aligned} \tag{3.4}$$

where $\widetilde{\mathcal{J}_{\alpha\beta}}$ are functionals of the original supercurrents $\mathcal{J}_{kl}$ (including both unconstrained $\mathcal{J}_{\alpha\beta}$ and constrained $\mathcal{J}_{\hat{m}\hat{n}}$ supercurrents) which satisfy the following restrictions on the constraints shell:

$$\widetilde{\mathcal{J}_{\alpha\beta}}|_{\{\Psi_{\hat{m}\hat{n}}=0\}} = \mathcal{J}_{\alpha\beta} \tag{3.5}$$

and are arbitrary otherwise. The supermatrix $\triangle^{\hat{i}\hat{j},\hat{k}\hat{l}}(Z_1,Z_2)$ is the inverse of the supermatrix

$$\triangle_{\hat{i}\hat{j},\hat{k}\hat{l}}(Z_1,Z_2) = (\Psi_{\hat{i}\hat{j}}(Z_1)\Psi_{\hat{k}\hat{l}}(Z_2))|_{\{\Psi_{\hat{m}\hat{n}}=0\}} \,, \tag{3.6}$$

i.e.

$$\frac{1}{2\pi i}\oint d\, Z_2 \triangle^{\hat{i}\hat{j},\hat{k}\hat{l}}(Z_1,Z_2)\triangle_{\hat{k}\hat{l},\hat{m}\hat{n}}(Z_2,Z_3) = \delta^{\hat{i}}_{\hat{m}}\delta^{\hat{j}}_{\hat{n}}\theta_{13}\bar{\theta}_{13}\delta(z_1-z_3)\,. \tag{3.7}$$

Any gauge invariant supercurrent can be represented as some functional of $\check{\mathcal{J}}_{\alpha\beta}$ and SOPEs between these functionals can be calculated using SOPEs (3.4).

It is a very complicated technical problem to calculate the inverse supermatrix $\triangle^{\hat{i}\hat{j},\hat{k}\hat{l}}(Z_1,Z_2)$ in the general case. To get around this difficulty, we use the following trick. By looking at (3.4), one can observe that for $\widetilde{\mathcal{J}_{\alpha\beta}}$ satisfying

$$(\widetilde{\mathcal{J}_{\alpha\beta}}(Z_1)\Psi_{\hat{m}\hat{n}}(Z_2))|_{\{\Psi_{\hat{m}\hat{n}}=0\}} = 0 \tag{3.8}$$

the second term in the r.h.s. of (3.4) is vanishing. We are free to choose $\widetilde{\mathcal{J}_{\alpha\beta}}$ to satisfy Eq. (3.8) as these functionals are *a priori* arbitrary up to the condition (3.5) which is obviously consistent with (3.8). Then the SOPEs with star between the gauge invariant supercurrents coincide with ordinary SOPEs between $\widetilde{\mathcal{J}_{\alpha\beta}}$ on the constraints shell and so can be calculated using SOPEs (2.1) for the original affine supercurrents,

$$(\mathcal{J}_{\alpha\beta}(Z_1)\mathcal{J}_{\gamma\sigma}(Z_2))^* \equiv (\widetilde{\mathcal{J}_{\alpha\beta}}(Z_1)\widetilde{\mathcal{J}_{\gamma\sigma}}(Z_2))|_{\{\Psi_{\hat{m}\hat{n}}=0\}} \,. \tag{3.9}$$

In this way, the task of constructing $N=2$ extended superalgebras reduces to that of constructing the functionals $\widetilde{\mathcal{J}_{\alpha\beta}}$ satisfying the restrictions (3.5), (3.8). Note that off the shell of constraints these objects can differ from the gauge invariant supercurrents $\check{\mathcal{J}}_{\alpha\beta}$ by terms depending on $\Psi_{\hat{m}\hat{n}}$.

Now let us discuss the general structure of such functionals. It is evident that only those of them which are *linear* in the total set of constraints $\Psi_{\hat{m}\hat{n}}$ can actually contribute to (3.9), because the SOPEs including any higher order monomial of $\Psi_{\hat{m}\hat{n}}$ are proportional to $\Psi_{\hat{m}\hat{n}}$ and so obviously vanish on the constraints shell $\{\Psi_{\hat{m}\hat{n}}=0\}$. The coefficients in these linear functionals can in general be nonlinear functionals of the remaining unconstrained supercurrents $\mathcal{J}_{\alpha\beta}$. These functionals can be local or non-local, depending on whether the superfield parameters $\Lambda_{\hat{m}\hat{n}}$ of the gauge transformation (3.3) relating the gauge-fixed supercurrents to the original ones are expressed in terms of the latter in a local or non-local way. In what follows we will always choose the gauges yielding local functionals, as they correspond to the most interesting extended $N=2$ superconformal algebras. Note that the Miura transformations relating superconformal algebras to the algebras of free superfields just realize the passing from the gauges of the first type to the second type.

Keeping in mind the above discussion, from now on we consider as a starting expression for $\widetilde{\mathcal{J}_{\alpha\beta}}$ linear functionals of constraints $\Psi_{\hat{m}\hat{n}}$ (and derivatives of the latter) with nonlinear in general coefficient-functions of $\mathcal{J}_{\alpha\beta}$. Taking for these coefficients the most general ansatz in terms of $\mathcal{J}_{\alpha\beta}$ with arbitrary constant coefficients, such that it preserves superspins and $u(1)$ charges with respect to the improved $N=2$ stress tensor (2.5), and substituting it into Eqs. (3.5), (3.8), one obtains the solution which proves to be unique up to some unessential coefficients which do not contribute to (3.9).

In the next section we will illustrate the formalism described above by the simplest example of hamiltonian reduction of the $N=2$ $sl(2|1)^{(1)}$ superalgebra.

4. Example: $N=2$ $sl(2|1)^{(1)}$ Affine Superalgebra

Let us apply the general procedure developed in the previous section to the superalgebra $N=2$ $sl(2|1)^{(1)}$. We will naturally come to the $N=2$ superspace formulation of standard $N=2$ SCA in this way.

In Appendix A, for completeness we give the explicit form of generators, structure constants and the Killing metric for the $sl(2|1)$ superalgebra in the complex basis described in Sect. 2, as well as the relations between affine supercurrents $\mathcal{J}_a, \mathcal{J}_{\bar{a}}$ in this basis and matrix elements $\mathcal{J}_{mn}$ introduced in Sect. 3. Substituting these formulas into (2.1), (2.3) and (2.5) one can obtain explicit expressions for the defining SOPEs of the $N=2$ affine extension of the $sl(2|1)$, for nonlinear constraints the relevant supercurrents satisfy, as well as for the improved Sugawara $N=2$ stress tensor. The last one has the following form:

$$\mathcal{T}_{sug} = \frac{1}{k}(\mathcal{J}_{12}\mathcal{J}_{21} - \mathcal{H}_1\mathcal{H}_{\bar{1}} - \mathcal{J}_{13}\mathcal{J}_{31} - \mathcal{J}_{23}\mathcal{J}_{32}) + \alpha_1\bar{\mathcal{D}}\mathcal{H}_1 + \alpha_{\bar{1}}\mathcal{D}\mathcal{H}_{\bar{1}}\,, \quad (4.1)$$

where two parameters, α_1 and $\alpha_{\bar{1}}$, give rise to a splitting of supercurrents into the grades with positive, zero and negative dimensions and $u(1)$ charges (see Table 1). Actually, this splitting is due to the existence of two grading operators: $(\alpha_1 t_2 + \alpha_{\bar{1}} t_{\bar{2}})/2$ and $\alpha_1 t_2 - \alpha_{\bar{1}} t_{\bar{2}}$, $t_i, t_{\bar{i}}$ being Cartan generators of $sl(2|1)$ in the coadjoint representation. The eigenvalues of the former are exactly ("dimension"- 1/2) in Table 1 and those of the latter are ("$u(1)$ charge" ± 1) where $+1$ is for barred supercurrents and -1 for unbarred ones.

Table 1.

scs	dim	$u(1)$
$\mathcal{H}_1^F$	1/2	1
$\mathcal{H}_{\bar{1}}^F$	1/2	-1
$\mathcal{J}_{12}^F$	$(1+\alpha_1-\alpha_{\bar{1}})/2$	$(1+\alpha_1+\alpha_{\bar{1}})$
$\mathcal{J}_{21}^F$	$(1-\alpha_1+\alpha_{\bar{1}})/2$	$(-1-\alpha_1-\alpha_{\bar{1}})$
$\mathcal{J}_{13}^B$	$(1+\alpha_1)/2$	$(1+\alpha_1)$
$\mathcal{J}_{31}^B$	$(1-\alpha_1)/2$	$(-1-\alpha_1)$
$\mathcal{J}_{23}^B$	$(1+\alpha_{\bar{1}})/2$	$(1-\alpha_{\bar{1}})$
$\mathcal{J}_{32}^B$	$(1-\alpha_{\bar{1}})/2$	$(-1+\alpha_{\bar{1}})$

In this and all subsequent tables we use the following abbreviations: "scs" for supercurrents, "dim" for superconformal dimensions and "$u(1)$" for $u(1)$ charges.

We also give the explicit form of the nonlinear constraints (2.3)

$$
\begin{aligned}
&\mathcal{D}\mathcal{H}_1 = 0, && \bar{\mathcal{D}}\mathcal{H}_{\bar{1}} = 0\,,\\
&\left(\mathcal{D} + \frac{1}{k}\mathcal{H}_1\right)\mathcal{J}_{12} = 0, && \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{21} = 0\,,\\
&\mathcal{D}\mathcal{J}_{13} - \frac{1}{k}\mathcal{J}_{12}\mathcal{J}_{23} = 0, && \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{31} - \frac{1}{k}\mathcal{J}_{21}\mathcal{J}_{32} = 0\,,\\
&\left(\mathcal{D} - \frac{1}{k}\mathcal{H}_1\right)\mathcal{J}_{23} = 0, && \bar{\mathcal{D}}\mathcal{J}_{32} = 0\,.
\end{aligned}
\tag{4.2}
$$

Now we are ready to consider a hamiltonian reduction of $N=2\,sl(2|1)^{(1)}$ which produces $N = 2$ SCA. To this end, we should first learn at which values of parameters α_1 and $\alpha_{\bar{1}}$ at least one of the bosonic supercurrents could have the spin and $u(1)$ charge characteristic of the $N=2$ stress tensor, i.e. 1, 0, respectively. It turns out to be possible with the following choice:

$$\alpha_1 = -1, \qquad \alpha_{\bar{1}} = 1\,. \tag{4.3}$$

In this case, besides the fermionic supercurrents $\mathcal{H}_1, \mathcal{H}_{\bar{1}}$ with the spin and $u(1)$ charge $1/2$ and ± 1, the $N=2$ $sl(2|1)^{(1)}$ superalgebra contains bosonic spin 0 $(\mathcal{J}_{13}, \mathcal{J}_{32})$ and spin 1 $(\mathcal{J}_{31}, \mathcal{J}_{23})$ ones with zero $u(1)$ charges, as well as the fermionic doublet $\mathcal{J}_{12}, \mathcal{J}_{21}$ with spins $-1/2, 3/2$ and $u(1)$ charges $1, -1$, respectively.

Secondly, we should put first-class constraints on some supercurrents at which at least one of two spin 1 supercurrents ($\mathcal{J}_{31}$ or $\mathcal{J}_{23}$) is unconstrained in order to be able to identify it with the $N=2$ unconstrained stress tensor. At first sight, it seems impossible to achieve this because from the beginning all the supercurrents are constrained by the conditions (4.2). Nevertheless, it can be done. Let us briefly explain the basic idea of how unconstrained $N=2$ superfields can come out in this way.

By looking at the constraints (4.2), one sees that they are quadratically nonlinear and their number precisely matches with that of supercurrrents. Moreover, in every constraint there is only one linear term with spinor covariant derivative on some supercurrent, and different constraints contain different linear terms, so they are in one-to-one correspondence with the consistent set of standard chiral and anti-chiral conditions. The last ones reduce the number of independent superfield components by the factor two. The same is evidently true for a nonlinear generalization of these constraints (4.2): the only new point is that the components which were forced to be zero in the case of chiral constraints become some functions of the remaining independent ones in the case of (4.2). However, an important difference of the latter from the linear constraints is the following. If we replace some bosonic supercurrents in (4.2) by nonzero constants, then in some constraints the nonlinear terms can produce *a linear* one without a spinor derivative on it. So, this constraint becomes algebraic with respect to the supercurrent entering it linearly and can be solved for the latter. Thus this supercurrent turns out to be eventually expressed in terms of other ones and their spinor covariant derivatives. Now among the remaining independent supercurrents one can find, in a number of cases, unconstrained $N=2$ superfields. This is just what comes about in the case at hand. An analogous resume could be drawn from the analysis of solutions of $N=2$ constraints (4.2) in terms of unconstrained $N=1$ superfields (2.9).

Keeping in mind the above remark, we choose first-class constraints as follows:

$$\mathscr{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 1 \\ * & * & * \\ * & 1 & * \end{pmatrix} . \tag{4.4}$$

They clearly preserve $N=2$ superconformal symmetry generated by $\mathscr{T}_{sug}$ (4.1), (4.3). This set of constraints is also consistent with Eqs. (4.2). Indeed, by substituting (4.4) into (4.2) we find that those constraints from (4.2) which include the spinor derivative of the supercurrents $\mathscr{J}_{13}, \mathscr{J}_{32}$ and $\mathscr{J}_{12}$ are satisfied identically while the constraint containing the spinor derivative of the $\mathscr{J}_{31}$ current becomes algebraic and expresses $\mathscr{J}_{21}$ in terms of $\mathscr{J}_{31}$,

$$\mathscr{J}_{21} = k\left(\bar{\mathscr{D}} - \frac{1}{k}\mathscr{H}_{\bar{1}}\right)\mathscr{J}_{31} . \tag{4.5}$$

The remaining constraints from the set (4.2) preserve their form on the constraints shell (4.4). Thus on the shell of constraints (4.4) no restrictions arise on the spin 1, $u(1)$ charge 0 bosonic supercurrent $\mathscr{J}_{31}$, so the latter is an unconstrained $N=2$ superfield and, as we will see soon, proves to be directly related to the $N=2$ superconformal stress tensor.

Let us note that the constraints (4.4) actually amount to the set of constraints imposed in [12] in $N=1$ superspace. This latter set can be shown to produce the above constraints without breaking $N=2$ supersymmetry through the explicit relation (2.9) between $N=1$ and $N=2$ supercurrents.

Constraints (4.4) can easily be checked to have zero mutual SOPEs on their shell, so they are first-class and give rise to a gauge invariance which can be used to gauge away three more entries in the supermatrix (4.4). Indeed, with respect to infinitesimal gauge transformations (3.3) generated by constraints (4.4) with the gauge parameters $\Lambda_{12}, \Lambda_{13}$ and Λ_{32} the currents $\mathscr{J}_{23}, \mathscr{H}_1$ and $\mathscr{H}_{\bar{1}}$ are transformed inhomogeneously,

$$\begin{aligned} &\delta_{\Lambda_{12}}\mathscr{J}_{23}(Z_2) = -\left(\mathscr{D} - \frac{1}{k}\mathscr{H}_1\right)\Lambda_{12} , \\ &\delta_{\Lambda_{13}}\mathscr{H}_1(Z_2) = \mathscr{D}\Lambda_{13} , \qquad \delta_{\Lambda_{32}}\mathscr{H}_{\bar{1}}(Z_2) = \bar{\mathscr{D}}\Lambda_{32} . \end{aligned} \tag{4.6}$$

One can explicitly check that these gauge transformations preserve the constraints (4.2). As a result, we can consistently fix the gauge as[4]

$$\mathscr{J}_{23} = 0, \qquad \mathscr{H}_1 = 0, \qquad \mathscr{H}_{\bar{1}} = 0 . \tag{4.7}$$

It is easy to check that the total set of constraints, i.e. constraints (4.4) and gauge fixing conditions (4.7), is second-class. Substituting the gauge fixing conditions (4.7)

---

[4] In this gauge there remains a residual gauge freedom with chiral Λ_{12}, Λ_{13} and anti-chiral Λ_{32}. However, the final expression for the $N=2$ stress tensor turns out to be invariant under this residual freedom.

and the expression (4.5) into the supermatrix (4.4) we obtain the expression for $N=2$ supercurrents $\mathscr{J}_{mn}$ in the highest weight (or Drinfeld–Sokolov [9]) gauge

$$\mathscr{J}^{DS}_{mn} = \begin{pmatrix} 0 & 0 & 1 \\ k\bar{\mathscr{D}}\mathscr{J}_{31} & 0 & 0 \\ \mathscr{J}_{31} & 1 & 0 \end{pmatrix}. \tag{4.8}$$

So the superalgebra which is produced from $N=2$ $sl(2|1)^{(1)}$ by hamiltonian reduction associated with the constraints (4.4) is generated by only one gauge invariant bosonic supercurrent $\widetilde{\mathscr{J}_{31}}$ which coincides with $\mathscr{J}_{31}$ on the shell of total set of constraints (see (3.5)).

Our next task is to find $\widetilde{\mathscr{J}_{31}}$ from the conditions (3.5), (3.8). This can be easily done by making use of the general procedure described in Sect. 3. As a result we obtain the following expression for $\widetilde{\mathscr{J}_{31}}$ up to unessential terms:

$$\begin{aligned}\widetilde{\mathscr{J}_{31}} &= (-\mathscr{J}_{12}\mathscr{J}_{21} + \mathscr{J}_{13}\mathscr{J}_{31} + \mathscr{J}_{23}) + k\bar{\mathscr{D}}\mathscr{H}_1 - k\mathscr{D}\mathscr{H}_{\bar{1}} \\ &\quad + k^3\bar{\mathscr{D}}\mathscr{J}'_{12} + k^2\mathscr{J}'_{13} - k^2\mathscr{J}'_{32}\,. \end{aligned} \tag{4.9}$$

Substituting this expression into (3.9), we get the SOPE of the superalgebra we are looking for. This SOPE coincides with the SOPE of $N=2$ SCA (2.6) with central charge $-2k$ after rescaling,

$$\mathscr{J}_{31} \to -k\mathscr{J}_{31}\,. \tag{4.10}$$

In the next section we will discuss various reductions of $N=2$ $sl(3|2)^{(1)}$ and deduce some new superfield extended $N=2$ SCAs in this way.

5. Hamiltonian Reductions of $N=2$ $sl(3|2)^{(1)}$ Affine Superalgebra

The reductions of $N=2$ $sl(3|2)^{(1)}$ we will consider in this section give rise to four new types of extensions of $N=2$ SCA. The first one is rather unusual in the sense that the $N=2$ stress tensor is a constrained supercurrent. The second possesses an unconstrained stress tensor, but contains spin 0 supercurrents, such that it turns out impossible to decouple dimension 0 component currents. We will concentrate on the third and fourth cases corresponding to $N=2\,u(2|1)$ and $N=2$ $u(3)$ SCAs, respectively, because these are "canonical" in the sense that the relevant $N=2$ stress tensor is unconstrained and there are no spin 0 supercurrents. We will also illustrate how the known $N=2$ W_3 [32] and $N=2$ $W_3^{(2)}$ [23] SCAs reappear in the hamiltonian reduction approach in $N=2$ superspace.

It is rather straightforward to find the structure constants and Killing metric in the complex basis for $sl(3|2)$, so we do not write them explicitly (see Appendix B). From the general expression for the improved Sugawara $N=2$ stress tensor (2.5) we obtain it for $N=2$ $sl(3|2)^{(1)}$ in the following form:

$$\begin{aligned}\mathscr{T}_{sug} &= \frac{1}{k}(-\mathscr{H}_1\mathscr{H}_{\bar{1}} - \mathscr{H}_2\mathscr{H}_{\bar{2}} + \mathscr{J}_{12}\mathscr{J}_{21} + \mathscr{J}_{13}\mathscr{J}_{31} + \mathscr{J}_{23}\mathscr{J}_{32} - \mathscr{J}_{14}\mathscr{J}_{41} \\ &\quad - \mathscr{J}_{15}\mathscr{J}_{51} - \mathscr{J}_{24}\mathscr{J}_{42} - \mathscr{J}_{25}\mathscr{J}_{52} - \mathscr{J}_{34}\mathscr{J}_{43} - \mathscr{J}_{35}\mathscr{J}_{53} - \mathscr{J}_{45}\mathscr{J}_{54} - \mathscr{H}_2\mathscr{H}_{\bar{1}}) \\ &\quad + \alpha_1\bar{\mathscr{D}}\mathscr{H}_1 + \alpha_{\bar{1}}\mathscr{D}\mathscr{H}_{\bar{1}} + \alpha_2\bar{\mathscr{D}}\mathscr{H}_2 + \alpha_{\bar{2}}\mathscr{D}\mathscr{H}_{\bar{2}}\,, \end{aligned} \tag{5.1}$$

where four parameters, $\alpha_1, \alpha_{\bar{1}}, \alpha_2, \alpha_{\bar{2}}$ split the supercurrents into the grades with positive, zero and negative dimensions and $u(1)$ charges (see Table 2).

Let us stress that the nonlinear constraints (2.3) for the case of $sl(3|2)$ can be easily read off using the structure constants of this superalgebra. They will play the important role in all the calculations in the remainder of this paper. Our main aim in this section will be to find extended $N=2$ SCAs which contain at least one

Table 2.

scs	dim	$u(1)$
$\mathcal{H}^F_{\bar{1}}$	1/2	-1
$\mathcal{H}^F_{1}$	1/2	1
$\mathcal{H}^F_{\bar{2}}$	1/2	-1
$\mathcal{H}^F_{2}$	1/2	1
$\mathcal{J}^F_{12}$	$(1+\alpha_1-\alpha_{\bar{1}}+\alpha_{\bar{2}})/2$	$(1+\alpha_1+\alpha_{\bar{1}}-\alpha_{\bar{2}})$
$\mathcal{J}^F_{21}$	$(1-\alpha_1+\alpha_{\bar{1}}-\alpha_{\bar{2}})/2$	$(-1-\alpha_1-\alpha_{\bar{1}}+\alpha_{\bar{2}})$
$\mathcal{J}^F_{13}$	$(1-\alpha_{\bar{1}}+\alpha_2)/2$	$(1+\alpha_{\bar{1}}+\alpha_2)$
$\mathcal{J}^F_{31}$	$(1+\alpha_{\bar{1}}-\alpha_2)/2$	$(-1-\alpha_{\bar{1}}-\alpha_2)$
$\mathcal{J}^F_{23}$	$(1-\alpha_1+\alpha_2-\alpha_{\bar{2}})/2$	$(1-\alpha_1+\alpha_2+\alpha_{\bar{2}})$
$\mathcal{J}^F_{32}$	$(1+\alpha_1-\alpha_2+\alpha_{\bar{2}})/2$	$(-1+\alpha_1-\alpha_2-\alpha_{\bar{2}})$
$\mathcal{J}^B_{14}$	$(1+\alpha_1)/2$	$(1+\alpha_1)$
$\mathcal{J}^B_{41}$	$(1-\alpha_1)/2$	$(-1-\alpha_1)$
$\mathcal{J}^B_{15}$	$(1-\alpha_{\bar{1}}+\alpha_2+\alpha_{\bar{2}})/2$	$(1+\alpha_{\bar{1}}+\alpha_2-\alpha_{\bar{2}})$
$\mathcal{J}^B_{51}$	$(1+\alpha_{\bar{1}}-\alpha_2-\alpha_{\bar{2}})/2$	$(-1-\alpha_{\bar{1}}-\alpha_2+\alpha_{\bar{2}})$
$\mathcal{J}^B_{24}$	$(1+\alpha_{\bar{1}}-\alpha_{\bar{2}})/2$	$(1-\alpha_{\bar{1}}+\alpha_{\bar{2}})$
$\mathcal{J}^B_{42}$	$(1-\alpha_{\bar{1}}+\alpha_{\bar{2}})/2$	$(-1+\alpha_{\bar{1}}-\alpha_{\bar{2}})$
$\mathcal{J}^B_{25}$	$(1-\alpha_1+\alpha_2)/2$	$(1-\alpha_1+\alpha_2)$
$\mathcal{J}^B_{52}$	$(1+\alpha_1-\alpha_2)/2$	$(-1+\alpha_1-\alpha_2)$
$\mathcal{J}^B_{34}$	$(1+\alpha_1+\alpha_{\bar{1}}-\alpha_2)/2$	$(1+\alpha_1-\alpha_{\bar{1}}-\alpha_2)$
$\mathcal{J}^B_{43}$	$(1-\alpha_1-\alpha_{\bar{1}}+\alpha_2)/2$	$(-1-\alpha_1+\alpha_{\bar{1}}+\alpha_2)$
$\mathcal{J}^B_{35}$	$(1+\alpha_{\bar{2}})/2$	$(1-\alpha_{\bar{2}})$
$\mathcal{J}^B_{53}$	$(1-\alpha_{\bar{2}})/2$	$(-1+\alpha_{\bar{2}})$
$\mathcal{J}^F_{45}$	$(1-\alpha_1-\alpha_{\bar{1}}+\alpha_2+\alpha_{\bar{2}})/2$	$(1-\alpha_1+\alpha_{\bar{1}}+\alpha_2-\alpha_{\bar{2}})$
$\mathcal{J}^F_{54}$	$(1+\alpha_1+\alpha_{\bar{1}}-\alpha_2-\alpha_{\bar{2}})/2$	$(-1+\alpha_1-\alpha_{\bar{1}}-\alpha_2+\alpha_{\bar{2}})$

bosonic unconstrained supercurrent with dimension 1 and vanishing $u(1)$ charge by applying the general procedure of the hamiltonian reduction to $N=2$ $sl(3|2)^{(1)}$ affine superalgebra.

In the next subsections, we will present only the basic results and make some comments without detailed explanations, because most of technical points are a direct generalization of those expounded in Sect. 4 on the simpler example of $N=2$ $sl(2|1)^{(1)}$.

5.1. $N=2$ W_3 SCA. In order to understand the reduction scheme in the case under consideration, we take as a first example $N=2$ W_3 SCA [32] and study how it is reproduced in our method.

The algebra $N=2$ W_3 has one extra spin 2 bosonic supercurrent besides the spin 1 $N=2$ stress tensor. This counting suggests that we should impose ten constraints which is the "maximal" set. The point is that requiring the constraints to be first-class restricts a possible number of such constraints. It can be easily checked that this requirement cannot be met if the number of constraints exceeds ten.

For the choice

$$\alpha_1 = -1, \qquad \alpha_{\bar{1}} = 2, \qquad \alpha_2 = -2, \qquad \alpha_{\bar{2}} = 1 \tag{5.2}$$

in Table 3, we give the list of "twisted" dimensions and $u(1)$ charges of those supercurrents which will be subjected to the reduction constraints and corresponding gauge fixing conditions (we use for them, respectively, the abbreviation "constr. scs" and "g.f. scs").

We impose the constraints on all the negative and zero dimension supercurrents as is summarized below:

$$\mathcal{J}_{mn}^{\mathrm{constr}} = \begin{pmatrix} * & 0 & 0 & 1 & 0 \\ * & * & 0 & * & 1 \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & 1 & * & * \end{pmatrix}. \tag{5.3}$$

As was repeatedly mentioned above, these first-class constraints generate gauge invariances. In the upper line of Table 3 we place the supercurrents which are subjected to the above constraints and are basically the generators of these invariances according to the general formula (3.3). The lower line collects the supercurrents

Table 3.

$u(1)$	1	0	0	1	1	1	0	0	0	0
dim	$-\frac{3}{2}$	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
constr. scs	$\mathcal{J}_{13}^F$	$\mathcal{J}_{15}^B$	$\mathcal{J}_{43}^F$	$\mathcal{J}_{12}^F$	$\mathcal{J}_{23}^F$	$\mathcal{J}_{45}^F$	$\mathcal{J}_{14}^B$	$\mathcal{J}_{25}^B$	$\mathcal{J}_{42}^F$	$\mathcal{J}_{53}^B$
g.f. scs	$\mathcal{J}_{34}^B$	$\mathcal{J}_{52}^B$	$\mathcal{J}_{32}^F$	$\mathcal{J}_{24}^B$	$\mathcal{J}_{35}^B$	$\mathcal{J}_{54}^F$	$\mathcal{H}_1^F$	$\mathcal{H}_2^F$	$\mathcal{H}_{\bar{1}}^F$	$\mathcal{H}_{\bar{2}}^F$
dim	2	1	$\frac{3}{2}$	1	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$u(1)$	0	0	-1	0	0	-1	1	1	-1	-1

which are gauged away by these invariances. For example, $\mathcal{J}_{34}$ can be gauged away using the gauge transformation generated by constraint $\mathcal{J}_{13}$ ($\mathcal{J}_{52}$ by $\mathcal{J}_{15}$ and so on). Note that four constraints of units in (5.3) are necessary to gauge away four dimension 1/2 supercurrents corresponding to Cartan elements.

As we see, only four supercurrents $\mathcal{J}_{21}, \mathcal{J}_{31}, \mathcal{J}_{41}, \mathcal{J}_{51}$ eventually survive. Substituting (5.3) into the nonlinear constraints (2.3) we find that $\mathcal{J}_{21}, \mathcal{J}_{31}$, before fixing the gauge, are expressed as follows:

$$\mathcal{J}_{21} = k\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{41}\,,$$
$$\mathcal{J}_{31} = k\left(\bar{\mathcal{D}} - \frac{1}{k}(\mathcal{H}_{\bar{1}} + \mathcal{H}_{\bar{2}})\right)\mathcal{J}_{51} - \mathcal{J}_{21}\mathcal{J}_{52} - \mathcal{J}_{41}\mathcal{J}_{54}\,. \tag{5.4}$$

After gauging away the unphysical degrees of freedom in accord with Table 3, we are left with the following supercurrent matrix $\mathcal{J}_{mn}$ in the highest weight gauge

$$\mathcal{J}^{DS}_{mn} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ k\bar{\mathcal{D}}\mathcal{J}_{41} & 0 & 0 & 0 & 1 \\ k\bar{\mathcal{D}}\mathcal{J}_{51} & 0 & 0 & 0 & 0 \\ \mathcal{J}_{41} & 1 & 0 & 0 & 0 \\ \mathcal{J}_{51} & 0 & 1 & 0 & 0 \end{pmatrix}. \tag{5.5}$$

Thus as an output we have two independent unconstrained supercurrents with zero $u(1)$ charges: a dimension 1 supercurrent $\mathcal{J}_{41}$ which is nothing but the $N=2$ stress tensor and a dimension 2 supercurrent $\mathcal{J}_{51}$.

We will not discuss here how to construct gauge invariant supercurrents and which SOPEs they satisfy, because all these formulas can be reproduced via a secondary hamiltonian reduction from $N=2$ $W_3^{(2)}$ SCA which will be discussed in the following subsection. Anticipating the result, the relevant set of SOPEs forms the classical $N=2$ W_3 SCA [32].

5.2. $N=2$ $W_3^{(2)}$ SCA. Let us now describe another reduction.

We wish to understand how $N=2$ $W_3^{(2)}$ SCA of Ref. [23] can be obtained within our procedure. Recall that this algebra is described in $N=2$ superspace by the spin 1/2, 2 bosonic and 1/2, 2 fermionic constrained supercurrents in addition to the spin 1 bosonic unconstrained $N=2$ stress tensor. To match this superfield content, we are led to impose nine constraints on the $N=2$ affine supercurrents. One could try to proceed by relaxing one of the constraints (5.3), still with the same choice of the splitting parameters (5.2). However, in this basis one finds no spin 2 fermionic supercurrents required by the superfield content of $N=2$ $W_3^{(2)}$ SCA. So we are led to choose $\alpha_i, \alpha_{\bar{i}}$ in another way (once again, the basic motivation for this choice is the presence of at least one spin 1 supercurrent with zero $u(1)$ charge after splitting)

$$\alpha_1 = -1, \qquad \alpha_{\bar{1}} = 1, \qquad \alpha_2 = -2, \qquad \alpha_{\bar{2}} = 0\,. \tag{5.6}$$

It turns out that this is the right choice to produce the $N=2$ $W_3^{(2)}$ SCA precisely in the form given in [23], one of the surviving supercurrents being the corresponding unconstrained $N=2$ stress tensor. Actually, the choices (5.2), (5.6) are closely related to each other: the relevant $N=2$ stress tensors differ by an improving term containing a spin 1/2 fermionic supercurrent. We will come back to this point later, while discussing the secondary reduction of $N=2$ $W_3^{(2)}$ SCA.

Proceeding as before, we list in Table 4 the dimensions and $u(1)$ charges of the constrained and gauge fixed supercurrents, and in Table 5 indicate the supercurrents surviving the whole set of the hamiltonian reduction second class constraints to be defined below (we denote these latter supercurrents as "surv. scs").

In Table 5 and in similar tables for other cases studied in this section we adopt the following convention: to the right from the double vertical line we place those of the surviving supercurrents (actually the single current $\mathcal{J}_{21}$ in the case at hand) which are expressed through other ones by the remnants of the nonlinear constraints (2.3) after imposing the hamiltonian reduction constraints. These latter supercurrents themselves (they still can be constrained, e.g., be chiral) are placed on the left.

From Table 4 we conclude that there are only three bosonic affine supercurrents with both spin and $u(1)$ charge equal to zero, namely, $\mathcal{J}_{14}$, $\mathcal{J}_{25}$ and $\mathcal{J}_{42}$. So we can put them equal to 1, while all the supercurrents with negative dimensions, as in the previous examples, equal to zero. We also equate to zero the fermionic supercurrent $\mathcal{J}_{23}$. Thus the constraints we impose are of the form

$$\mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 0 & 1 & 0 \\ * & * & 0 & * & 1 \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & * & * & * \end{pmatrix}. \tag{5.7}$$

Table 4.

$u(1)$	0	0	1	-1	1	0	0	0	0
dim	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
constr. scs	$\mathcal{J}_{13}^F$	$\mathcal{J}_{15}^B$	$\mathcal{J}_{12}^F$	$\mathcal{J}_{43}^B$	$\mathcal{J}_{45}^F$	$\mathcal{J}_{23}^F$	$\mathcal{J}_{14}^B$	$\mathcal{J}_{25}^B$	$\mathcal{J}_{42}^B$
g.f. scs	$\mathcal{J}_{34}^B$	$\mathcal{J}_{52}^B$	$\mathcal{J}_{24}^B$	$\mathcal{J}_{32}^F$	$\mathcal{J}_{54}^F$	$\mathcal{J}_{35}^B$	$\mathcal{H}_1^F$	$\mathcal{H}_2^F$	$\mathcal{H}_{\bar{1}}^F$
dim	$\frac{3}{2}$	1	1	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$u(1)$	1	0	0	0	-1	1	1	1	-1

Table 5.

surv. scs	$\mathcal{J}_{53}^B$	$\mathcal{H}_{\bar{2}}^F$	$\mathcal{J}_{41}^B$	$\mathcal{J}_{31}^F$	$\mathcal{J}_{51}^B$	$\mathcal{J}_{21}^F$
dim	$\frac{1}{2}$	$\frac{1}{2}$	1	2	2	$\frac{3}{2}$
$u(1)$	-1	-1	0	0	0	-1

By plugging (5.7) into the nonlinear constraints (2.3), we can solve one of them for $\mathcal{J}_{21}$ and express the latter in terms of $\mathcal{J}_{41}$,

$$\mathcal{J}_{21} = k\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{41} \,. \tag{5.8}$$

As the next step we should fix gauges. Gauge fixing procedure can be performed using the same arguments as in the previous examples and we eventually arrive at the following $\mathcal{J}^{DS}_{mn}$:

$$\mathcal{J}^{DS}_{mn} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ k\bar{\mathcal{D}}\mathcal{J}_{41} & \mathcal{H}_{\bar{2}} & 0 & 0 & 1 \\ \mathcal{J}_{31} & 0 & 0 & 0 & 0 \\ \mathcal{J}_{41} & 1 & 0 & 0 & 0 \\ \mathcal{J}_{51} & 0 & \mathcal{J}_{53} & 0 & \mathcal{H}_{\bar{2}} \end{pmatrix} . \tag{5.9}$$

Now we are ready to construct five independent gauge invariant supercurrents by exploiting the general procedure expounded in Sect. 3. It is a matter of lengthy but straightforward computation to explicitly find them and to verify that they satisfy the condition (3.5). In view of the complexity of the relevant formulas, we do not present them here.

It is also direct to calculate the star SOPEs between these supercurrents using the rule (3.9) and the relations (2.1). The $N=2$ stress tensor is given by

$$\mathcal{T} = -\frac{1}{k}\widetilde{\mathcal{J}_{41}} = -\frac{1}{k}\mathcal{J}_{41} \,, \tag{5.10}$$

where the second equality is fulfilled on the shell of constraints. It has the central charge $-2k$ and coincides with $\mathcal{T}_{sug}$ (5.1), (5.6) on the constraints shell. After the redefinitions

$$\mathcal{J}_{53} \to \mathcal{J}_{53}, \qquad \mathcal{H}_{\bar{2}} \to \mathcal{H}_{\bar{2}}, \qquad \mathcal{J}_{31} \to \frac{1}{k^3}\mathcal{J}_{31}, \qquad \mathcal{J}_{51} \to \frac{1}{k^3}\mathcal{J}_{51} \,, \tag{5.11}$$

all the supercurrents except for $\mathcal{H}_{\bar{2}}$ are superprimary with respect to the stress tensor (5.10) (see Eq. (2.8)), and have the spins $1/2, 1/2, 2$ and 2, respectively, while $\mathcal{H}_{\bar{2}}$ is quasi-superprimary (from now on, we omit the index "*", keeping in mind that all such SOPEs are computed according to the rule (3.9))

$$\mathcal{T}(Z_1)\mathcal{H}_{\bar{2}}(Z_2) = \frac{\theta_{12}}{z_{12}^2}2k + \left[\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{1}{2} - \frac{\theta_{12}}{z_{12}}\mathcal{D} + \frac{\bar{\theta}_{12}}{z_{12}}\bar{\mathcal{D}} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\partial - \frac{1}{z_{12}}\right]\mathcal{H}_{\bar{2}} \,. \tag{5.12}$$

The rest of SOPEs coincides with those quoted in [23] and for this reason we do not give them here.

So we end up with the following five $N=2$ supercurrents: a general spin 1 $\mathcal{T}$, spin 1/2 antichiral fermionic $\mathcal{H}_{\bar{2}}$ and bosonic $\mathcal{J}_{53}$, constrained spin 2 fermionic $\mathcal{J}_{31}$ and bosonic $\mathcal{J}_{51}$ ones.

We also write down the constraints which stem from the original nonlinear constraints on the affine supercurrents,

$$\bar{\mathscr{D}}\mathscr{H}_{\bar{2}} = 0, \qquad \bar{\mathscr{D}}\mathscr{J}_{53} = 0\,,$$
$$\left(\bar{\mathscr{D}} - \frac{1}{k}\mathscr{H}_{\bar{2}}\right)\mathscr{J}_{31} = 0, \qquad \left(\bar{\mathscr{D}} - \frac{1}{k}\mathscr{H}_{\bar{2}}\right)\mathscr{J}_{51} - \frac{1}{k}\mathscr{J}_{31}\mathscr{J}_{53} = 0\,. \qquad (5.13)$$

By construction, all the above SOPEs are compatible with these constraints. These SOPEs and constraints constitute the superfield description of $N=2$ $W_3^{(2)}$ superalgebra given in [23].

As shown in [23], we can obtain $N=2$ W_3 SCA from $N=2$ $W_3^{(2)}$ SCA by means of secondary hamiltonian reduction [33] (by the primary hamiltonian reduction we mean the one which proceeds directly from the affine (super)algebra).

With respect to the new stress tensor $\mathscr{T}_{\text{new}}$,

$$\mathscr{T}_{\text{new}} = \mathscr{T} + \mathscr{D}\mathscr{H}_{\bar{2}}\,, \qquad (5.14)$$

$\mathscr{J}_{53}$ has vanishing spin and $u(1)$ charge. Then we can put nonzero constraint on $\mathscr{J}_{53}$,

$$\mathscr{J}_{53} - 1 = 0\,, \qquad (5.15)$$

and gauge away $\mathscr{H}_{\bar{2}}$,

$$\mathscr{H}_{\bar{2}} = 0\,. \qquad (5.16)$$

These additional constraints are consistent with the first and second of Eqs. (5.13), respectively. One observes that now $\mathscr{J}_{31}$ is expressed from the fourth of Eqs. (5.13) as

$$\mathscr{J}_{31} = k\left(\bar{\mathscr{D}} - \frac{1}{k}\mathscr{H}_{\bar{2}}\right)\mathscr{J}_{51}\,, \qquad (5.17)$$

after which the third of Eqs. (5.13) is satisfied identically. Then we are left with the same $\mathscr{J}_{mn}^{DS}$ as in (5.5). Thus the surviving independent supercurrents are $\mathscr{T}_{\text{new}}$ and $\mathscr{J}_{51}$ and it remains to construct the appropriate gauge invariant supercurrents and to compute their SOPEs using the rule (3.9). The dimension of $\mathscr{J}_{51}$ and its $u(1)$ charge with respect to $\mathscr{T}_{\text{new}}$ are the same as in Table 5, i.e. 2 and 0, which are characteristic of the second supercurrent of $N=2$ W_3 SCA. According to [23], the resulting superalgebra is precisely the $N=2$ W_3 SCA [32]. Of course, we could arrive at the same SOPEs directly in the framework of the primary hamiltonian reduction procedure described in the previous subsection.

Let us remark that $\mathscr{T}_{\text{new}}$ (5.14) exactly corresponds to the previous choice of the splitting parameters (5.2), in the sense that the dimensions and $u(1)$ charges of all the supercurrents with respect to it are the same as in Subsect. 5.1. This implies that the bases (5.6) and (5.2) are related through the shift $\sim \mathscr{D}\mathscr{H}_{\bar{2}}$ of the respective $N=2$ stress tensors. We could equally derive $N=2$ $W_3^{(2)}$ SCA sticking to the choice (5.2) and relaxing one of the constraints of units (on $\mathscr{J}_{53}$) directly in the supermatrix (5.3). However, in the corresponding basis the $N=2$ $W_3^{(2)}$ supercurrents are even not quasi-superprimary. To put this superalgebra in the standard form given in [23] one should pass to the stress tensor $\mathscr{T}$ (5.10) by the relation (5.14).

It is worth noticing that one can produce eight reduction constraints by relaxing the constraint on $\mathcal{J}_{23}$ in the supermatrix (5.7). Then the surviving supercurrents have two extra supercurrents $\mathcal{J}_{23}, \mathcal{J}_{35}$ in addition to the superfield contents of $N=2$ $W_3^{(2)}$ SCA.

In the next subsection we will consider more examples of extended $N=$ 2 conformal superalgebras obtained from $N=2$ $sl(3|2)^{(1)}$ via $N=2$ superfield hamiltonian reduction.

5.3. New Extended $N=2$ SCAs. From now on we will concentrate on those examples of hamiltonian reduction in $N=2$ superspace which generate new extended $N=2$ SCAs.

The next natural step is to consider the cases in which the number of the reduction constraints is less than nine. Let us first describe the case with five constraints (this number is the minimal one at which the constraints can still be chosen to be first-class). As before, the reason why we choose the specific values for splitting parameters as below stems from the demand that among the surviving supercurrents there is at least one bosonic supercurrent with spin 1 and $u(1)$ charge zero.

For the choice

$$\alpha_1 = -1, \qquad \alpha_{\bar{1}} = 0, \qquad \alpha_2 = 0, \qquad \alpha_{\bar{2}} = 1\,, \tag{5.18}$$

we list the dimensions and $u(1)$ charges of supercurrents in Tables 6 and 7.

Table 6.

$u(1)$	-1	0	2	0	0
dim	$-\frac{1}{2}$	0	0	0	0
constr. scs	$\mathcal{J}^F_{54}$	$\mathcal{J}^B_{14}$	$\mathcal{J}^B_{24}$	$\mathcal{J}^B_{34}$	$\mathcal{J}^B_{53}$
g.f. scs	$\mathcal{J}^B_{43}$	$\mathcal{H}^F_{\bar{1}}$	$\mathcal{J}^F_{12}$	$\mathcal{J}^F_{13}$	$\mathcal{H}^F_{\bar{2}}$
dim	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$u(1)$	0	1	-1	1	-1

Table 7.

surv. scs	$\mathcal{J}^B_{51}$	$\mathcal{J}^B_{52}$	$\mathcal{J}^F_{21}$	$\mathcal{J}^F_{23}$	$\mathcal{H}^F_{\bar{1}}$	$\mathcal{H}^F_{2}$	$\mathcal{J}^B_{15}$	$\mathcal{J}^B_{35}$	$\mathcal{J}^B_{41}$	$\mathcal{J}^B_{42}$	$\mathcal{J}^B_{25}$	$\mathcal{J}^F_{31}$	$\mathcal{J}^F_{32}$	$\mathcal{J}^F_{45}$
dim	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
$u(1)$	0	-2	1	-1	-1	1	0	0	0	-2	2	-1	-3	1

We can choose the appropriate constraints as follows

$$\mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & * & * & 1 & * \\ * & * & * & 0 & * \\ * & * & * & 0 & * \\ * & * & * & * & * \\ * & * & 1 & 0 & * \end{pmatrix} . \tag{5.19}$$

One observes that it is not a subset of constraints discussed in the previous subsections, (5.3), (5.7). Further we fix the gauge according to Table 6 and quote the surviving supercurrents in Table 7. There are three supercurrents expressible at the expense of the remaining ones, which can be seen by substituting (5.19) into the nonlinear constraints (2.3),

$$\begin{aligned} \mathcal{J}_{31} &= k\left(\bar{\mathcal{D}} - \frac{1}{k}(\mathcal{H}_{\bar{1}} + \mathcal{H}_{\bar{2}})\right)\mathcal{J}_{51} - \mathcal{J}_{21}\mathcal{J}_{52} , \\ \mathcal{J}_{32} &= k\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{52} , \\ \mathcal{J}_{45} &= k\left(\mathcal{D} + \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{15} - \mathcal{J}_{12}\mathcal{J}_{25} - \mathcal{J}_{13}\mathcal{J}_{35} . \end{aligned} \tag{5.20}$$

Thus we come to the following $\mathcal{J}_{mn}^{DS}$:

$$\mathcal{J}_{mn}^{DS} = \begin{pmatrix} \mathcal{H}_{\bar{1}} & 0 & 0 & 1 & \mathcal{J}_{15} \\ \mathcal{J}_{21} & 0 & \mathcal{J}_{23} & 0 & \mathcal{J}_{25} \\ k(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}})\mathcal{J}_{51} - \mathcal{J}_{21}\mathcal{J}_{52} & k\bar{\mathcal{D}}\mathcal{J}_{52} & \mathcal{H}_2 & 0 & \mathcal{J}_{35} \\ \mathcal{J}_{41} & \mathcal{J}_{42} & 0 & \mathcal{H}_{\bar{1}} & k\mathcal{D}\mathcal{J}_{15} \\ \mathcal{J}_{51} & \mathcal{J}_{52} & 1 & 0 & \mathcal{H}_2 \end{pmatrix} . \tag{5.21}$$

The supercurrents $\mathcal{J}_{51}, \mathcal{J}_{52}$, and $\mathcal{J}_{15}$ remain unconstrained.

Once we know the gauge invariant supercurrents, it is straightforward to deduce their algebra. The construction of these gauge invariant quantities is the crucial (and most difficult) step of our approach. With the above choice of five constraints, it is rather lengthy and cumbersome to find the correct ansatz for gauge invariant supercurrents because two spin 0 supercurrents $\mathcal{J}_{51}, \mathcal{J}_{52}$ are present (let us recall that $\widetilde{\mathcal{J}_{\alpha\beta}}$ are some nonlinear functionals of $\mathcal{J}_{\alpha\beta}$ and their derivatives). As the first step we write down $\widetilde{\mathcal{J}_{\alpha\beta}}$ as a lowest order monomial in $\mathcal{J}_{51}, \mathcal{J}_{52}$, and check whether it satisfies the conditions (3.5), (3.8). If this is not the case, we include next order terms in $\mathcal{J}_{51}, \mathcal{J}_{52}$, etc., until the conditions (3.5), (3.8) are satisfied.

Finally we obtain the gauge invariant supercurrents $\widetilde{\mathcal{J}_{\alpha\beta}}$ in the following form:

$$
\begin{aligned}
\widetilde{\mathcal{J}_{51}} &= \mathcal{J}_{34} + 2\mathcal{J}_{51} - k\mathcal{D}\mathcal{J}_{54} + \mathcal{H}_2\mathcal{J}_{54} + \mathcal{J}_{23}\mathcal{J}_{52}\mathcal{J}_{54} + \mathcal{J}_{24}\mathcal{J}_{52} - \mathcal{J}_{51}\mathcal{J}_{53}\,, \\
\widetilde{\mathcal{J}_{52}} &= 2\mathcal{J}_{52} - \mathcal{J}_{14}\mathcal{J}_{52}\,, \\
\widetilde{\mathcal{J}_{21}} &= \mathcal{J}_{21} + k\bar{\mathcal{D}}\mathcal{J}_{24} + \mathcal{J}_{14}\mathcal{J}_{21} - \mathcal{J}_{21}\mathcal{J}_{53} + \mathcal{J}_{23}\mathcal{H}_{\bar{1}}\mathcal{J}_{54} - \mathcal{J}_{24}\mathcal{H}_{\bar{1}} + k\bar{\mathcal{D}}\mathcal{J}_{23}\mathcal{J}_{54}\,, \\
\widetilde{\mathcal{J}_{23}} &= \mathcal{J}_{23} + \mathcal{J}_{23}\mathcal{J}_{14} - \mathcal{J}_{23}\mathcal{J}_{53}\,, \\
\widetilde{\mathcal{H}_{\bar{1}}} &= \mathcal{H}_{\bar{1}} + k\bar{\mathcal{D}}\mathcal{J}_{14}\,, \\
\widetilde{\mathcal{H}_2} &= \mathcal{H}_2 + k\mathcal{D}\mathcal{J}_{53}\,, \\
\widetilde{\mathcal{J}_{15}} &= \mathcal{J}_{43} - k\bar{\mathcal{D}}\mathcal{J}_{13} + \mathcal{J}_{15}\mathcal{J}_{53}\,, \\
\widetilde{\mathcal{J}_{35}} &= -k\mathcal{D}\mathcal{H}_{\bar{2}} + \mathcal{H}_2\mathcal{H}_{\bar{2}} - k\mathcal{H}_2\bar{\mathcal{D}}\mathcal{J}_{14} - \mathcal{H}_2\mathcal{J}_{15}\mathcal{J}_{54} \\
&\quad -\mathcal{J}_{15}\mathcal{J}_{34} + k\mathcal{J}_{15}\mathcal{D}\mathcal{J}_{54} + \mathcal{J}_{35}\mathcal{J}_{53} + \mathcal{J}_{45}\mathcal{J}_{54} - k^2\mathcal{J}'_{14}\,, \\
\widetilde{\mathcal{J}_{41}} &= k\bar{\mathcal{D}}\mathcal{H}_{\bar{1}} + k\mathcal{H}_{\bar{1}}\mathcal{D}\mathcal{J}_{53} + \mathcal{H}_1\mathcal{H}_{\bar{1}} - \mathcal{J}_{12}\mathcal{J}_{21} - \mathcal{J}_{13}\mathcal{H}_{\bar{2}}\mathcal{J}_{51} - \mathcal{J}_{13}\mathcal{J}_{31}\mathcal{J}_{53} \\
&\quad +\mathcal{J}_{14}\mathcal{J}_{41} + \mathcal{J}_{23}\mathcal{J}_{42}\mathcal{J}_{54} + \mathcal{J}_{24}\mathcal{J}_{42} - \mathcal{J}_{43}\mathcal{J}_{51} + k\bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{J}_{51} \\
&\quad +k^2\mathcal{J}'_{53} - \mathcal{J}_{13}\mathcal{J}_{41}\mathcal{J}_{54}\,, \\
\widetilde{\mathcal{J}_{42}} &= -k\bar{\mathcal{D}}\mathcal{J}_{12} - \mathcal{J}_{13}\mathcal{H}_{\bar{2}}\mathcal{J}_{52} - \mathcal{J}_{13}\mathcal{J}_{32}\mathcal{J}_{53} - \mathcal{J}_{13}\mathcal{J}_{42}\mathcal{J}_{54} \\
&\quad +\mathcal{J}_{42}\mathcal{J}_{53} - \mathcal{J}_{43}\mathcal{J}_{52} + k\bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{J}_{52}\,, \\
\widetilde{\mathcal{J}_{25}} &= \mathcal{J}_{14}\mathcal{J}_{25} - \mathcal{J}_{15}\mathcal{J}_{24} + \mathcal{J}_{23}\mathcal{H}_{\bar{2}} - k\mathcal{J}_{23}\bar{\mathcal{D}}\mathcal{J}_{14} + k\mathcal{J}_{23}\bar{\mathcal{D}}\mathcal{J}_{53} - \mathcal{J}_{23}\mathcal{J}_{15}\mathcal{J}_{54}\,.
\end{aligned}
\tag{5.22}
$$

We also construct the $N=2$ stress tensor

$$
\mathcal{T} = -\frac{1}{k}\left[\mathcal{J}_{35} + \mathcal{J}_{41} + \mathcal{H}_2\mathcal{H}_{\bar{1}} + \mathcal{J}_{15}\mathcal{J}_{51} - k\mathcal{J}_{23}\bar{\mathcal{D}}\mathcal{J}_{52} + \mathcal{J}_{25}\mathcal{J}_{52}\right] \tag{5.23}
$$

with central charge $2k$. The remnants of nonlinear irreducibility constraints are given by

$$
\begin{aligned}
&\bar{\mathcal{D}}\mathcal{H}_{\bar{1}} = 0, && \mathcal{D}\mathcal{H}_2 = 0\,, \\
&\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{21} = 0, && \left(\mathcal{D} + \frac{1}{k}\mathcal{H}_2\right)\mathcal{J}_{23} = 0\,, \\
&\left(\mathcal{D} - \frac{1}{k}\mathcal{H}_2\right)\mathcal{J}_{35} = 0, && \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{41} + \frac{1}{k}\mathcal{J}_{21}\mathcal{J}_{42} = 0\,, \\
&\bar{\mathcal{D}}\mathcal{J}_{42} = 0, && \mathcal{D}\mathcal{J}_{25} - \frac{1}{k}\mathcal{J}_{23}\mathcal{J}_{35} = 0\,.
\end{aligned}
\tag{5.24}
$$

The above gauge invariant supercurrents form some extended $N=2$ SCA, in particular, the stress tensor (5.23) generates the standard $N=2$ SCA. Here we do not present this superalgebra explicitly and leave its study to the future, limiting ourselves only to some comments concerning its irreducible $N=1$ supercurrent content (see below). The reason is that it does not meet one of the criterions by which we limited from the beginning our study in this paper. Namely, the $N=2$ stress tensor (5.23) is constrained because the linear terms in (5.23) $\mathcal{J}_{35}, \mathcal{J}_{41}$ are constrained. The only unconstrained bosonic supercurrent with the spin and $u(1)$ charge appropriate for $N=2$ stress tensor, $\mathcal{J}_{15}$, enters nonlinearly into $\mathcal{T}$ (5.23), so Eq. (5.23) does not imply an invertible relation between $\mathcal{T}$ and $\mathcal{J}_{15}$.

We would like to mention that the supercurrents

$$\mathcal{H}_2, \qquad \mathcal{H}_{\bar{1}} \tag{5.25}$$

are quasi-superprimary, and

$$\mathcal{J}_{51},\ \mathcal{J}_{52},\ \mathcal{J}_{21},\ \mathcal{J}_{23},\ \mathcal{J}_{15},\ \mathcal{J}_{42},\ \mathcal{J}_{25}-\frac{k}{2}\bar{\mathcal{D}}\mathcal{J}_{23},\ \mathcal{J}_{35}-\mathcal{J}_{41}-k(\bar{\mathcal{D}}\mathcal{H}_2+\mathcal{D}\mathcal{H}_{\bar{1}}) \tag{5.26}$$

are superprimary with respect to $\mathcal{T}$ (5.23).

Before going further, let us comment on the $N=1$ superfield formulation of this unusual superalgebra. Solving the constraints (5.24) through $N=1$ superfields like this has been done for the constrained $N=2$ affine supercurrents in Eq. (2.9); we find that it is generated by 14 independent $N=1$ supercurrents:

$$h_2,\ h_{\bar{1}},\ j^1_{15},\ j^2_{15},\ j_{21},\ j_{23},\ j_{25},\ j_{35},\ j_{42},\ j_{41},\ j^1_{51},\ j^2_{51},\ j^1_{52},\ j^2_{52}\ ,$$

which are basically the first components in the θ^1 decomposition of the related $N=2$ supercurrents, except for $j^2_{15}, j^2_{51}, j^2_{52}$ which are second components (the corresponding $N=2$ supercurrents are unconstrained). After substituting the solution of the constraints (5.24) into the $N=1$ components of the stress-tensor (5.23),

$$\mathcal{T}=\mathcal{T}^1+\theta^1\mathcal{T}^2\,, \tag{5.27}$$

we obtain

$$\mathcal{T}^1=-\frac{1}{k}\left[j_{35}+j_{41}+h_2h_{\bar{1}}+j^1_{15}j^1_{51}-\frac{k}{2}j_{23}(iDj^1_{52}+j^2_{52})+j_{25}j^1_{52}\right]\,, \tag{5.28}$$

$$\begin{aligned}\mathcal{T}^2=-\frac{1}{k}\Bigg[&\left(iD+\frac{2}{k}h_2\right)j_{35}-\left(iD-\frac{2}{k}h_{\bar{1}}\right)j_{41}-\frac{2}{k}j_{21}j_{42}+ih_2Dh_{\bar{1}}+i(Dh_2)h_{\bar{1}}\\&+j^1_{15}j^2_{51}+j^2_{15}j^1_{51}+\frac{k}{2}j_{23}(Dj^2_{52}+j'_{52})-\left(\frac{ik}{2}Dj_{23}-h_2j_{23}\right)(iDj^1_{52}+j^2_{52})\\&+j_{25}j^2_{52}+\left(iDj_{25}+\frac{2}{k}j_{23}j_{35}\right)j^1_{52}\Bigg]\,.\end{aligned} \tag{5.29}$$

We see that the $N=1$ stress-tensor $\mathcal{T}^1$ is unconstrained and elementary as it starts with a combination of independent $N=1$ supercurrents. On the other hand, the supercurrent $\mathcal{T}^2$ completing $\mathcal{T}^1$ to the whole $N=2$ stress-tensor turns out to be composite: for it one gets a kind of $N=1$ Sugawara construction in terms of the remaining $N=1$ supercurrents. This composite nature of $\mathcal{T}^2$ is just the $N=1$

superfield manifestation of the above mentioned fact that the $N=2$ stress-tensor is constrained in the present case. To our knowledge, such an unusual situation was not earlier encountered in the study of $N=1$ superfield hamiltonian reduction [12].

Now we turn to another choice of five constraints which leads to an unconstrained $N=2$ stress tensor. For

$$\alpha_1 = -1, \qquad \alpha_{\bar{1}} = 1, \qquad \alpha_2 = 0, \qquad \alpha_{\bar{2}} = 0 \tag{5.30}$$

we have the spins and $u(1)$ charges as is given in Tables 8, 9.

With this choice of parameters, we impose the following constraints:

$$\mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 0 & 1 & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & 1 & 0 & * & * \\ * & * & * & * & * \end{pmatrix} . \tag{5.31}$$

Then, fixing gauges according to Table 8, for surviving supercurrents we have Table 9. In Table 9, last three supercurrents are expressed through the remaining ones by the relations

$$\mathcal{J}_{45} = k\left(\mathcal{D} + \frac{1}{k}\mathcal{H}_1\right)\mathcal{J}_{15} ,$$

$$\mathcal{J}_{21} = k\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{1}}\right)\mathcal{J}_{41} ,$$

$$\mathcal{J}_{54} = k\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{52} - \mathcal{J}_{32}\mathcal{J}_{53} . \tag{5.32}$$

Table 8.

$u(1)$	1	2	0	0	1
dim	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$
constr. scs	$\mathcal{J}_{12}^F$	$\mathcal{J}_{13}^F$	$\mathcal{J}_{42}^B$	$\mathcal{J}_{14}^B$	$\mathcal{J}_{43}^B$
g.f. scs	$\mathcal{J}_{24}^B$	$\mathcal{J}_{34}^B$	$\mathcal{H}_{\bar{1}}^F$	$\mathcal{H}_1^F$	$\mathcal{J}_{32}^F$
dim	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
$u(1)$	0	-1	-1	1	-2

Table 9.

surv. scs	$\mathcal{J}_{15}^B$	$\mathcal{J}_{52}^B$	$\mathcal{J}_{53}^B$	$\mathcal{J}_{35}^B$	$\mathcal{H}_2^F$	$\mathcal{H}_{\bar{2}}^F$	$\mathcal{J}_{25}^B$	$\mathcal{J}_{51}^B$	$\mathcal{J}_{23}^F$	$\mathcal{J}_{41}^B$	$\mathcal{J}_{31}^F$	$\mathcal{J}_{54}^F$	$\mathcal{J}_{45}^F$	$\mathcal{J}_{21}^F$
dim	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
$u(1)$	2	-2	-1	1	1	-1	2	-2	2	0	-2	-3	3	-1

The supercurrents $\mathcal{J}_{15}, \mathcal{J}_{41}$, and $\mathcal{J}_{52}$ are unconstrained. Then one gets the following $\mathcal{J}_{mn}^{DS}$:

$$\mathcal{J}_{mn}^{DS} = \begin{pmatrix} 0 & 0 & 0 & 1 & \mathcal{J}_{15} \\ k\bar{\mathcal{D}}\mathcal{J}_{41} & \mathcal{H}_{\bar{2}} & \mathcal{J}_{23} & 0 & \mathcal{J}_{25} \\ \mathcal{J}_{31} & 0 & \mathcal{H}_2 & 0 & \mathcal{J}_{35} \\ \mathcal{J}_{41} & 1 & 0 & 0 & k\mathcal{D}\mathcal{J}_{15} \\ \mathcal{J}_{51} & \mathcal{J}_{52} & \mathcal{J}_{53} & k(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}})\mathcal{J}_{52} & \mathcal{H}_2 + \mathcal{H}_{\bar{2}} \end{pmatrix} \tag{5.33}$$

The gauge invariant supercurrents are given by the following expressions:

$$\begin{aligned}
\widetilde{\mathcal{J}_{15}} &= 2\mathcal{J}_{15} - \mathcal{J}_{15}\mathcal{J}_{42} + k\bar{\mathcal{D}}\mathcal{J}_{12}\mathcal{J}_{15} , \\
\widetilde{\mathcal{J}_{52}} &= 2\mathcal{J}_{52} - \mathcal{J}_{12}\mathcal{J}_{54} - \mathcal{J}_{14}\mathcal{J}_{52} , \\
\widetilde{\mathcal{J}_{53}} &= \mathcal{J}_{53} - \mathcal{J}_{13}\mathcal{J}_{54} - \mathcal{J}_{13}\mathcal{H}_{\bar{2}}\mathcal{J}_{52} - \mathcal{J}_{43}\mathcal{J}_{52} + k\bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{J}_{52} , \\
\widetilde{\mathcal{J}_{35}} &= \mathcal{J}_{35} + \mathcal{H}_2\mathcal{J}_{15}\mathcal{J}_{32} - \mathcal{J}_{15}\mathcal{J}_{34} - k\mathcal{J}_{15}\mathcal{D}\mathcal{J}_{32} - \mathcal{J}_{45}\mathcal{J}_{32} , \\
\widetilde{\mathcal{H}_2} &= \mathcal{H}_2 , \\
\widetilde{\mathcal{H}_{\bar{2}}} &= \mathcal{H}_{\bar{1}} + \mathcal{H}_{\bar{2}} , \\
\widetilde{\mathcal{J}_{25}} &= \mathcal{J}_{13}\mathcal{J}_{35}\mathcal{H}_{\bar{2}} + \mathcal{J}_{14}\mathcal{J}_{25} - \mathcal{J}_{15}\mathcal{J}_{24} + k\mathcal{J}_{15}\mathcal{D}\mathcal{H}_{\bar{1}} + \mathcal{J}_{23}\mathcal{J}_{15}\mathcal{J}_{32} + \mathcal{J}_{35}\mathcal{J}_{43} \\
&\quad + \mathcal{J}_{45}\mathcal{H}_{\bar{1}} - k\bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{J}_{35} - k\bar{\mathcal{D}}\mathcal{J}_{14}\mathcal{J}_{45} - k^2\mathcal{J}_{14}'\mathcal{J}_{15} , \\
\widetilde{\mathcal{J}_{51}} &= \mathcal{H}_1\mathcal{J}_{54} - \mathcal{H}_2\mathcal{J}_{32}\mathcal{J}_{53} - \mathcal{J}_{23}\mathcal{J}_{32}\mathcal{J}_{52} + \mathcal{J}_{24}\mathcal{J}_{52} + \mathcal{J}_{34}\mathcal{J}_{53} + \mathcal{J}_{42}\mathcal{J}_{51} \\
&\quad - k\mathcal{D}\mathcal{H}_{\bar{1}}\mathcal{J}_{52} + k\mathcal{D}\mathcal{J}_{32}\mathcal{J}_{53} + k\mathcal{D}\mathcal{J}_{42}\mathcal{J}_{54} + k^2\mathcal{J}_{12}'\mathcal{J}_{54} + k^2\mathcal{J}_{14}'\mathcal{J}_{52} \\
&\quad - k\bar{\mathcal{D}}\mathcal{J}_{12}\mathcal{J}_{51} , \\
\widetilde{\mathcal{J}_{23}} &= k\mathcal{D}\mathcal{J}_{43} + \mathcal{H}_2\mathcal{J}_{43} - k\mathcal{J}_{12}\bar{\mathcal{D}}\mathcal{J}_{23} + \mathcal{J}_{12}\mathcal{J}_{23}\mathcal{H}_{\bar{2}} - k\mathcal{J}_{13}\mathcal{D}\mathcal{H}_{\bar{2}} \\
&\quad + \mathcal{J}_{23}\mathcal{J}_{14} - k\bar{\mathcal{D}}\mathcal{H}_2\mathcal{J}_{13} + k\bar{\mathcal{D}}\mathcal{J}_{12}\mathcal{J}_{23} + k^2\mathcal{J}_{13}', \\
\widetilde{\mathcal{J}_{41}} &= \mathcal{J}_{24} + k\bar{\mathcal{D}}\mathcal{H}_1 + k^3\bar{\mathcal{D}}\mathcal{J}_{12}' - k\mathcal{D}\mathcal{H}_{\bar{1}} - \mathcal{J}_{12}\mathcal{J}_{21} \\
&\quad - \mathcal{J}_{13}\mathcal{J}_{31} + \mathcal{J}_{14}\mathcal{J}_{41} - \mathcal{J}_{23}\mathcal{J}_{32} + k^2\mathcal{J}_{14}' - k^2\mathcal{J}_{42}', \\
\widetilde{\mathcal{J}_{31}} &= k\bar{\mathcal{D}}\mathcal{J}_{34} + \mathcal{J}_{31}\mathcal{J}_{42} - \mathcal{J}_{32}\mathcal{J}_{41} - \mathcal{J}_{34}\mathcal{H}_{\bar{2}} - k\bar{\mathcal{D}}\mathcal{H}_2\mathcal{J}_{32} \\
&\quad - k\bar{\mathcal{D}}\mathcal{J}_{12}\mathcal{J}_{31} - k\mathcal{D}\mathcal{H}_{\bar{2}}\mathcal{J}_{32} - k^2\mathcal{J}_{32}' .
\end{aligned} \tag{5.34}$$

The whole set of irreducibility constraints for surviving supercurrents is as follows:

$$\mathcal{D}\mathcal{H}_2 = 0, \qquad \bar{\mathcal{D}}\mathcal{H}_{\bar{2}} = 0, \qquad \bar{\mathcal{D}}\mathcal{J}_{53} = 0\,,$$
$$\left(\mathcal{D} - \frac{1}{k}\mathcal{H}_2\right)\mathcal{J}_{35} = 0, \qquad \left(\mathcal{D} + \frac{1}{k}\mathcal{H}_2\right)\mathcal{J}_{23} = 0\,,$$
$$\mathcal{D}\mathcal{J}_{25} - \frac{1}{k}\mathcal{J}_{23}\mathcal{J}_{35} = 0, \qquad \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{31} = 0\,,$$
$$\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{51} + \frac{1}{k}(\mathcal{J}_{21}\mathcal{J}_{52} + \mathcal{J}_{31}\mathcal{J}_{53} + \mathcal{J}_{41}\mathcal{J}_{54}) = 0\,. \tag{5.35}$$

The stress tensor has the following form:

$$\mathcal{T} = -\frac{1}{k}[\mathcal{H}_2\mathcal{H}_{\bar{2}} + \mathcal{J}_{41} + \mathcal{J}_{15}\mathcal{J}_{51} + \mathcal{J}_{25}\mathcal{J}_{52} + \mathcal{J}_{35}\mathcal{J}_{53} + k\mathcal{D}\mathcal{J}_{15}(k\bar{\mathcal{D}}\mathcal{J}_{52} - \mathcal{H}_{\bar{2}}\mathcal{J}_{52})]\,, \tag{5.36}$$

and possesses the central charge $-2k$. On the constraints shell the Sugawara $N=2$ stress tensor coincides with $\mathcal{T}$. With respect to $\mathcal{T}$, the following combinations of supercurrents:

$$\mathcal{J}_{15},\ \mathcal{J}_{52},\ \mathcal{J}_{53},\ \mathcal{J}_{23},\ \mathcal{H}_2,\ \mathcal{H}_{\bar{2}},\ \mathcal{J}_{35},\ \mathcal{J}_{31},\ \mathcal{J}_{25} - \frac{k^2}{2}([\mathcal{D},\bar{\mathcal{D}}]\mathcal{J}_{15} - \mathcal{J}'_{15})\,,$$
$$\mathcal{J}_{51} - \frac{k^2}{2}[\mathcal{D},\bar{\mathcal{D}}]\mathcal{J}_{52} - \frac{k^2}{2}\mathcal{J}'_{52} + k\mathcal{D}\mathcal{H}_{\bar{2}}\mathcal{J}_{52} + k\mathcal{J}_{52}\mathcal{T} \tag{5.37}$$

are superprimary.

It is straightforward to derive the complete set of SOPEs between the above supercurrents $\mathcal{J}_{\alpha\beta}$'s (5.37). The $N=2$ stress tensor (5.36) entering into this $N=2$ SCA is *unconstrained* since the linear term $\mathcal{J}_{41}$ in (5.36) is unconstrained. We do not give here the SOPEs between the surviving supercurrents because these are very complicated due to the presence of dimension zero supercurrents $\mathcal{J}_{15}, \mathcal{J}_{52}$. Let us only point out that in the present case one cannot decouple two fields of dimension 0 after passing to the component form of the superalgebra.

In the next subsection we will show that the above unpleasant features of SCA under consideration disappear after the appropriate secondary hamiltonian reduction of it. The resulting SCA does not contain any spin 0 supercurrents; all the involved supercurrents are superprimary with respect to the corresponding $N=2$ stress tensor. This reduction is accomplished by adding two more constraints to the set (5.31) and so corresponds to imposing some seven constraints on the original supermatrix of $N=2$ $sl(3|2)^{(1)}$ affine supercurrents.

5.4. $N=2$ $u(2|1)$ SCA. In this subsection we show that there exists a natural reduction of the second of extended $N=2$ SCAs considered in the previous subsection, such that it yields a $N=2$ extension of the $u(2|1)$ SCA of Ref. [24].

The $u(2|1)$ SCA is some graded version of the $u(3)$ KB SCA and is generated by 16 component currents, the number of bosonic and fermionic ones being the same. The spins of them are greater than 1/2. The details of this algebra will be given later, the only point we wish to mention at once is that there is no standard supersymmetry subalgebra in this SCA. Anticipating our results, the $N=2$ supersymmetric extension of this SCA, $N=2$ $u(2|1)$, contains four extra spin 1/2 currents: two of them are bosonic, others fermionic. This current content immediately implies that the number of the hamiltonian reduction constraints should be seven. One could start directly from the $N=2$ $sl(3|2)^{(1)}$ current algebra, i.e. make use of the primary hamiltonian reduction procedure. However, it is simpler to deduce the same results in an equivalent way, applying a secondary reduction to the extended $N=2$ SCA described in the end of the previous subsection.

Thus we start with the same choice of splitting parameters (5.30) and wish to strengthen the set of constraints (5.31) by adding two more. A natural desire is to get rid of the unwanted spin 0 supercurrents, viz. $\mathcal{J}_{15}$, $\mathcal{J}_{25}$ (see Table 9). It turns out that they both are eliminated by enforcing the constraint

$$\mathcal{J}_{15} = 0\,. \tag{5.38}$$

Then we can gauge away $\mathcal{J}_{52}$ using the gauge transformation generated by this new constraint:

$$\mathcal{J}_{52} = 0\,. \tag{5.39}$$

We also note that (5.38), via the relations (5.32), automatically implies

$$\mathcal{J}_{45} = 0\,. \tag{5.40}$$

So the final supermatrix of constraints is given by

$$\mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 0 & 1 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & * & * & * \end{pmatrix}. \tag{5.41}$$

As in previous examples, we list the constrained, gauge fixed and surviving supercurrents in Tables 10 and 11.

After substituting (5.38), (5.39) into (5.33), the relevant $\mathcal{J}_{mn}^{DS}$ takes the following form:

$$\mathcal{J}_{mn}^{DS} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ k\bar{\mathcal{D}}\mathcal{J}_{41} & \mathcal{H}_{\bar{2}} & \mathcal{J}_{23} & 0 & \mathcal{J}_{25} \\ \mathcal{J}_{31} & 0 & \mathcal{H}_2 & 0 & \mathcal{J}_{35} \\ \mathcal{J}_{41} & 1 & 0 & 0 & 0 \\ \mathcal{J}_{51} & 0 & \mathcal{J}_{53} & 0 & \mathcal{H}_2 + \mathcal{H}_{\bar{2}} \end{pmatrix}. \tag{5.42}$$

Table 10.

$u(1)$	1	2	0	0	1	2	3
dim	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$
constr. scs	$\mathcal{J}_{12}^F$	$\mathcal{J}_{13}^F$	$\mathcal{J}_{42}^B$	$\mathcal{J}_{14}^B$	$\mathcal{J}_{43}^B$	$\mathcal{J}_{15}^B$	$\mathcal{J}_{45}^F$
g.f. scs	$\mathcal{J}_{24}^B$	$\mathcal{J}_{34}^B$	$\mathcal{H}_{\bar{1}}^F$	$\mathcal{H}_1^F$	$\mathcal{J}_{32}^F$	$\mathcal{J}_{52}^B$	$\mathcal{J}_{54}^F$
dim	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
$u(1)$	0	-1	-1	1	-2	-2	-3

Table 11.

surv. scs	$\mathcal{J}_{53}^B$	$\mathcal{J}_{35}^B$	$\mathcal{H}_2^F$	$\mathcal{H}_{\bar{2}}^F$	$\mathcal{J}_{25}^B$	$\mathcal{J}_{51}^B$	$\mathcal{J}_{23}^F$	$\mathcal{J}_{41}^B$	$\mathcal{J}_{31}^F$	$\mathcal{J}_{21}^F$
dim	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	$\frac{3}{2}$
$u(1)$	-1	1	1	-1	2	-2	2	0	-2	-1

All the elementary supercurrents here, except for $\mathcal{J}_{41}$, are still subjected to the constraints which are obtained by substituting (5.38), (5.39) into (5.35):

$$\mathcal{D}\mathcal{H}_2 = 0, \qquad \bar{\mathcal{D}}\mathcal{H}_{\bar{2}} = 0\,,$$

$$\left(\mathcal{D} - \frac{1}{k}\mathcal{H}_2\right)\mathcal{J}_{35} = 0, \qquad \bar{\mathcal{D}}\mathcal{J}_{53} = 0\,,$$

$$\left(\mathcal{D} + \frac{1}{k}\mathcal{H}_2\right)\mathcal{J}_{23} = 0, \qquad \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{31} = 0\,,$$

$$\mathcal{D}\mathcal{J}_{25} - \frac{1}{k}\mathcal{J}_{23}\mathcal{J}_{35} = 0, \qquad \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{51} - \frac{1}{k}\mathcal{J}_{31}\mathcal{J}_{53} = 0\,. \tag{5.43}$$

Using the same techniques as before, we get the appropriate expressions for gauge invariant supercurrents in terms of the original ones (forming the previous SCA with five constraints). We do not give them explicitly. We only note that the SOPEs between $\mathcal{J}_{kl}$ appearing in the r.h.s. of these expressions can be found by using the SOPEs of the second superalgebra presented in subsect. 5.3. The $N=2$ stress tensor is given by

$$\mathcal{T} = -\frac{1}{k}[\mathcal{H}_2\mathcal{H}_{\bar{2}} + \mathcal{J}_{41} + \mathcal{J}_{35}\mathcal{J}_{53}] \tag{5.44}$$

with central charge $-2k$. On the shell of constraints the Sugawara $N=2$ stress tensor coincides with this stress tensor and contains linearly the supercurrent $\mathcal{J}_{41}$, so $\mathcal{T}$ (5.44) is unconstrained.

Then the $N=2$ $u(2|1)$ SCA (the reason why we call it this will be soon clear) besides the general spin 1 supercurrent $\mathcal{T}$, $N=2$ stress tensor, contains the following eight constrained $N=2$ supercurrents: spin 1/2 $\mathcal{H}_2$, $\mathcal{J}_{53}$, $\mathcal{J}_{35}$ and $\mathcal{H}_{\bar{2}}$, spin 1 $\mathcal{J}_{51}, \mathcal{J}_{25}$, $\mathcal{J}_{23}$, and $\mathcal{J}_{31}$. All these supercurrents are superprimary with respect to $\mathcal{T}$. After rescaling

$$\mathcal{J}_{25} \to \frac{1}{k}\mathcal{J}_{25}, \qquad \mathcal{J}_{23} \to \frac{1}{k}\mathcal{J}_{23}, \qquad \mathcal{J}_{31} \to \frac{1}{k}\mathcal{J}_{31}, \qquad \mathcal{J}_{51} \to \frac{1}{k}\mathcal{J}_{51}, \quad (5.45)$$

the rest of the nonvanishing SOPEs are as follows:

$$\begin{aligned}
\mathcal{H}_2(Z_1)\mathcal{H}_{\bar{2}}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{k}{2} - \frac{1}{z_{12}}k, \\
\mathcal{H}_{\bar{2}}(Z_1)\mathcal{J}_{35}(Z_2) &= -\frac{\theta_{12}}{z_{12}}\mathcal{J}_{35}, \\
\mathcal{H}_{\bar{2}}(Z_1)\mathcal{J}_{53}(Z_2) &= \frac{\theta_{12}}{z_{12}}\mathcal{J}_{53}, \\
\mathcal{J}_{35}(Z_1)\mathcal{J}_{53}(Z_2) &= -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{k}{2} + \frac{1}{z_{12}}k + \frac{\theta_{12}}{z_{12}}\mathcal{H}_2 + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\bar{\mathcal{D}}\mathcal{H}_2,
\end{aligned} \quad (5.46)$$

$$\mathcal{H}_2(Z_1)\begin{cases}
\mathcal{J}_{23}(Z_2) = \frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{23}, \\
\mathcal{J}_{25}(Z_2) = \frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{25}, \\
\mathcal{J}_{31}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{31}, \\
\mathcal{J}_{51}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{51},
\end{cases}$$

$$\mathcal{H}_{\bar{2}}(Z_1)\begin{cases}
\mathcal{J}_{23}(Z_2) = \frac{\theta_{12}}{z_{12}}\mathcal{J}_{23}, \\
\mathcal{J}_{31}(Z_2) = -\frac{\theta_{12}}{z_{12}}\mathcal{J}_{31},
\end{cases}$$

$$\mathcal{J}_{35}(Z_1)\begin{cases}
\mathcal{J}_{23}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{25} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}[\mathcal{H}_2\mathcal{J}_{25} + \mathcal{J}_{35}\mathcal{J}_{23}], \\
\mathcal{J}_{25}(Z_2) = \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathcal{J}_{35}\mathcal{J}_{25}, \\
\mathcal{J}_{31}(Z_2) = -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathcal{J}_{35}\mathcal{J}_{31}, \\
\mathcal{J}_{51}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{31} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}[\mathcal{H}_2\mathcal{J}_{31} - \mathcal{J}_{35}\mathcal{J}_{51}],
\end{cases}$$

$$\mathcal{J}_{53}(Z_1)\begin{cases}
\mathcal{J}_{25}(Z_2) = \frac{\theta_{12}}{z_{12}}\mathcal{J}_{23}, \\
\mathcal{J}_{31}(Z_2) = -\frac{\theta_{12}}{z_{12}}\mathcal{J}_{51},
\end{cases}$$

$$\mathcal{J}_{23}(Z_1)\mathcal{J}_{25}(Z_2) = -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathcal{J}_{23}\mathcal{J}_{25},$$

$$\mathcal{J}_{23}(Z_1)\mathcal{J}_{31}(Z_2) = \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^3}k - \frac{1}{z_{12}^2}k + \frac{\bar{\theta}_{12}}{z_{12}^2}\mathcal{H}_{\bar{2}} + \frac{\theta_{12}}{z_{12}^2}\mathcal{H}_2$$

$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\left[-\frac{1}{2}\mathcal{T} + \frac{3}{2}\bar{\mathcal{D}}\mathcal{H}_2 + \frac{1}{2}\mathcal{D}\mathcal{H}_{\bar{2}} + \frac{1}{2k}\mathcal{H}_2\mathcal{H}_{\bar{2}} - \frac{1}{k}\mathcal{J}_{35}\mathcal{J}_{53}\right]$$

$$+\frac{1}{z_{12}}\left[\mathcal{T} - \mathcal{D}\mathcal{H}_{\bar{2}} + \frac{1}{k}\mathcal{H}_2\mathcal{H}_{\bar{2}} + \frac{2}{k}\mathcal{J}_{35}\mathcal{J}_{53} - \bar{\mathcal{D}}\mathcal{H}_2\right]$$

$$+\frac{\bar{\theta}_{12}}{z_{12}}\left[\bar{\mathcal{D}}\mathcal{T} - \frac{1}{k}\mathcal{H}_{\bar{2}}\mathcal{T} + \frac{1}{k}\mathcal{H}_{\bar{2}}\mathcal{D}\mathcal{H}_{\bar{2}} - \frac{2}{k^2}\mathcal{J}_{35}\mathcal{H}_{\bar{2}}\mathcal{J}_{53}\right.$$

$$\left.+\frac{2}{k}\bar{\mathcal{D}}\mathcal{H}_2\mathcal{H}_{\bar{2}} + \frac{2}{k}\bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{J}_{53} + \mathcal{H}_{\bar{2}}'\right]$$

$$+\frac{\theta_{12}}{z_{12}}\left[-\frac{1}{k}\mathcal{H}_2\mathcal{T} + \frac{1}{k}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{H}_2 \dot{+} \frac{1}{k}\mathcal{H}_2\mathcal{D}\mathcal{H}_{\bar{2}} - \frac{2}{k^2}\mathcal{H}_2\mathcal{J}_{35}\mathcal{J}_{53} + \mathcal{H}_2'\right]$$

$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[\bar{\mathcal{D}}\mathcal{H}_2' + \frac{1}{k}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{T} - \frac{1}{k^2}\mathcal{H}_2\mathcal{H}_{\bar{2}}\mathcal{T} + \frac{1}{k^2}\mathcal{H}_2\mathcal{H}_{\bar{2}}\mathcal{D}\mathcal{H}_{\bar{2}}\right.$$

$$-\frac{2}{k^3}\mathcal{H}_2\mathcal{J}_{35}\mathcal{H}_{\bar{2}}\mathcal{J}_{53} + \frac{1}{k^2}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{H}_2\mathcal{H}_{\bar{2}} + \frac{2}{k^2}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{J}_{53} + \frac{1}{k}\mathcal{J}_{23}\mathcal{J}_{31}$$

$$-\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_2\mathcal{T} + \frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{H}_2 + \frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_2\mathcal{D}\mathcal{H}_{\bar{2}} - \frac{2}{k^2}\bar{\mathcal{D}}\mathcal{H}_2\mathcal{J}_{35}\mathcal{J}_{53}$$

$$\left.+\frac{1}{k}(\mathcal{H}_2\mathcal{H}_{\bar{2}})'\right],$$

$$\mathcal{J}_{23}(Z_1)\mathcal{J}_{51}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}^2}\mathcal{J}_{53} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\left[-\frac{1}{2}\mathcal{D}\mathcal{J}_{53} - \frac{3}{2k}\mathcal{H}_2\mathcal{J}_{53}\right]$$

$$+\frac{1}{z_{12}}\left[\mathcal{D}\mathcal{J}_{53} + \frac{1}{k}\mathcal{H}_2\mathcal{J}_{53}\right]$$

$$+\frac{\bar{\theta}_{12}}{z_{12}}\left[-\frac{1}{k}\mathcal{H}_{\bar{2}}\mathcal{D}\mathcal{J}_{53} + \frac{2}{k^2}\mathcal{H}_2\mathcal{H}_{\bar{2}}\mathcal{J}_{53} + \frac{2}{k^2}\mathcal{J}_{35}\mathcal{J}_{53}\mathcal{J}_{53} + \frac{1}{k}\mathcal{T}\mathcal{J}_{53}\right.$$

$$\left.-\frac{1}{k}\mathcal{D}\mathcal{H}_{\bar{2}}\mathcal{J}_{53} - \mathcal{J}_{53}'\right] - \frac{\theta_{12}}{z_{12}}\frac{1}{k}\mathcal{H}_2\mathcal{D}\mathcal{J}_{53}$$

$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[-\frac{1}{k^2}\mathcal{H}_2\mathcal{H}_{\bar{2}}\mathcal{D}\mathcal{J}_{53}+\frac{2}{k^3}\mathcal{H}_2\mathcal{J}_{35}\mathcal{J}_{53}\mathcal{J}_{53}+\frac{1}{k^2}\mathcal{H}_2\mathcal{T}\mathcal{J}_{53}\right.$$

$$-\frac{1}{k^2}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{H}_2\mathcal{J}_{53}-\frac{1}{k^2}\mathcal{H}_2\mathcal{D}\mathcal{H}_{\bar{2}}\mathcal{J}_{53}-\frac{1}{k}(\mathcal{H}_2\mathcal{J}_{53})'+\frac{1}{k}\mathcal{J}_{23}\mathcal{J}_{51}$$

$$\left.-\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_2\mathcal{D}\mathcal{J}_{53}\right]$$

$$\mathcal{J}_{25}(Z_1)\mathcal{J}_{31}(Z_2)=-\frac{\theta_{12}}{z_{12}^2}\mathcal{J}_{35}+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\left[-\frac{3}{2}\bar{\mathcal{D}}\mathcal{J}_{35}+\frac{1}{k}\mathcal{J}_{35}\mathcal{H}_{\bar{2}}\right]$$

$$+\frac{1}{z_{12}}\bar{\mathcal{D}}\mathcal{J}_{35}+\frac{\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{H}_{\bar{2}}$$

$$+\frac{\theta_{12}}{z_{12}}\left[\frac{1}{k^2}\mathcal{H}_2\mathcal{J}_{35}\mathcal{H}_{\bar{2}}+\frac{1}{k}\mathcal{J}_{35}\mathcal{T}-\frac{1}{k}\mathcal{J}_{35}\mathcal{D}\mathcal{H}_{\bar{2}}+\frac{2}{k^2}\mathcal{J}_{35}\mathcal{J}_{35}\mathcal{J}_{53}\right.$$

$$\left.-\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_2\mathcal{J}_{35}-\mathcal{J}'_{35}\right]$$

$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[-\bar{\mathcal{D}}\mathcal{J}'_{35}-\frac{1}{k^2}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{H}_{\bar{2}}+\frac{1}{k}\mathcal{J}_{35}\bar{\mathcal{D}}\mathcal{T}-\frac{1}{k^2}\mathcal{J}_{35}\mathcal{H}_{\bar{2}}\mathcal{T}\right.$$

$$+\frac{1}{k^2}\mathcal{J}_{35}\mathcal{H}_{\bar{2}}\mathcal{D}\mathcal{H}_{\bar{2}}-\frac{2}{k^3}\mathcal{J}_{35}\mathcal{J}_{35}\mathcal{H}_{\bar{2}}\mathcal{J}_{53}$$

$$+\frac{4}{k^2}\mathcal{J}_{35}\bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{J}_{53}+\frac{1}{k}(\mathcal{J}_{35}\mathcal{H}_{\bar{2}})'$$

$$\left.-\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_2\bar{\mathcal{D}}\mathcal{J}_{35}+\frac{2}{k^2}\bar{\mathcal{D}}\mathcal{H}_2\mathcal{J}_{35}\mathcal{H}_{\bar{2}}+\frac{1}{k}\bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{T}-\frac{1}{k}\bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{D}\mathcal{H}_{\bar{2}}\right],$$

$$\mathcal{J}_{25}(Z_1)\mathcal{J}_{51}(Z_2)=\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^3}k-\frac{1}{z_{12}^2}k+\frac{\bar{\theta}_{12}}{z_{12}^2}\mathcal{H}_{\bar{2}}$$

$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\left[-\frac{1}{2}\mathcal{T}+\frac{1}{2}\mathcal{D}\mathcal{H}_{\bar{2}}-\frac{1}{2k}\mathcal{H}_2\mathcal{H}_{\bar{2}}-\frac{2}{k}\mathcal{J}_{35}\mathcal{J}_{53}\right]$$

$$+\frac{1}{z_{12}}\left[\mathcal{T}-\mathcal{D}\mathcal{H}_{\bar{2}}+\frac{1}{k}\mathcal{H}_2\mathcal{H}_{\bar{2}}+\frac{2}{k}\mathcal{J}_{35}\mathcal{J}_{53}\right]$$

$$+\frac{\bar\theta_{12}}{z_{12}}\left[\bar{\mathscr{D}}\mathscr{T}-\frac{1}{k}\mathscr{H}_{\bar 2}\mathscr{T}+\frac{1}{k}\mathscr{H}_{\bar 2}\mathscr{D}\mathscr{H}_{\bar 2}-\frac{2}{k^2}\mathscr{J}_{35}\mathscr{H}_{\bar 2}\mathscr{J}_{53}\right.$$

$$\left.+\frac{1}{k}\bar{\mathscr{D}}\mathscr{H}_2\mathscr{H}_{\bar 2}+\frac{1}{k}\bar{\mathscr{D}}\mathscr{J}_{35}\mathscr{J}_{53}+\mathscr{H}'_{\bar 2}\right]$$

$$+\frac{\theta_{12}}{z_{12}}\left[\frac{1}{k^2}\mathscr{H}_2\mathscr{J}_{35}\mathscr{J}_{53}+\frac{1}{k}\mathscr{J}_{35}\mathscr{D}\mathscr{J}_{53}\right]$$

$$+\frac{\theta_{12}\bar\theta_{12}}{z_{12}}\left[\frac{2}{k^3}\mathscr{H}_2\mathscr{J}_{35}\mathscr{H}_{\bar 2}\mathscr{J}_{53}-\frac{1}{k^2}\mathscr{H}_2\bar{\mathscr{D}}\mathscr{J}_{35}\mathscr{J}_{53}+\frac{1}{k}\mathscr{J}_{23}\mathscr{J}_{31}\right.$$

$$-\frac{1}{k^2}\mathscr{J}_{35}\mathscr{H}_{\bar 2}\mathscr{D}\mathscr{J}_{53}+\frac{2}{k^3}\mathscr{J}_{35}\mathscr{J}_{35}\mathscr{J}_{53}\mathscr{J}_{53}+\frac{1}{k^2}\mathscr{J}_{35}\mathscr{T}\mathscr{J}_{53}$$

$$\left.+\frac{1}{k}\bar{\mathscr{D}}\mathscr{J}_{35}\mathscr{D}\mathscr{J}_{53}-\frac{1}{k}(\mathscr{J}_{35}\mathscr{J}_{53})'-\frac{1}{k^2}\mathscr{J}_{35}\mathscr{D}\mathscr{H}_{\bar 2}\mathscr{J}_{53}\right]. \tag{5.47}$$

It can be checked that our algebra satisfies all the Jacobi identities. The supercurrents $\mathscr{H}_2, \mathscr{H}_{\bar 2}, \mathscr{J}_{35}, \mathscr{J}_{53}$ form the $N=2$ $u(1|1)^{(1)}$ current algebra as a subalgebra (their SOPEs form a closed set as is seen from Eq. (5.46)).

Let us show that the $N=2$ $W_3^{(2)}$ SCA constructed in Subsect. 5.2 using primary hamiltonian reduction can be equally obtained via a secondary hamiltonian reduction from the $N=2$ $u(2|1)$ SCA. The existence of such a possibility follows already from the fact that the constraints (5.41) form a subclass of the $N=2$ $W_3^{(2)}$ constraints (5.7).

With respect to the new stress tensor $\mathscr{T}_{\rm new}$,

$$\mathscr{T}_{\rm new}=\mathscr{T}-2\bar{\mathscr{D}}\mathscr{H}_2\,, \tag{5.48}$$

the spins ($u(1)$ charges) of $\mathscr{J}_{25}$ and $\mathscr{J}_{23}$ are 0(0). In this new basis, we can add two extra constraints such that

$$\mathscr{J}_{25}=1,\qquad \mathscr{J}_{23}=0\,, \tag{5.49}$$

and, as usual, make use of the gauge freedom associated with these constraints for gauging away two more supercurrents

$$\mathscr{H}_2=0,\qquad \mathscr{J}_{35}=0\,. \tag{5.50}$$

Using (5.49), (5.50) one can check that (5.43) precisely reduces to (5.13) and $\mathscr{J}_{mn}^{DS}$ coincides with (5.9). It can be easily checked that the dimensions and $u(1)$ charges of the surviving supercurrents $\mathscr{J}_{53}, \mathscr{H}_{\bar 2}, \mathscr{J}_{51}, \mathscr{J}_{31}$ with respect to $\mathscr{T}_{\rm new}$ change and take the same values as in Table 5. After finding gauge invariant supercurrents which we did not write down explicitly, the reduced algebra becomes the algebra $N=2$ $W_3^{(2)}$ SCA elaborated in Subsect. 5.2.

Let us analyze in some detail the component structure of the extended $N=2$ SCA constructed here.

After solving the constraints (5.43) for the involved supercurrents we are left with the following set of $(10+10)$ currents: one Virasoro spin 2 stress tensor, two bosonic and four fermionic spin 3/2 currents, five bosonic and four fermionic spin 1 currents, two bosonic and two fermionic spin 1/2 currents. For the time being we do not give the precise relation of these currents to the components of supercurrents, we only note that four spin 1/2 currents appear as the $\theta, \bar{\theta}$ independent parts of $\mathcal{J}_{35}, \mathcal{J}_{53}, \mathcal{H}_{\bar{2}}, \mathcal{H}_{\bar{2}}$. The Virasoro stress tensor, the pair of fermionic spin 3/2 currents and one bosonic spin 1 current form $N=2$ SCA as a subalgebra, while the remainder of currents are spread over $N=2$ multiplets.

It is not too enlightening to present the OPEs between these latter currents. For a better understanding of what we have obtained, it is more appropriate to pass, by means of some nonlinear invertible transformation, to another basis of the constituent currents in which the $N=2$ multiplet structure becomes implicit but the spin 1/2 currents commute with all other ones and so can be factored out. The possibility of such a factorization agrees with the general statement of Ref. [15]. Below we give the explicit correspondence between the modified currents (commuting with the spin 1/2 ones) and the initial supercurrents

$$kJ_1{}^2 = \mathcal{J}_{25}| \,,$$

$$kJ_2{}^1 = \mathcal{J}_{51}| \,,$$

$$-kJ_1{}^3 = \mathcal{J}_{23}| \,,$$

$$kJ_3{}^1 = \mathcal{J}_{31}| \,,$$

$$k(J_1{}^1 - J_2{}^2 - J_3{}^3) = (-k\mathcal{T} - \mathcal{H}_2\mathcal{H}_{\bar{2}} - \mathcal{J}_{35}\mathcal{J}_{53})| \,,$$

$$J_3{}^2 = \bar{\mathcal{D}}\mathcal{J}_{35}| \,,$$

$$-J_2{}^3 = \left(\mathcal{D}\mathcal{J}_{53} + \frac{1}{k}\mathcal{J}_{53}\mathcal{H}_2\right)\Big| \,,$$

$$J_3{}^3 = \left(\mathcal{D}\mathcal{H}_{\bar{2}} - \frac{1}{k}\mathcal{J}_{35}\mathcal{J}_{53}\right)\Big| \,,$$

$$J_2{}^2 + J_3{}^3 = \bar{\mathcal{D}}\mathcal{H}_2| \,,$$

$$T = \left(-\frac{1}{2k}[k[\mathcal{D},\bar{\mathcal{D}}]\mathcal{T} - \mathcal{H}_2'\mathcal{H}_{\bar{2}} + \mathcal{H}_2\mathcal{H}_{\bar{2}}' - \mathcal{J}_{35}'\mathcal{J}_{53} + \mathcal{J}_{35}\mathcal{J}_{53}']\right)\Big| \,,$$

$$-i\frac{k}{\sqrt{2}}G^2 = \left(\bar{\mathcal{D}}\mathcal{J}_{25} + \frac{1}{k}\mathcal{J}_{25}\mathcal{H}_{\bar{2}}\right)\Big| \,,$$

$$-i\frac{k}{\sqrt{2}}\bar{G}_2 = \mathcal{D}\mathcal{J}_{51}| \,,$$

$$-i\frac{k}{\sqrt{2}}G^3 = \left(\bar{\mathcal{D}}\mathcal{J}_{23} + \frac{1}{k}\mathcal{H}_{\bar{2}}\mathcal{J}_{23} - \frac{1}{k}\mathcal{J}_{53}\mathcal{J}_{25}\right)\Bigg| \,,$$

$$-i\frac{k}{\sqrt{2}}\bar{G}_3 = \left(\mathcal{D}\mathcal{J}_{31} - \frac{1}{k}\mathcal{H}_2\mathcal{J}_{31} + \frac{1}{k}\mathcal{J}_{51}\mathcal{J}_{35}\right)\Bigg| \,,$$

$$-i\frac{k}{\sqrt{2}}\bar{G}_1 = \left(-k\mathcal{D}\mathcal{T} + \mathcal{H}_2\mathcal{D}\mathcal{H}_{\bar{2}} - \frac{1}{k}\mathcal{H}_2\mathcal{J}_{35}\mathcal{J}_{53} - \mathcal{J}_{35}\mathcal{D}\mathcal{J}_{53}\right)\Bigg| \,,$$

$$-i\frac{k}{\sqrt{2}}G^1 = (-k\bar{\mathcal{D}}\mathcal{T} - \bar{\mathcal{D}}\mathcal{H}_2\mathcal{H}_{\bar{2}} - \bar{\mathcal{D}}\mathcal{J}_{35}\mathcal{J}_{53})| \,, \qquad (5.51)$$

where $|$ means the $\theta, \bar{\theta}$ independent part of the corresponding supercurrents. After decoupling spin 1/2 currents the quotient algebra includes the Virasoro stress tensor T, two bosonic and four fermionic spin 3/2 currents, respectively, $G^3, \bar{G}_3$ and $G^a, \bar{G}_a, (a=1,2)$, five bosonic and four fermionic spin 1 currents, respectively ${J_b}^a, (a,b=1,2)$, ${J_3}^3$ and ${J_3}^a, {J_a}^3$, $a=1,2$. Nine spin 1 currents turn out to generate the $u(2|1)$ current algebra.[5] Spin 3/2 currents transform under fundamental and conjugate representaions of $u(2|1)$ for upper and lower positions of indices. Their OPEs contain a quadratic nonlinearity in the $u(2|1)$ currents. All the currents are primary with respect to T.

A simple inspection shows that this quotient algebra is none other than the $Z_2 \times Z_2$ graded extension of the $u(2|1)$ current superalgebra, $u(2|1)$ SCA [24], which is some graded version of the $u(3)$ KB SCA (the precise correspondence comes out with the choice $k=-\kappa, m=2, n=1$ in the general formulas of [24]). In contrast to the original $N=2$ algebra with the spin 1/2 currents added, the quotient algebra does not contain the standard linear $N=2$ SCA as a subalgebra; respectively, the $N=2$ multiplet structure of the currents turns out to be lost. Thus we see that adding the spin 1/2 currents to the $u(2|1)$ SCA makes it possible to extend it to some extended $N=2$ SCA, and this is why we call the latter $N=2$ $u(2|1)$ SCA. The relation between this SCA and its quotient by the spin 1/2 currents strongly resembles, say, the relation between linear $N=3$ SCA and nonlinear $so(3)$ KB SCA [15]. The essential difference consists, however, in that both $N=2$ $u(2|1)$ SCA and its quotient are *nonlinear* algebras. Nonetheless, we can say that the first algebra is still "more linear" compared to the second one, because passing to it linearizes two of four nonlinear supersymmetries of $u(2|1)$ SCA.

Let us also recall that in the component version of hamiltonian reduction of $sl(3|2)^{(1)}$, when we constrain both $sl(3)$ and $sl(2)$ blocks, $N=2$ W_3 or $N=2$ $W_3^{(2)}$ SCAs come out. It is also known that we can obtain $u(3)$ KB SCA by imposing constraints only on the $sl(2)$ block [19, 27]. In terms of component currents, $u(2|1)$ SCA corresponds to the reduction when constraints are placed only on the $sl(3)$ block of the 5×5 $sl(3|2)^{(1)}$ supermatrix of currents.

In the next subsection we show that there exists another kind of hamiltonian reduction of $N=2$ $sl(3|2)^{(1)}$ with the same number (7) of constraints. It yields

[5] We give the relations of $u(m|n)$ current algebra in Appendix C.

some nonlinear extended $N=2$ SCA which by the same reasoning as above can be called $N=2$ $u(3)$ SCA.

5.5. $N=2$ $u(3)$ SCA. Using exactly the same arguments as given in the previous subsections, we can continue our reduction procedure. We want to construct an $N=2$ extension of $u(3)$ KB SCA which has 16 component currents: that is, 10 bosonic currents and 6 fermionic ones. The minimal way to equalize the number of bosonic and fermionic currents is to add 4 extra fermionic currents. This implies that the number of the relevant reduction constraints should again be equal to 7.

We choose

$$\alpha_1 = 1, \qquad \alpha_{\bar{1}} = 0, \qquad \alpha_2 = 0, \qquad \alpha_{\bar{2}} = -1, \tag{5.52}$$

and list the dimensions and $u(1)$ charges of supercurrents in Tables 12 and 13.

We impose the following reduction constraints

$$\mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & * & * & 0 \\ * & * & 0 & * & 1 \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & * & * & * \end{pmatrix}. \tag{5.53}$$

These constraints are a subset of those we imposed in the $N=2$ $W_3^{(2)}$ case. This implies, by the way, that we can produce $N=2$ $W_3^{(2)}$ (or $N=2$ W_3) SCA by secondary hamiltonian reduction starting with these seven constraints and imposing two (three) more constraints. As usual, the gauge fixing procedure goes in accord with Table 12 and, as the result, we are left with the set of surviving currents indicated in Table 13.

Table 12.

$u(1)$	1	0	-2	0	2	-1	3
dim	$-\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
constr. scs	$\mathcal{J}_{45}^F$	$\mathcal{J}_{42}^B$	$\mathcal{J}_{43}^B$	$\mathcal{J}_{25}^B$	$\mathcal{J}_{15}^B$	$\mathcal{J}_{23}^F$	$\mathcal{J}_{12}^F$
g.f. scs	$\mathcal{J}_{24}^B$	$\mathcal{H}_{\bar{1}}^F$	$\mathcal{J}_{32}^F$	$\mathcal{H}_2^F$	$\mathcal{J}_{41}^B$	$\mathcal{J}_{35}^B$	$\mathcal{J}_{21}^F$
dim	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
$u(1)$	0	-1	1	1	-2	2	-3

Table 13.

surv. scs	$\mathcal{J}_{13}^F$	$\mathcal{J}_{31}^F$	$\mathcal{H}_1^F$	$\mathcal{H}_{\bar{2}}^F$	$\mathcal{J}_{52}^B$	$\mathcal{J}_{14}^B$	$\mathcal{J}_{34}^B$	$\mathcal{J}_{51}^B$	$\mathcal{J}_{53}^B$	$\mathcal{J}_{54}^F$
dim	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	$\frac{3}{2}$
$u(1)$	1	-1	1	-1	0	2	2	-2	-2	-1

Using the nonlinear irreducibility constraints, we may express $\mathcal{J}_{54}$ through the other supercurrents

$$\mathcal{J}_{54} = k\left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{52} - \mathcal{J}_{32}\mathcal{J}_{53}\,, \tag{5.54}$$

and finally arrive at the following $\mathcal{J}^{DS}_{mn}$:

$$\mathcal{J}^{DS}_{mn} = \begin{pmatrix} 0 & 0 & \mathcal{J}_{13} & \mathcal{J}_{14} & 0 \\ 0 & \mathcal{H}_{\bar{2}} + \mathcal{H}_1 & 0 & 0 & 1 \\ \mathcal{J}_{31} & 0 & 0 & \mathcal{J}_{34} & 0 \\ 0 & 1 & 0 & \mathcal{H}_1 & 0 \\ \mathcal{J}_{51} & \mathcal{J}_{52} & \mathcal{J}_{53} & k(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}})\mathcal{J}_{52} & \mathcal{H}_{\bar{2}} \end{pmatrix}. \tag{5.55}$$

The remnants of the irreducibility constraints read

$$\mathcal{D}\mathcal{H}_1 = 0, \qquad \bar{\mathcal{D}}\mathcal{H}_{\bar{2}} = 0\,,$$

$$\left(\mathcal{D} + \frac{1}{k}\mathcal{H}_1\right)\mathcal{J}_{13} = 0, \qquad \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{31} = 0\,,$$

$$\bar{\mathcal{D}}\mathcal{J}_{53} = 0, \qquad \left(\mathcal{D} - \frac{1}{k}\mathcal{H}_1\right)\mathcal{J}_{34} = 0\,,$$

$$\mathcal{D}\mathcal{J}_{14} - \frac{1}{k}\mathcal{J}_{13}\mathcal{J}_{34} = 0, \qquad \left(\bar{\mathcal{D}} - \frac{1}{k}\mathcal{H}_{\bar{2}}\right)\mathcal{J}_{51} - \frac{1}{k}\mathcal{J}_{31}\mathcal{J}_{53} = 0\,. \tag{5.56}$$

The computation of gauge invariant supercurrents is not very hard due to the absence of dimension 0 supercurrents among the surviving currents. The unconstrained $N=2$ stress tensor is given by

$$\mathcal{T} = \frac{1}{k}\mathcal{J}_{13}\mathcal{J}_{31} - \frac{1}{k}\mathcal{J}_{52} + \bar{\mathcal{D}}\mathcal{H}_1 - \mathcal{D}\mathcal{H}_{\bar{2}} \tag{5.57}$$

with central charge $2k$. All the supercurrents are superprimary with respect to $\mathcal{T}$.

After rescaling

$$\mathcal{J}_{14} \to \frac{1}{k}\mathcal{J}_{14}, \qquad \mathcal{J}_{34} \to \frac{1}{k}\mathcal{J}_{34}, \qquad \mathcal{J}_{51} \to \frac{1}{k}\mathcal{J}_{51}, \qquad \mathcal{J}_{53} \to \frac{1}{k}\mathcal{J}_{53}\,, \tag{5.58}$$

we can write down the remaining SOPEs in the following form:

$$\mathcal{H}_1(Z_1)\mathcal{H}_{\bar{2}}(Z_2) = \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{k}{2} - \frac{1}{z_{12}}k\,,$$

$$\mathcal{H}_1(Z_1)\mathcal{J}_{13}(Z_2) = \frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{13}\,,$$

$$\mathcal{H}_1(Z_1)\mathcal{J}_{31}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}}\mathcal{J}_{31}\,,$$

$$\mathscr{H}_{\bar{2}}(Z_1)\mathscr{J}_{13}(Z_2) = \frac{\theta_{12}}{z_{12}}\mathscr{J}_{13}\,,$$

$$\mathscr{H}_{\bar{2}}(Z_1)\mathscr{J}_{31}(Z_2) = -\frac{\theta_{12}}{z_{12}}\mathscr{J}_{31}\,,$$

$$\mathscr{J}_{13}(Z_1)\mathscr{J}_{31}(Z_2) = -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{k}{2} + \frac{1}{z_{12}}k - \frac{\bar{\theta}_{12}}{z_{12}}\mathscr{H}_{\bar{2}} - \frac{\theta_{12}}{z_{12}}\mathscr{H}_1$$
$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[-\bar{\mathscr{D}}\mathscr{H}_1 - \frac{1}{k}\mathscr{H}_1\mathscr{H}_{\bar{2}} + \frac{1}{k}\mathscr{J}_{13}\mathscr{J}_{31}\right], \qquad (5.59)$$

$$\mathscr{H}_1(Z_1)\begin{cases}\mathscr{J}_{14}(Z_2) = \frac{\bar{\theta}_{12}}{z_{12}}\mathscr{J}_{14} \\ \mathscr{J}_{51}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}}\mathscr{J}_{51}\end{cases},$$

$$\mathscr{H}_{\bar{2}}(Z_1)\begin{cases}\mathscr{J}_{34}(Z_2) = -\frac{\theta_{12}}{z_{12}}\mathscr{J}_{34} \\ \mathscr{J}_{53}(Z_2) = \frac{\theta_{12}}{z_{12}}\mathscr{J}_{53}\end{cases},$$

$$\mathscr{J}_{13}(Z_1)\begin{cases}\mathscr{J}_{14}(Z_2) = -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathscr{J}_{13}\mathscr{J}_{14}\,, \\ \mathscr{J}_{34}(Z_2) = -\frac{\bar{\theta}_{12}}{z_{12}}\mathscr{J}_{14} - \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathscr{H}_1\mathscr{J}_{14}\,, \\ \mathscr{J}_{51}(Z_2) = \frac{\bar{\theta}_{12}}{z_{12}}\mathscr{J}_{53} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\left[\mathscr{H}_1\mathscr{J}_{53} + \mathscr{J}_{13}\mathscr{J}_{51}\right],\end{cases}$$

$$\mathscr{J}_{31}(Z_1)\begin{cases}\mathscr{J}_{14}(Z_2) = \frac{\theta_{12}}{z_{12}}\mathscr{J}_{34} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathscr{J}_{34}\mathscr{H}_{\bar{2}}\,, \\ \mathscr{J}_{34}(Z_2) = \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathscr{J}_{31}\mathscr{J}_{34}\,, \\ \mathscr{J}_{53}(Z_2) = -\frac{\theta_{12}}{z_{12}}\mathscr{J}_{51} - \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\left[\mathscr{H}_{\bar{2}}\mathscr{J}_{51} + \mathscr{J}_{31}\mathscr{J}_{53}\right],\end{cases}$$

$$\mathscr{J}_{14}(Z_1)\mathscr{J}_{34}(Z_2) = -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k}\mathscr{J}_{14}\mathscr{J}_{34}\,,$$

$$\mathscr{J}_{14}(Z_1)\mathscr{J}_{51}(Z_2) = -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^3}k + \frac{1}{z_{12}^2}k - \frac{\bar{\theta}_{12}}{z_{12}^2}\mathscr{H}_{\bar{2}}$$
$$+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\left[-\frac{1}{2}\mathscr{T} - \frac{1}{2}\mathscr{D}\mathscr{H}_{\bar{2}} - \frac{1}{2k}\mathscr{H}_1\mathscr{H}_{\bar{2}} + \frac{2}{k}\mathscr{J}_{13}\mathscr{J}_{31}\right]$$
$$+\frac{1}{z_{12}}\left[\mathscr{T} + \mathscr{D}\mathscr{H}_{\bar{2}} + \frac{1}{k}\mathscr{H}_1\mathscr{H}_{\bar{2}} - \frac{2}{k}\mathscr{J}_{13}\mathscr{J}_{31}\right]$$
$$+\frac{\bar{\theta}_{12}}{z_{12}}\left[\bar{\mathscr{D}}\mathscr{T} - \frac{1}{k}\mathscr{H}_{\bar{2}}\mathscr{T} - \frac{1}{k}\mathscr{H}_{\bar{2}}\mathscr{D}\mathscr{H}_{\bar{2}} - \frac{1}{k^2}\mathscr{J}_{13}\mathscr{H}_{\bar{2}}\mathscr{J}_{31}\right.$$

$$
\begin{aligned}
&+\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_1\mathcal{H}_{\bar 2}-\frac{1}{k}\bar{\mathcal{D}}\mathcal{J}_{13}\bar{\mathcal{J}}_{13}-\mathcal{H}'_{\bar 2}\Big]-\frac{\theta_{12}}{z_{12}}\frac{1}{k}\mathcal{D}(\mathcal{J}_{13}\mathcal{J}_{31})\\
&+\frac{\theta_{12}\bar\theta_{12}}{z_{12}}\Big[-\frac{1}{k^3}\mathcal{H}_1\mathcal{J}_{13}\mathcal{H}_{\bar 2}\mathcal{J}_{31}-\frac{1}{k^2}\mathcal{H}_1\bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{J}_{31}+\frac{1}{k^2}\mathcal{J}_{13}\mathcal{J}_{31}\mathcal{T}\\
&+\frac{1}{k^2}\mathcal{J}_{13}\mathcal{D}\mathcal{H}_{\bar 2}\mathcal{J}_{31}+\frac{1}{k}(\mathcal{J}_{13}\mathcal{J}_{31})'+\frac{1}{k}\mathcal{J}_{34}\mathcal{J}_{53}+\frac{1}{k}\bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{D}\mathcal{J}_{31}\Big],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{J}_{14}(Z_1)\mathcal{J}_{53}(Z_2)=&\frac{\theta_{12}}{z_{12}^2}\mathcal{J}_{13}+\frac{\theta_{12}\bar\theta_{12}}{z_{12}^2}\Big[\frac{3}{2}\bar{\mathcal{D}}\mathcal{J}_{13}-\frac{1}{2k}\mathcal{J}_{13}\mathcal{H}_{\bar 2}\Big]\\
&+\frac{1}{z_{12}}\Big[-\bar{\mathcal{D}}\mathcal{J}_{13}+\frac{1}{k}\mathcal{J}_{13}\mathcal{H}_{\bar 2}\Big]+\frac{\bar\theta_{12}}{z_{12}}\frac{1}{k}\bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{H}_{\bar 2}\\
&+\frac{\theta_{12}}{z_{12}}\Big[-\frac{1}{k^2}\mathcal{H}_1\mathcal{J}_{13}\mathcal{H}_{\bar 2}+\frac{1}{k}\mathcal{J}_{13}\mathcal{T}-\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_1\mathcal{J}_{13}+\mathcal{J}'_{13}\Big]\\
&+\frac{\theta_{12}\bar\theta_{12}}{z_{12}}\Big[\bar{\mathcal{D}}\mathcal{J}'_{13}-\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_1\bar{\mathcal{D}}\mathcal{J}_{13}-\frac{1}{k^2}\bar{\mathcal{D}}(\mathcal{H}_1\mathcal{J}_{13}\mathcal{H}_{\bar 2})\\
&+\frac{1}{k}\bar{\mathcal{D}}(\mathcal{J}_{13}\mathcal{T})\Big],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{J}_{34}(Z_1)\mathcal{J}_{51}(Z_2)=&-\frac{\bar\theta_{12}}{z_{12}^2}\mathcal{J}_{31}+\frac{\theta_{12}\bar\theta_{12}}{z_{12}^2}\Big[-\frac{1}{2}\mathcal{D}\mathcal{J}_{31}+\frac{3}{2k}\mathcal{H}_1\mathcal{J}_{31}\Big]\\
&+\frac{1}{z_{12}}\Big[\mathcal{D}\mathcal{J}_{31}-\frac{1}{k}\mathcal{H}_1\mathcal{J}_{31}\Big]+\frac{\theta_{12}}{z_{12}}\frac{1}{k}\mathcal{H}_1\mathcal{D}\mathcal{J}_{31}\\
&+\frac{\bar\theta_{12}}{z_{12}}\Big[-\frac{1}{k^2}\mathcal{H}_1\mathcal{H}_{\bar 2}\mathcal{J}_{31}-\frac{1}{k}\mathcal{J}_{31}\mathcal{T}-\frac{1}{k}\mathcal{D}\mathcal{H}_{\bar 2}\mathcal{J}_{31}-\mathcal{J}'_{31}\Big]\\
&+\frac{\theta_{12}\bar\theta_{12}}{z_{12}}\Big[\frac{1}{k^2}\mathcal{H}_1\mathcal{J}_{31}\mathcal{T}-\frac{1}{k^2}\mathcal{H}_1\bar{\mathcal{D}}\mathcal{H}_1\mathcal{J}_{31}+\frac{1}{k^2}\mathcal{H}_1\mathcal{D}\mathcal{H}_{\bar 2}\mathcal{J}_{31}\\
&+\frac{1}{k}(\mathcal{H}_1\mathcal{J}_{31})'-\frac{1}{k}\mathcal{J}_{34}\mathcal{J}_{51}+\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_1\mathcal{D}\mathcal{J}_{31}\Big],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{J}_{34}(Z_1)\mathcal{J}_{53}(Z_2)=&-\frac{\theta_{12}\bar\theta_{12}}{z_{12}^3}k+\frac{1}{z_{12}^2}k+\frac{\theta_{12}}{z_{12}^2}\mathcal{H}_1\\
&+\frac{\theta_{12}\bar\theta_{12}}{z_{12}^2}\Big[-\frac{1}{2}\mathcal{T}+\frac{3}{2}\bar{\mathcal{D}}\mathcal{H}_1-\frac{1}{2k}\mathcal{H}_1\mathcal{H}_{\bar 2}\Big]\\
&+\frac{1}{z_{12}}\Big[\mathcal{T}-\bar{\mathcal{D}}\mathcal{H}_1+\frac{1}{k}\mathcal{H}_1\mathcal{H}_{\bar 2}\Big]+\frac{\bar\theta_{12}}{z_{12}}\Big[\bar{\mathcal{D}}\mathcal{T}+\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_1\mathcal{H}_{\bar 2}\Big]\\
&+\frac{\theta_{12}}{z_{12}}\Big[\frac{1}{k}\mathcal{H}_1\mathcal{T}-\frac{1}{k}\mathcal{H}_1\bar{\mathcal{D}}\mathcal{H}_1+\mathcal{H}'_1\Big]\\
&+\frac{\theta_{12}\bar\theta_{12}}{z_{12}}\Big[\bar{\mathcal{D}}\mathcal{H}'_1-\frac{1}{k}\mathcal{H}_1\bar{\mathcal{D}}\mathcal{T}+\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_1\mathcal{T}-\frac{1}{k}\bar{\mathcal{D}}\mathcal{H}_1\bar{\mathcal{D}}\mathcal{H}_1\Big].
\end{aligned}
\tag{5.60}
$$

Let us summarize the $N=2$ $u(3)$ SCA. It contains the unconstrained spin 1 $N=2$ stress tensor $\mathcal{T}$, the spin 1/2 chiral and anti-chiral supercuremts $\mathcal{H}_1$ and $\mathcal{H}_{\bar{2}}$, the spin 1/2 supercurrents $\mathcal{J}_{13}$ and $\mathcal{J}_{31}$ subjected to the nonlinear chirality constraints, the spin 1 anti-chiral supercurrent $\mathcal{J}_{53}$ and the spin 1 constrained supercurrents $\mathcal{J}_{14}, \mathcal{J}_{51}, \mathcal{J}_{34}$. All these supercurrents are bosonic (fermionic) for integer (half-integer) spin. The supercurrents $\mathcal{H}_1, \mathcal{H}_{\bar{2}}, \mathcal{J}_{13}, \mathcal{J}_{31}$ possess a closed set of SOPEs (see Eqs. (5.59)) and form the $N=2$ $u(2) = u(2|0)$ current subalgebra.

We would like to note that in [28] an $N=1$ superfield extension of $u(3)$ KB SCA has been found. The field content of both $N=1$ $u(3)$ SCA of Ref. [28] and our superalgebra is the same (modulo different choices of the basis for the constituent currents), but the novel point is that we have succeeded in arranging the relevant currents into $N=2$ supermultiplets (by putting them into properly constrained $N=2$ supercurrents) and thereby revealed $N=2$ supersymmetry of this superlagebra which was hidden in the formulation of Ref. [28].

Let us now consider a secondary Hamiltonian reduction of $N=2$ $u(3)$ SCA to $N=2$ $W_3^{(2)}$ SCA. It goes as follows. With respect to the new stress tensor $\mathcal{T}_{\text{new}}$,

$$\mathcal{T}_{\text{new}} = \mathcal{T} - 2\bar{\mathcal{D}}\mathcal{H}_1 - \mathcal{D}\mathcal{H}_{\bar{2}}\,, \tag{5.61}$$

the supercurrent $\mathcal{J}_{14}$ has zero spin and $u(1)$ charge, while the spin and $u(1)$ charge of $\mathcal{J}_{13}$ are equal, respectively, to $-1/2$ and -1. Thus we can impose two first-class constraints,

$$\mathcal{J}_{14} = 1, \qquad \mathcal{J}_{13} = 0\,. \tag{5.62}$$

Gauge fixing procedure for either constraints can be done as usual. So we fix the gauge by

$$\mathcal{H}_1 = 0, \qquad \mathcal{J}_{34} = 0\,. \tag{5.63}$$

Using (5.62), (5.63) we see that (5.56) is reduced to (5.13). The dimensions and $u(1)$ charges of the surviving supercurrents $\mathcal{J}_{31}, \mathcal{H}_{\bar{2}}, \mathcal{J}_{51}, \mathcal{J}_{53}$ with respect to $\mathcal{T}_{\text{new}}$ coincide with those in Table 5.

Let us come back to a discussion of $N=2$ $u(3)$ SCA. A simple inspection of its current content shows that there are four spin 1/2 currents in it besides the set of 16 currents with higher spins. As in the case of $N=2$ $u(2|1)$ SCA, they can be factored out by passing to a new basis where they (anti)commute with the remainder of the currents. After decoupling of these spin 1/2 currents our $N=2$ $u(3)$ SCA reproduces $u(3)$ KB SCA [13, 14].

Let us recall the current content of $u(3)$ KB SCA. It is generated by 16 currents: Virasoro stress tensor T_{KB}, six spin 3/2 currents G^a_{KB} and $\bar{G}_{a\,KB}$, and nine spin 1 currents forming the $u(3)$ affine current algebra, namely, the $u(1)$ current H_{KB} and eight $su(3)$ currents $J_b{}^a{}_{KB}$ with zero trace ($J_a{}^a{}_{KB} = 0$). Indices a, b are running from 1 to 3 and correspond to the fundamental 3 and its conjugate $\bar{3}$ representations of $su(3)$ (for upper and lower positions, respectively).

Below we give the precise correspondence between these $u(3)$ KB SCA currents and components of the original set of $N=2$ $u(3)$ SCA supercurrents

$$J^3{}_{2,KB} = \left(\bar{\mathcal{D}}\mathcal{J}_{13} - \frac{1}{k}\mathcal{J}_{13}\mathcal{H}_{\bar{2}}\right)\Big|\,,$$

$$-J^2{}_{3,KB} = \left(\mathcal{D}\mathcal{J}_{31} - \frac{1}{k}\mathcal{H}_1\mathcal{J}_{31}\right)\Big| ,$$

$$-\frac{1}{3}H_{KB} - J^2{}_{2,KB} = \left(\bar{\mathcal{D}}\mathcal{H}_1 - \frac{1}{k}\mathcal{J}_{13}\mathcal{J}_{31}\right)\Big| ,$$

$$\frac{1}{3}H_{KB} - J^1{}_{1,KB} - J^2{}_{2,KB} = \left(\mathcal{D}\mathcal{H}_{\bar{2}} - \frac{1}{k}\mathcal{J}_{13}\mathcal{J}_{31}\right)\Big| ,$$

$$-kJ^1{}_{2,KB} = \mathcal{J}_{14}| ,$$

$$\frac{k}{\sqrt{2}}\bar{G}_{2,KB} = \left(\bar{\mathcal{D}}\mathcal{J}_{14} + \frac{1}{k}\mathcal{H}_{\bar{2}}\mathcal{J}_{14}\right)\Big| ,$$

$$-kJ^1{}_{3,KB} = \mathcal{J}_{34}| ,$$

$$\frac{k}{\sqrt{2}}\bar{G}_{3,KB} = \left(\bar{\mathcal{D}}\mathcal{J}_{34} + \frac{1}{k}\mathcal{J}_{31}\mathcal{J}_{14}\right)\Big| ,$$

$$-kJ^2{}_{1,KB} = \mathcal{J}_{51}| ,$$

$$\frac{k}{\sqrt{2}}G^2_{KB} = \mathcal{D}\mathcal{J}_{51}| ,$$

$$-\frac{k}{3}H_{KB} - kJ^1{}_{1,KB} = (-k\mathcal{T} + \mathcal{J}_{13}\mathcal{J}_{31} + k\bar{\mathcal{D}}\mathcal{H}_1 - k\mathcal{D}\mathcal{H}_{\bar{2}} - \mathcal{H}_1\mathcal{H}_{\bar{2}})| ,$$

$$\frac{k}{\sqrt{2}}G^1_{KB} = \left(-k\mathcal{D}\mathcal{T} - \frac{1}{k}\mathcal{H}_1\mathcal{J}_{13}\mathcal{J}_{31} - \mathcal{J}_{13}\mathcal{D}\mathcal{J}_{31} + \mathcal{H}_1\mathcal{D}\mathcal{H}_{\bar{2}}\right)\Big| ,$$

$$\frac{k}{\sqrt{2}}\bar{G}_{1,KB} = \left(-k\bar{\mathcal{D}}\mathcal{T} + \bar{\mathcal{D}}\mathcal{J}_{13}\mathcal{J}_{31} - \frac{1}{k}\mathcal{J}_{13}\mathcal{H}_{\bar{2}}\mathcal{J}_{31} - \bar{\mathcal{D}}\mathcal{H}_1\mathcal{H}_{\bar{2}}\right)\Big| ,$$

$$T_{KB} = \left(-\frac{1}{2k}[k[\mathcal{D},\bar{\mathcal{D}}]\mathcal{T} + \mathcal{H}_1\mathcal{H}'_{\bar{2}} - \mathcal{H}'_1\mathcal{H}_{\bar{2}} + \mathcal{J}'_{13}\mathcal{J}_{31} - \mathcal{J}_{13}\mathcal{J}'_{31}]\right)\Big| ,$$

$$-kJ^3{}_{1,KB} = \mathcal{J}_{53}| ,$$

$$\frac{k}{\sqrt{2}}G^3{}_{KB} = \left(\mathcal{D}\mathcal{J}_{53} + \frac{1}{k}\mathcal{H}_1\mathcal{J}_{53} + \frac{1}{k}\mathcal{J}_{13}\mathcal{J}_{51}\right)\Big| . \tag{5.64}$$

The OPEs of these currents are a particular case of OPEs of $u(m|n)$ SCA given in Appendix C, Eqs. (C.1), with the following correspondence:

$$k = \kappa, \qquad T_{KB} = T, \qquad H_{KB} = J^a{}_a ,$$

$$J^a{}_{b,KB} = J^a{}_b - \frac{1}{3}\delta^a_b J^c{}_c, \qquad G^a_{KB} = iG^a, \qquad \bar{G}_{a,KB} = i\bar{G}_a , \tag{5.65}$$

and $m = 3,\ n = 0$.

It is worth noticing that $G^1_{KB}, \bar{G}_{1,KB}$ are related to the two fermionic components of the linear $N=2$ superconformal stress tensor, $\mathcal{T}$, through nonlinear transformations. So, two of six supersymmetries of $u(3)$ KB SCA are linearized by passing to $N=2$ $u(3)$ SCA(viz., by adding four spin 1/2 fermionic currents), but four of them remain nonlinear.

6. Conclusion and Outlook

In this paper we constructed $N=2$ $sl(n\,|\,n-1)^{(1)}$ current superalgebras and developed a general scheme of classical hamiltonian reduction in $N=2$ superspace. We applied it to the $N=2$ extension of the affine superalgebra $sl(3|2)^{(1)}$. As the main result, we deduced some new extensions of $N=2$ SCA, $N=2$ $u(2|1)$ and $N=2$ $u(3)$ SCAs. Within our scheme, these two new algebras turn out to be more fundamental than the previously explored $N=2$ $W_3^{(2)}$, $N=2$ W_3 SCAs in the sense that the latter can be generated by secondary hamiltonian reductions from the former. The following diagram depicts basic points of our reduction procedure:

$$\begin{array}{c} N=2\ sl(3|2)^{(1)} \\ \swarrow \quad \searrow \\ N=2\ u(2|1)\ SCA \qquad N=2\ u(3)\ SCA \\ \searrow \quad \swarrow \\ N=2\ W_3^{(2)}\ SCA \\ \downarrow \\ N=2\ W_3\ SCA \end{array}$$

There are several problems to be worked out and questions which at present are open.

Quantizing W algebras associated with arbitray embeddings of $sl(2)$ into (super) algebras has been studied in [34]. These results were extended to $N=1$ affine Lie superalgebras in superspace formalism [35]. It is interesting to see whether the quantization of our superconformal algebras can be carried out in the $N=2$ superfield formalism.

It would be also interesting to study how $N=2$ W_4 [36], and $N=2$ extensions (yet to be constructed) of some other reductions of $sl(4)$ could come out in the framework of hamiltonian reduction applied to the $N=2$ $sl(4|3)^{(1)}$ superalgebra.

There exist some other superalgebras which have a completely fermionic simple root system and admit $osp(1|2)$ principal embedding: $osp(2n\pm 1|2n), osp(2n|2n)$, $osp(2n+2|2n)$ $n \geqq 1$ and $D(2,1;\alpha)$ $\alpha \neq 0,-1$ [37]. It is natural to apply our general procedure to these superalgebras and see whether they admit $N=2$ superfield extensions.

It is also rather straightforward to construct free superfield realizations for $N=2$ $u(2|1)$ and $N=2$ $u(3)$ SCAs. An interesting related problem is to understand how these latter algebras reappear in the $N=2$ superfield Toda and WZNW setting.[6]

It is a rather exciting task to extend the techniques developed here to the $N=4$ case, and, as a first step, to regain "small" $N=4$ SCA within the hamiltonian reduction framework in a manifestly supersymmetric $N=4$ superfield fashion.

Acknowledgements. We would like to thank M. Magro, E. Ragoucy, A. Semikhatov, P. Sorba, F. Toppan and, especially, F. Delduc and S. Krivonos for many useful and clarifying discussions. We are grateful to V. Ogievetsky for his interest in this work. We also thank the Referee for a number of useful suggestions. Two of us (E.I. & A.S.) acknowledge partial support from the

[6] For $N=2$ W_n this is discussed in [38].

Russian Foundation of Fundamental Research, grant 93-02-03821, and the International Science Foundation, grant M9T300.

Appendix A: Notations for $sl(2|1)$ Superalgebra

The generators of the $sl(2|1)$ superalgebra in the complex basis introduced in Sect. 2 for the fundamental representation are given by

$$t_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad t_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$t_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad t_{\bar{1}} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad t_{\bar{2}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$t_{\bar{3}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad t_{\bar{4}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{A.1}$$

where Cartan generators $t_2, t_{\bar{2}}$ together with $t_1, t_{\bar{1}}$ form the bosonic subalgebra $sl(2) \oplus u(1)$, while the generators $t_3, t_{\bar{3}}, t_4, t_{\bar{4}}$ are fermionic roots. In this basis the structure constants of $sl(2|1)$ are

$$\begin{aligned}
&f^{1}{}_{2,1} = -1, \quad f^{\bar{1}}{}_{2,\bar{1}} = 1, \quad f^{3}{}_{2,3} = -1, \quad f^{\bar{3}}{}_{2,\bar{3}} = 1, \\
&f^{1}{}_{\bar{2},1} = 1, \quad f^{\bar{1}}{}_{\bar{2},\bar{1}} = -1, \quad f^{\bar{4}}{}_{\bar{2},\bar{4}} = 1, \quad f^{4}{}_{\bar{2},4} = -1, \\
&f^{2}{}_{1,\bar{1}} = -1, \quad f^{\bar{2}}{}_{1,\bar{1}} = 1, \quad f^{\bar{4}}{}_{1,\bar{3}} = -1, \quad f^{3}{}_{1,4} = 1, \\
&f^{\bar{3}}{}_{\bar{1},\bar{4}} = -1, \quad f^{4}{}_{\bar{1},3} = -1, \quad f^{4}{}_{\bar{1},3} = 1, \\
&f^{1}{}_{3,\bar{3}} = 1, \quad f^{1}{}_{3,\bar{4}} = 1, \quad f^{\bar{1}}{}_{4,\bar{3}} = 1, \quad f^{2}{}_{4,\bar{4}} = 1,
\end{aligned} \tag{A.2}$$

and nonzero elements of the Killing metric are given by

$$g_{1\bar{1}} = -g_{2\bar{2}} = g_{3\bar{3}} = g_{4\bar{4}} = 1. \tag{A.3}$$

The explicit relations between affine supercurrents $\mathcal{J}_a, \mathcal{J}_{\bar{a}}$ in this basis and the entries $\mathcal{J}_{mn}$ of the $sl(2|1)$ superlagebra valued affine supercurrent introduced in Sect. 3 are as follows:

$$\begin{aligned}
&\mathcal{J}_1 = \mathcal{J}_{12}, \quad \mathcal{J}_2 \equiv \mathcal{H}_1 = \mathcal{J}_{11}, \quad \mathcal{J}_3 = \mathcal{J}_{13}, \quad \mathcal{J}_4 = \mathcal{J}_{23}, \\
&\mathcal{J}_{\bar{1}} = \mathcal{J}_{21}, \quad \mathcal{J}_{\bar{2}} \equiv \mathcal{H}_{\bar{1}} = \mathcal{J}_{22}, \quad \mathcal{J}_{\bar{3}} = \mathcal{J}_{31}, \quad \mathcal{J}_{\bar{4}} = \mathcal{J}_{32}.
\end{aligned} \tag{A.4}$$

Appendix B: Notations for $sl(3|2)$ Superalgebra

We choose four Cartan generators $t_1, t_{\bar{1}}, t_2, t_{\bar{2}}$ of the $sl(3|2)$ superalgebra in the fundamental representation in the following form:

$$t_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad t_{\bar{1}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$t_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad t_{\bar{2}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{B.1}$$

Each of the remaining 10 unbarred generators t_a, $a = 3, 4, \ldots, 12$ is represented by a 5×5 supermatrix with the only non-zero entry 1 on the intersection of the m^{th} line and n^{th} row $(m = 1, 2, 3, 4, n > m)$. The barred generators $t_{\bar{a}}$ have their nonzero entries 1 in the bottom triangular part. Using the explicit form of the $sl(3|2)$ generators we can find their (anti)commutators, taking into account their statistics (the generators with non-zero entries inside the diagonal 3×3 and 2×2 blocks are bosonic, all others are fermionic). In the complex basis, they satisfy the following graded commutators:

$$[t_a, t_b\} = F^c{}_{ab} t_c, \qquad [t_{\bar{a}}, t_{\bar{b}}\} = F^{\bar{c}}{}_{\bar{a}\bar{b}} t_{\bar{c}}, \qquad [t_a, t_{\bar{b}}\} = F^c{}_{a\bar{b}} t_c + F^{\bar{c}}{}_{a\bar{b}} t_{\bar{c}} \,. \tag{B.2}$$

From this we can read off all the structure constants $F^C{}_{AB}$ which are 1 or -1 (remember that $f^C{}_{AB} = (-1)^{(d_A+1)d_B} F^C{}_{AB}$). The Killing metric $g_{a\bar{b}}$ is given by $Str(t_a t_{\bar{b}})$, where we take the usual convention for supertrace. Just as an example, we write down nonzero elements of $g_{a\bar{b}}$ for the subset (B.1),

$$-g_{1\bar{1}} = g_{1\bar{2}} = -g_{2\bar{2}} = 1 \,. \tag{B.3}$$

Appendix C: $u(m|n)$ SCAs [24] in Terms of Currents

This algebra includes the Virasoro stress tensor T, $2n$ spin 3/2 bosonic currents, $G^a, \bar{G}_a$, $2m$ spin 3/2 fermionic currents, G^b, $\bar{G}_b$, $a = m+1, m+2, \ldots, m+n$, $b = 1, 2, \ldots, m$, $(m^2 + n^2)$ spin 1 bosonic currents, $J^c{}_d$, $c, d = 1, 2, \ldots, m$, $J^e{}_f$, $e = m+1, m+2, \ldots, m+n$, $f = n+1, n+2, \ldots, m+n$, and $2mn$ spin 1 fermionic ones, $J^g{}_h$, $J^i{}_j$, $g = m+1, m+2, \ldots, m+n$, $h = 1, 2, \ldots, n$, $i = n+1, n+2, \ldots, m+n$, $j = 1, 2, \ldots, m$. The total set of $(m+n)^2$ spin 1 currents forms the $u(m|n)$ current algebra. Spin 3/2 currents transform under fundamental and conjugate representaions of $u(m|n)$, for upper and lower positions of the indices, respectively.

These currents satisify the following OPEs:

$$T(z)T(w) = \frac{1}{(z-w)^4}3\kappa + \frac{1}{(z-w)^2}2T + \frac{1}{(z-w)}T' ,$$

$$T(z)J^a{}_b(w) = \frac{1}{(z-w)^2}J^a{}_b + \frac{1}{(z-w)}J^{a\prime}{}_b ,$$

$$T(z)G^a(w) = \frac{1}{(z-w)^2}\frac{3}{2}G^a + \frac{1}{(z-w)}G^{a\prime} ,$$

$$T(z)\bar{G}_a(w) = \frac{1}{(z-w)^2}\frac{3}{2}\bar{G}_a + \frac{1}{(z-w)}\bar{G}'_a ,$$

$$J^a{}_b(z)J^c{}_d(w) = \frac{1}{(z-w)^2}\left[\frac{1}{2-(m-n)}(-1)^{(d_a+1)(d_b+1)+(d_c+1)(d_d+1)}\delta^a_b\delta^c_d + (-1)^{(d_a+d_b+d_c+1)(d_d+1)}\delta^a_d\delta^c_b\right]\kappa + \frac{1}{(z-w)}[(-1)^{(d_a+1)(d_b+d_c)}\delta^c_b J^a{}_d - (-1)^{(d_a+1)(d_d+1)+(d_b+1)(d_c+1)+(d_d+1)(d_b+1)+(d_d+1)(d_c+1)}\delta^a_d J^c{}_b] ,$$

$$J^a{}_b(z)G^c(w) = \frac{1}{(z-w)}\delta^c_b G^a ,$$

$$J^a{}_b(z)\bar{G}_c(w) = -\frac{1}{(z-w)}\delta^a_c(-1)^{(d_b+1)d_c}\bar{G}_b ,$$

$$\begin{aligned} G^a(z)\bar{G}_b(w) = & -\frac{1}{(z-w)^3}(-1)^{(d_a+1)(d_b+1)}\delta^a_b 4\kappa \\ & + \frac{1}{(z-w)^2}[2(-1)^{(d_a+1)(d_b+1)}\delta^a_b J^c{}_c - 4J^a{}_b] \\ & + \frac{1}{(z-w)}\Big[(-1)^{(d_a+1)(d_b+1)}\delta^a_b J^{c\prime}{}_c - 2J^{a\prime}{}_b - 2(-1)^{(d_a+1)(d_b+1)}\delta^a_b T \\ & - \frac{1}{\kappa}(-1)^{(d_a+1)(d_b+1)}\delta^a_b J^c{}_c J^d{}_d + \frac{2}{\kappa}J^c{}_c J^a{}_b \\ & + \left(-\frac{1}{\kappa}\delta^a_c\delta^d_b + \frac{1}{2\kappa}(-1)^{(d_a+1)(d_b+1)}\delta^a_b\delta^c_d\right) \\ & \times((-1)^{d_e+1}J^c{}_e J^e{}_d + (-1)^{(d_c+1)(d_d+d_e)+(d_d+1)(d_e+1)}J^e{}_d J^c{}_e)\Big] . \end{aligned} \quad \text{(C.1)}$$

Appendix D: A Different Realization of $sl(n|n-1)$

We can realize the superalgebra $sl(n|n-1)$ in a different, though equivalent way by the $(2n-1)\times(2n-1)$ supermatrix whose entries $\mathcal{J}_{\tilde{k}\tilde{l}}$ are related to those $\mathcal{J}_{kl}$

in the standard realization according to the following rule [5]:

$$\widetilde{k} = 2k-1, \qquad \widetilde{l} = 2l-1 \quad \text{if } 1 \leqq k,\ l \leqq n\,,$$
$$\widetilde{k} = 2(k-n), \qquad \widetilde{l} = 2(l-n) \quad \text{if } n < k,\ l \leqq 2n-1\,. \tag{D.1}$$

This parametrization corresponds to choosing the system of purely fermionic simple roots in $sl(n|n-1)$. It is very convenient when studying embeddings of $sl(2|1)$ into $sl(n|n-1)$: the former is identified with proper 3×3 blocks in the $sl(n|n-1)$ supermatrix.[7]

Using this convention, the set of hamiltonian reduction constraints we dealt with in the $sl(2|1)$ case can be rewritten in the following suggestive way:

$$N=2\,SCA:\ \mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 1 \\ * & * & * \\ * & 1 & * \end{pmatrix} (4.4) \Rightarrow \begin{pmatrix} * & 1 & 0 \\ * & * & 1 \\ * & * & * \end{pmatrix}. \tag{D.2}$$

This picture shows that the constraints are concentrated in the upper triangular part of the supercurrent matrix, and this is true as well for the $sl(3|2)$ constraints except for (5.19). We first present the matrices of constraints for the cases of $N=2$ W_3, $N=2$ $W_3^{(2)}$, $N=2$ $u(2|1)$ and $N=2$ $u(3)$ SCAs:

$$N=2\,W_3:\quad \mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 0 & 1 & 0 \\ * & * & 0 & * & 1 \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & 1 & * & * \end{pmatrix} (5.3) \Rightarrow \begin{pmatrix} * & 1 & 0 & 0 & 0 \\ * & * & 1 & 0 & 0 \\ * & * & * & 1 & 0 \\ * & * & * & * & 1 \\ * & * & * & * & * \end{pmatrix}, \tag{D.3}$$

$$N=2\,W_3^{(2)}:\quad \mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 0 & 1 & 0 \\ * & * & 0 & * & 1 \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & * & * & * \end{pmatrix} (5.7) \Rightarrow \begin{pmatrix} * & 1 & 0 & 0 & 0 \\ * & * & 1 & 0 & 0 \\ * & * & * & 1 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}, \tag{D.4}$$

$$N=2\,u(2|1):\quad \mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & 0 & 1 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & * & * & * \end{pmatrix} (5.41) \Rightarrow \begin{pmatrix} * & 1 & 0 & 0 & 0 \\ * & * & 1 & 0 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}, \tag{D.5}$$

$$N=2\,u(3):\quad \mathcal{J}_{mn}^{\text{constr}} = \begin{pmatrix} * & 0 & * & * & 0 \\ * & * & 0 & * & 1 \\ * & * & * & * & * \\ * & 1 & 0 & * & 0 \\ * & * & * & * & * \end{pmatrix} (5.53) \Rightarrow \begin{pmatrix} * & * & 0 & 0 & * \\ * & * & 1 & 0 & 0 \\ * & * & * & 1 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}. \tag{D.6}$$

[7] We are grateful to F. Delduc for explaining us the merits of this realization.

The supermatrices of constraints for two "noncanonical" cases described in Subsect. 5.3, respectively with the constrained $N=2$ stress tensor and/or spin 0 supercurrents present, are given by

$$\mathscr{I}^{\rm constr}_{mn} = \begin{pmatrix} * & * & * & 1 & * \\ * & * & * & 0 & * \\ * & * & * & 0 & * \\ * & * & * & * & * \\ * & * & 1 & 0 & * \end{pmatrix} (5.19) \Rightarrow \begin{pmatrix} * & 1 & * & * & * \\ * & * & * & * & * \\ * & 0 & * & * & * \\ * & 0 & * & * & 1 \\ * & 0 & * & * & * \end{pmatrix}, \tag{D.7}$$

$$\mathscr{I}^{\rm constr}_{mn} = \begin{pmatrix} * & 0 & 0 & 1 & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & 1 & 0 & * & * \\ * & * & * & * & * \end{pmatrix} (5.31) \Rightarrow \begin{pmatrix} * & 1 & 0 & * & 0 \\ * & * & 1 & * & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}. \tag{D.8}$$

These pictures clearly demonstrate the relations between different reductions in accord with the diagram of Sect. 6. Also it is seen from them that it is natural to treat all the considered cases in the language of $sl(2|1)$ embeddings. The case of $N=2$ W_3 corresponds to the principal embedding of $sl(2|1)$ into $sl(3|2)$ while the $N=2$ $u(2|1)$ and $N=2$ $u(3)$ ones to two inequivalent non-principal embeddings. It would be interesting to understand from an analogous point of view the cases (D.4), (D.7), (D.8). It seems that in this way one could explain some peculiar features of them (lacking the superprimary basis in the $N=2$ $W_3^{(2)}$ case, the presence of the constrained $N=2$ stress tensor and/or spin 0 supercurrents in the two remaining cases). Note that the complete classification of $sl(2|1)$ embeddings, at the component level and in the string theory context, is undertaken in [39].

References

1. Witten, E.: Topological Sigma Models. Commun. Math. Phys. **118**, 411 (1988)
2. Lerche, W., Vafa, C., Warner, N.P.: Chiral Rings in $N=2$ Superconformal Theories. Nucl. Phys. **B324**, 427 (1989)
3. Eguchi, T., Yang, S.-K.: $N=2$ Superconformal Models as Topological Field Theories. Mod. Phys. Lett. **A5**, 1693 (1990)
4. Gato-Rivera, B., Semikhatov, A.M.: $d \leqq 1 \cup d \geqq 25$ and W constraints from BRST invariance in the $c \neq 3$ topological algebra. Phys. Lett. **B293**, 72 (1992)
5. Bershadsky, M., Lerche, W., Nemeschansky, D., Warner, N.P.: Extended $N=2$ Superconformal Structure of Gravity and W-Gravity Coupled to Matter. Nucl. Phys. **B401**, 304 (1993)
6. Ito, K., Kanno, H.: Lie Superalgebra and Extended Topological Conformal Symmetry in Non-critical W_3 Strings. Preprint UTHEP-277, hep-th/9405049
7. Boresch, A., Landsteiner, K., Lerche, W., Sevrin, A.: Superstrings from Hamiltonian Reduction. Nucl. Phys. **B436**, 609 (1995)
8. Landsteiner, K., Lerche, W., Sevrin, A.: Topological Strings from WZW Models. Phys. Lett. **B352**, 286 (1995)

9. Drinfeld, V., Sokolov, V.: Lie algebras and equations of Korteweg–de Vries type. J. Sov. Math. **30**, 1975 (1984)
10. Feher, L., O'Raifeartaigh, L., Ruelle, P., Tsutsui, I., Wipf, A.: On the general structure of hamiltonian reductions of the WZNW theory. Phys. Rep. **222**, 1 (1992)
11. Bais, F.A., Tjin, T., van Driel, P.: Covariantly coupled chiral algebras. Nucl. Phys. **B357**, 632 (1991)
12. Delduc, F., Ragoucy, E., Sorba, P.: Super-Toda theories and W algebras from Superspace Wess–Zumino–Witten models. Commun. Math. Phys. **146**, 403 (1992)
13. Knizhnik, V.G.: Superconformal algebras in two dimensions. Theor. Math. Phys. **66**, 68 (1986)
14. Bershadsky, M.: Superconformal algebras in two dimensions with arbitrary N. Phys. Lett. **B174**, 285 (1986)
15. Goddard, P., Schwimmer, A.: Factoring out free fermions and superconformal algebras. Phys. Lett. **B214**, 209 (1988)
16. Ademollo, M., et al.: Supersymmetric strings and Color Confinement. Phys. Lett. **B62**, 105 (1976); Dual string with $U(1)$ colour symmetry. Nucl. Phys. **B111**, 77 (1976); Dual string models with non-Abelian colour and flavour symmetries. Nucl. Phys. **B114**, 297 (1976)
17. Polyakov, A.: Gauge transformations and diffeomorphisms. Int. J. Mod. Phys. **A5**, 833 (1990)
18. Bershadsky, M.: Conformal field theories via hamiltonian reduction. Commun. Math. Phys. **139**, 71 (1991)
19. Romans, L.J.: Quasi-superconformal algebras in two dimensions and hamiltonian reduction. Nucl. Phys. **B357**, 549 (1991)
20. Fuchs, J.: Bosonic superconformal algebras from Hamiltonian reduction. Phys. Lett. **B262**, 249 (1991)
21. Krivonos, S., Sorin, A.: The $N=2$ super-$W_3^{(2)}$ algebra. Preprint LNF-94/014 (P), hep-th/9402064
22. Ahn, C., Krivonos, S., Sorin, A.: The Full Structure of Quantum $N=2$ Super-$W_3^{(2)}$ Algebra. Mod. Phys. Lett. **A10**, 1299 (1995)
23. Ivanov, E., Krivonos, S., Sorin, A.: $N=2$ super-$W_3^{(2)}$ algebra in superfields. Mod. Phys. Lett. **A10**, 2439 (1995)
24. Defever, F., Troost, W., Hasiewicz, Z.: Superconformal algebras with quadratic nonlinearity. Phys. Lett. **B273**, 51 (1991)
25. Fradkin, E.S., Linetsky, V.Ya.: Classification of Superconformal Algebras with quadratic nonlinearity. Phys. Lett. **B291**, 71 (1992)
26. Bowcock, P.: Exceptional superconformal algebras. Nucl. Phys. **B381**, 415 (1992)
27. Ito, K., Madsen, J.O.: Hamiltonian reduction and classical extended superconformal algebras. Phys. Lett. **B283**, 223 (1992)
28. Ragoucy, E.: $OSp(1|2)$ and $Sl(2)$ reductions in generalised super-Toda models and factorization of spin 1/2 fields. Nucl. Phys. **B411**, 778 (1994)
29. Hull, C.M., Spence, B.: $N=2$ Current Algebra And Coset Models. Phys. Lett. **B241**, 357 (1990)
30. Frappat, L., Ragoucy, E., Sorba, P.: W-algebras and super algebras from constrained WZW models: A group theoretical classification. Commun. Math. Phys. **157**, 499 (1993)
31. Roček, M., Ahn, C., Schoutens, K., Sevrin, A.: Superspace WZW Models and Black Holes. In: Workshop on Superstrings and Related Topics. Trieste, Aug. 1991, Preprint IASSNS-HEP-91/69, ITP-SB-91-49, LBL-31325, UCB-PTH-91/50, hep-th/9110035
32. Lu, H., Pope, C.N., Romans, L.J., Shen, X., Wang, X.-J.: Polyakov Construction of the $N=2$ Super-W_3 Algebra. Phys. Lett. **264B**, 91 (1991); Nemeschansky, D., Yankielowicz, S.: $N=2$ W-Algebra, Kazama–Suzuki Models and Drinfeld–Sokolov Reduction. Preprint USC-91/005; Ivanov, E., Krivonos, S.: Superfield Realizations of $N=2$ Super-W_3. Phys. Lett. **B291**, 63 (1992); **B301**, 454E (1993)
33. Delduc, F., Frappat, L., Ragoucy, E., Sorba, P.: Gauged W algebras. Phys. Lett. **B335**, 151 (1992)
34. Sevrin, A., Troost, W.: Extensions of the Virasoro algebra and gauged WZW models. Phys. Lett. **B315**, 304 (1993); de Boer, J., Tjin, T.: The Relation Between Quantum W Algebras and Lie Algebras. Commun. Math. Phys. **160**, 317 (1994)
35. Madsen, J.O., Ragoucy, E.: Quantum Hamiltonian Reduction in Superspace Formalism. Nucl. Phys. **B429**, 277 (1994)

36. Yung, C.M., Warner, R.C.: The $N=2$ super W_4 algebra and its associated generalized KdV hierarchies. Preprint UTAS-PHYS-92-32, hep-th/9301077; Blumenhagen, R., Wißkirchen, A.: Extension of the $N=2$ Virasoro algebra by two primary fields of dimension 2 and 3. Phys. Lett. **B343**, 168 (1995)
37. Leites, D.A., Saveliev, M.V., Serganova, V.V.: In: Group Theoretical Methods in Physics, Amsterdam: North Holland, 1985; preprint IHEP 85–81, Serpukhov 1985
38. Delduc, F., Magro, M.: Gauge invariant formulation of $N=2$ Toda and KdV systems in extended superspace. Preprint ENSLAPP-L-559, hep-th/9512220
39. Ragoucy, E., Sevrin, A., Sorba, P.: Strings from $N=2$ gauged Wess–Zumino–Witten models. Preprint ENSLAPP-555/95, VUB-TH.395, hep-th/9511049

Communicated by R.H. Dijkgraaf