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Selection Rules for Topology Change*

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Abstract. It is shown that there are restrictions on the possible changes of topology of space sections of the universe if this topology change takes place in a compact region which has a Lorentzian metric and spinor structure. In particular, it is impossible to create a single wormhole or attach a single handle to a spacetime but it is kinematically possible to create such wormholes in pairs. Another way of saying this is that there is a $\mathbb{Z}_2$ invariant for a closed oriented 3-manifold Σ which determines whether Σ can be the spacelike boundary of a compact manifold M which admits a Lorentzian metric and a spinor structure. We evaluate this invariant in terms of the homology groups of Σ and find that it is the mod2 Kervaire semi-characteristic.

Introduction

There has been great interest recently in the possibility that the topology of space may change in a semi-classical theory of quantum gravity in which one assumes the existence of an everywhere non-singular Lorentzian metric $g^L_{\alpha\beta}$ of signature $-+++$. In particular, Thorne, Frolov, Novikov and others have speculated that an advanced civilization might at some time in our future be able to change the topology of space sections of the universe so that they developed a wormhole or handle [1–3]. If one were to be able to control such a topology change, it would have to occur in a compact region of spacetime without singularities at which the equations broke down and without extra unpredictable information entering the spacetime from infinity. Thus if we assume, for convenience, that space is compact now, then the suggestion amounts to saying that the 4-dimensional spacetime manifold M, which we assume to be smooth and connected, is compact with boundary $\partial M = \Sigma$ consisting of 2 connected components, one of which has topology S^3 and the other of which has topology $S^1 \times S^2$, and both are spacelike

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with respect to the Lorentzian metric $g^L_{\alpha\beta}$. If $(M, g^L_{\alpha\beta})$ is assumed time-oriented, which we will justify later, then the S^3 component should be the past boundary of M and the $S^1 \times S^2$ component should be the future boundary of M. Spacetimes of this type have previously been thought to be of no physical interest because a theorem of Geroch [4] states that they must contain closed timelike curves. In the last few years, however, people have begun to consider seriously whether such causality violating spacetimes might be permitted by the laws of physics. One of the main results of this paper is that even if causality violations are allowed, there is an even greater obstacle to considering such a spacetime as physically reasonable – it does not admit an $SL(2, \mathbb{C})$ spinor structure and therefore it is simply not possible on purely kinematical grounds to contemplate a civilization, no matter how advanced constructing a wormhole of this type, provided one assumes that the existence of two-component Weyl fermions is an essential ingredient of any successful theory of nature. We will discuss later the extent to which one might circumvent this result by appealing to more exotic possibilities such as $Spin^c$ structures.

It appears, however, that there is no difficulty in imagining an advanced civilization constructing a pair of wormholes, i.e. that the final boundary is the connected sum of 2 copies of $S^1 \times S^2$, $S^1 \times S^2 \# S^1 \times S^2$. Thus one may interpret our results as providing a new topological conservation law for wormholes, they must be conserved modulo 2. More generally we are able to associate with any closed orientable 3-manifold Σ a topological invariant, call it u (for universe) such that $u=0$ if Σ

(1) bounds a smooth connected compact Lorentz 4-manifold M which admits an $SL(2, \mathbb{C})$ spinor structure;
(2) is spacelike with respect to the Lorentz metric $g^L_{\alpha\beta}$,

and $u=1$ otherwise.

We shall show that this invariant is additive modulo 2 under disjoint union of 3-manifolds,

$$u(\Sigma_1 \cup \Sigma_2) = u(\Sigma_1) + (\Sigma_2) \bmod 2 .$$

Under the connected sum it satisfies

$$u(\Sigma_1 \# \Sigma_2) = u(\Sigma_1) + u(\Sigma_2) + 1 \bmod 2 .$$

The connected sum, $X \# Y$ of two manifolds X, Y of the same dimension n is obtained by removing an n-ball B^n and from X and Y and gluing the two manifolds together across the common S^{n-1} boundary component so created. We shall also show that $u(S^3)=1$, and $u(S^1 \times S^2)=0$. The result that one cannot create a single wormhole then follows immediately from the formula for disjoint unions while the fact that one can create pairs of wormholes follows from the formula for connected sums. Another consequence of these formulae is that for the disjoint union of k S^3's, $u=k$ modulo 2. In particular, this prohibits the "creation from nothing" of a single S^3 universe with a Lorentz metric and spinor structure.

Our invariant u may be expressed in terms of rather more familiar topological invariants of 3-manifolds. In fact,

$$u = \dim_{\mathbb{Z}_2}(H_0(\Sigma; \mathbb{Z}_2) \oplus H_1(\Sigma; \mathbb{Z}_2)) \bmod 2 ,$$

where $H_0(\Sigma; \mathbb{Z}_2)$ is the zero$^{\text{th}}$ and $H_1(\Sigma; \mathbb{Z}_2)$ the first homology group of Σ with $\mathbb{Z}_2$ coefficients. Thus $\dim_{\mathbb{Z}_2} H_0(\Sigma; \mathbb{Z}_2) \bmod 2$ counts the number of connected components modulo 2. The right-hand side of this expression for u is sometimes referred to as the mod 2 Kervaire semi-characteristic.

So far we have considered the case where the space sections of the universe are closed. We can extend these results to cases where the space sections of the universe may be non-compact but the topology change takes place in a compact region bounded by a timelike tube. Such spacetimes may be obtained from the ones we have considered by removing a tubular neighbourhood of a timelike curve.

It seems that a selection rule of this type derived in this paper occurs only if one insists on an everywhere non-singular Lorentzian metric. If one gives up the Lorentzian metric and passes to a Riemannian metric or if one adopts a "first order formalism" in which one treats the vierbein field as the primary variable and allows the legs of the vierbein to become linearly dependent at some points in spacetime then our selection rule would not necessarily apply. However, in the context of asking what an advanced civilization is capable of neither of these possibilities seems reasonable. At the quantum level, however, both are rather natural and in view of the existence of a number of examples there seems to be little reason to doubt that the topology of space can fluctuate at the quantum level. For the purposes of the present paper we will adhere to the assumption of an everywhere non-singular Lorentz metric.

Spin-Cobordism and Lorentz-Cobordism

Every closed oriented 3-manifold admits a $Spin(3) \equiv SU(2)$ spin structure. If the 3-manifold is not simply connected the spin structure is not unique. The set of spin structures is in $1-1$-correspondence with elements of $H^1(\Sigma; \mathbb{Z}_2)$, the first cohomology group of the 3-manifold Σ with $\mathbb{Z}_2$ coefficients. Given a closed oriented 3-manifold Σ one can always find a spin-cobordism, that is there always exists a compact orientable 4-manifold M with boundary $\partial M = \Sigma$ and such that M admits a $Spin(4) \equiv SU(2) \times SU(2)$ spin structure which when restricted to the boundary Σ coincides with any given spin structure on Σ [5].

A closed 3-manifold Σ is said to admit a Lorentz-cobordism if one can find a compact 4-manifold M whose boundary $\partial M = \Sigma$ together with an everywhere non-singular Lorentzian metric with respect to which the boundary Σ is spacelike. A necessary and sufficient condition for a Lorentz-cobordism is that the manifold M should admit a line field $\mathbf{V}$, i.e. a pair $(\mathbf{V}, -\mathbf{V})$ at each point, where $\mathbf{V}$ is a non-zero vector which is transverse to the boundary ∂M. To show this one uses the fact that any compact manifold admits a Riemannian metric $g^R_{\alpha\beta}$. If one has a line field $\mathbf{V}$, one can define a Lorentzian metric $g^L_{\alpha\beta}$ by

$$g^{L\alpha\beta} = g^{R\alpha\beta} - 2V^\alpha V^\beta / (g^R_{\alpha\beta} V^\alpha V^\beta) .$$

Alternatively, given a Lorentzian metric $g^L_{\alpha\beta}$ one can diagonalize it with respect to the Riemannian metric $g^R_{\alpha\beta}$. One can choose $\mathbf{V}$ to be the eigenvector with negative eigenvalue. The Lorentzian metric $g^L_{\alpha\beta}$ will be time-orientable if and only if one can choose a consistent sign for $\mathbf{V}$. For physical reasons we shall generally assume time-orientability. If $M, g^L_{\alpha\beta}$ is not time-orientable, it will have a double cover that is, with twice as many boundary components.

If one has a time-orientable Lorentz-cobordism, the various connected components of the boundary lie either in the past or in the future. Thus one might think that one should specify in the boundary data for a Lorentz-cobordism a specification of which connected components lie in the future and which lie in the past. However, it is not difficult to show that given a time-oriented Lorentz-cobordism for which a particular component lies in, say the future, one can construct another time-oriented Lorentz-cobordism for which that component lies in the past and the remaining components are as they were in the first Lorentz-cobordism. The construction is as follows. Let Σ be the component in question. Consider the Riemannian product metric on $\Sigma \times I$, where I is the closed interval $-1 \leqq t \leqq 1$. Now by virtue of being a closed orientable 3-manifold Σ admits an everywhere non-vanishing vector field $\mathbf{U}$ which may be normalized to have unit length with respect to the metric on Σ. To give $\Sigma \times I$ a time-orientable Lorentz metric we choose as our everywhere non-vanishing unit timelike vector field $\mathbf{V}$:

$$\mathbf{V} = a(t)\frac{\partial}{\partial t} + b(t)\mathbf{U},$$

where $a^2 + b^2 = 1$ and $a(t)$ passes smoothly and monotonically from -1 at $t = +1$ to $+1$ at $t = 1$. Thus $\mathbf{V}$ is outward directed on both boundary components. One can now attach a copy of $\Sigma \times I$ with this metric, or its time reversed version, to the given Lorentz-cobordism so reversing the direction of time at the boundary desired component. Of course, one will have to arrange that the metrics match smoothly but this is always possible. Considered in its own right the spacetime we have just used could serve as a model for the "creation from nothing" of a pair of twin universes. In general, it will not be geodesically complete and it contains closed timelike curves inside the Cauchy Horizons which occur at the two values of t for which $a^2 = b^2$. However, it is a perfectly valid Lorentz-cobordism.

If a Lorentzian spacetime admits an $SL(2, \mathbb{C})$ spinor structure it must be both orientable and time-orientable and in addition admit a $Spin(4)$ structure [9, 10]. For example, since any closed orientable 3-manifold is a spin manifold, the time reversing product metric we constructed above admits an $SL(2, \mathbb{C})$ structure. By contrast the next example, which could be said to represent the creation of a single, i.e. connected, universe from nothing, does not admit an $SL(2, \mathbb{C})$ spinor structure because it is not time-orientable. Let Σ be a closed connected orientable Riemannian 3-manifold admitting a free involution Γ which is an isometry of the 3-metric on Σ. A Lorentz-cobordism for Σ is obtained by taking $\Sigma \times I$ as before but now with the product Lorentzian metric, i.e. with $a = 1$ and $b = 0$. One now identifies points under the free $\mathbb{Z}_2$ action which is the composition of the involution Γ acting on Σ and reversal of the time coordinate t on the interval I, $-1 \leqq t \leqq 1$. Because its double cover has no closed time like curves, the identified space has none either. Of course, it may be that two points x^α and x'^α lying on a timelike curve γ in $\Sigma \times I$ are images of one another under the involution Γ. On the identified space $(\Sigma \times I)/\Gamma$ the timelike curve γ will thus intersect itself. However, the two tangent vectors at the identified point lie in different halves of the light cone at that point. Thus a particle moving along such a curve may set out into the future and subsequently return from the future or *vice versa*. This is not what is meant by a closed timelike curve because if such a curve has a discontinuity in its tangent vector at some point the two tangent vectors must lie in the same half of the light cone at that point.

The special case when Σ is the standard round 3-sphere and the involution Γ is the antipodal map gives a Lorentz-cobordism for a single S^3 universe. If one modifies the product metric by multiplying the metric on Σ by a square of scale factor which is a non-vanishing even function of time one obtains a Friedman-Lemaitre-Robertson-Walker metric. Identifying points in the way described above is referred to as the "elliptic interpretation". A particular case arises when one considers de-Sitter spacetime. If one regards this as a quadric in 5-dimensional Minkowski spacetime the identification is of antipodal points on the quadric. In this case there are no timelike or lightlike curves joining antipodal points, however, there remains a number of difficulties with this interpretation from the point of view of physics [11], not the least of which is the absence of a spinor structure. In fact, as we shall see below, this problem is quite general: there is no spin-Lorentz-cobordism for a single S^3 universe.

A necessary and sufficient condition for the existence of a line field transverse to the boundary ∂M of a compact manifold M is, by a theorem of Hopf, the vanishing of the Euler characteristic $\chi(M)$. Given an oriented cobordism M of Σ, one can obtain another cobordism by taking the connected sum of M and a compact four manifold without boundary. Under connected sums of 4-manifolds the Euler characteristic obeys the equation

$$\chi(M_1 \# M_2) = \chi(M-1) + \chi(M_2) - 2\,.$$

Thus we can increase the Euler characteristic by two by taking the connected sum with $S^2 \times S^2$ and decrease it by two by taking the connected sum with $S^1 \times S^3$. Therefore, if we start with a spin-cobordism for which the Euler characteristic is even we may, by taking connected sums, obtain an orientable spin-cobordism with zero Euler characteristic and hence a spin-Lorentz-cobordism. On the other hand, if the initial spin-cobordism had odd Euler characteristic we would be obliged to take connected sums with closed 4-manifolds with odd Euler characteristic in order to obtain a Lorentz-cobordism. Examples of such manifolds are $\mathbb{RP}^4$ which has Euler characteristic 1 and $\mathbb{CP}^2$ which has Euler characteristic 3. However, the former is not orientable while the latter, though orientable, is not a spin-manifold. In fact, quite generally, it is easy to see that any four-dimensional closed spin manifold must have even Euler characteristic and thus it is not possible, by taking connected sums, to find a spin-Lorentz-cobordism if the initial spin-cobordism had odd Euler characteristic. To see that a closed spin 4-manifold has even Euler characteristic recall from Hodge theory that on a closed orientable 4-manifold one has, using Poincaré duality:

$$\chi = 2 - 2b_1 + b_2^+ + b_2^-\,,$$

where b_1 is the first Betti number and b_2^+ and b_2^- are the dimensions of the spaces of harmonic 2-forms which are self-dual or anti-self-dual, respectively. On the other hand, from the Atiyah-Singer theorem the index of the Dirac operator with respect to some, and hence all, Riemannian metrics on a closed 4-manifold is given by

$$\text{index}(\text{Dirac}) = (b_2^+ - b_2^-)/8\,.$$

The index of the Dirac operator is always an integer, in fact on a closed 4-manifold it is always an even integer. It follows therefore that for a spin 4-manifold χ must be even. The arguments we have just given suggest, but do not prove, that the Euler characteristic of any spin-cobordism for a closed 3-manifold Σ is a property only of Σ. This is in fact true, as we shall show in the next section. It then follows from our discussion above that we may identify our invariant $u(\Sigma)$ with the Euler characteristic mod2 of any spin cobordism for Σ.

Even without the results of the next section it is easy to evaluate our invariant $u(\Sigma)$ for a number of 3-manifolds of interest using comparatively elementary arguments. Suppose there were a spin-Lorentz-cobordism M for S^3. Then one could glue M across the S^3 to a four-ball, B^4. The Euler characteristic of the resulting closed manifold would be the Euler characteristic of M, which is zero, plus the Euler characteristic of the four-ball, which is one. It is clear that the unique spin structure induced on the boundary would extend to the interior of the 4-ball and so one obtains a contradiction. The same contradiction would result if we took the disjoint union of an odd number of S^3's. If we take the disjoint union of an even number of S^3's it is easy to construct spin-Lorentz-cobordisms. Thus although there exists a spin-Lorentz-cobordism with two S^3's in the past and two in the future, our results show that one cannot slice this spin-Lorentz-cobordism by a spacelike hypersurface diffeomorphic to S^3 which disconnects the spacetime. If this were possible we would have obtained a spin-Lorentz-cobordism for three S^3's which is impossible. In the language of particle physics: there is a 4-fold vertex but no 3-fold vertex.

If we regard $S^1 \times S^2$ as the boundary of $S^1 \times B^3$, where B^3 is the 3-ball we may fill it in with $S^1 \times B^3$. There are two possible spin structures to consider but in both cases they extend to the interior and one obtains a spin-cobordism with vanishing Euler characteristic. Starting with the flat product Riemannian metric on $S^1 \times B^3$ it is easy to find an everywhere non-vanishing unit vector field $\mathbf{V}$ which is outward pointing on the boundary: one simply takes a linear combination of the radial vector field on the 3-ball and the standard rotational vector field on the circle S^1 with radius-dependent coefficients such that the coefficient of the radial vector field vanishes at the origin of the 3-ball and the coefficient of the circular vector field vanishes on the boundary of the 3-ball. As with our product example above the resulting spacetime will, in general, be incomplete and have closed timelike curves but it is a valid spin-Lorentz-cobordism.

These results are sufficient to justify the claim in the introduction that wormholes must be created in pairs according to the Lorentzian point of view. One can also establish easily enough, using suitable connected sums of spin-Lorentz-cobordisms, that our invariant $u(\Sigma)$ is well defined and has the stated behaviour under disjoint union and connected sum of 3-manifolds as long as one fixes a spin structure on the boundary. However, our invariant is independent of the choice of spin structure on the boundary, as we have seen in the examples given above. In order not to have to keep track of the spin structure on the boundary it is advantageous to proceed in a slightly different fashion by using some $\mathbb{Z}_2$-cohomology theory. This we shall do in the next section.

The Euler Characteristic and the Kervaire Semi-Characteristic

The calculations which follow owe a great deal to conversations with Michael Atiyah, Nigel Hitchin, and Graeme Segal for which we are grateful. We begin by recalling the following exact sequence of homomorphisms of cohomology groups for an orientable cobordism M of a closed orientable 3-manifold Σ, the coefficient group being $\mathbb{Z}_2$:

$$0 \to H^0(M) \to H^0(\Sigma) \to H^1(M,\Sigma) \to H^1(M) \to H^1(\Sigma) \to H^2(M,\Sigma) \to H^2(M) \to \dots .$$

Now if we define W to be the image of $H^2(M,\Sigma)$ in $H^2(M)$ under the last homomorphism, and we use Lefshetz-Poincaré duality between relative cohomology and absolute homology groups together with the fact that the compact manifold M is connected we obtain the following exact sequence:

$$0\to\mathbb{Z}_2\to H^0(\Sigma)\to H_3(M)\to H^1(M)\to H^1(\Sigma)\to H_2(M)\to W\to 0\,.$$

By virtue of exactness, the alternating sum of the ranks, or equivalently the dimensions of these vector spaces over $\mathbb{Z}_2$, must vanish. Now the Euler characteristic $\chi(M)$ is given by:

$$\chi(M)=\sum_{i=0}^{i=4}(-1)^i \dim H_i(M;\mathbb{Z}_2)$$

while the $\mathbb{Z}_2$ Kervaire semi-characteristic $s(\Sigma)$ is given by:

$$s(\Sigma)=\dim H^0(\Sigma;\mathbb{Z}_2)+\dim H^1(\Sigma;\mathbb{Z}_2)\,.$$

If dimensions are taken modulo 2 we may reverse any of the signs in these expressions to obtain the relation:

$$\chi(M)-s(\Sigma)=\dim W \bmod 2\,.$$

So far we have not used the condition that the compact 4-manifold M is a spin manifold. To do so we consider the cup product, $\cup$ which gives a map:

$$H^2(M,\Sigma)\times H^2(M)\to H^4(M)\,.$$

For a compact connected 4-manifold $H^4(M;\mathbb{Z}_2)\equiv\mathbb{Z}_2$ so the cup product provides a well defined $\mathbb{Z}_2$ valued bilinear form Q on the image of $H^2(M,\Sigma)$ in $H^2(M)$ under the same homomorphism as above. In other words Q is non-degenerate on the vector space W. [A symmetric bilinear form Q on a vector-space W is non-degenerate if and only if $Q(x,y)=0\ \forall x\in W \Rightarrow y=0$.]

The obstruction to the existence of a spin structure, the second Stiefel-Whitney class $w_2\in H^2(M;\mathbb{Z}_2)$, is characterized by [12]:

$$w_2\cup x=x\cup x \qquad \forall x\in H^2(M;\mathbb{Z}_2)\,.$$

Thus if M is a spin manifold w_2 must vanish and hence

$$Q(x,x)=x\cup x=0 \qquad \forall x\in H^2(M;\mathbb{Z}_2)\,.$$

Now over $\mathbb{Z}_2$, a symmetric bilinear form which vanishes on the diagonal is the same thing as skew-symmetric bilinear form. But a skew-symmetric bilinear form over any field must have even rank and since Q is non-degenerate this implies that the dimension of W must be even. Indeed, one may identify the dimension of W modulo two as the second Stiefel-Whitney class in this situation. We have thus established that for an orientable spin-cobordism

$$\chi(M)=s(\Sigma) \bmod 2$$

and hence:

$$u(\Sigma)=s(\Sigma) \bmod 2\,.$$

Thus, for example, $u(\mathbb{RP}^3)=0$ since it is connected and $H_1(\mathbb{RP}^3;\mathbb{Z})=\mathbb{Z}_2$. It is straightforward to check this example directly by regarding $\mathbb{RP}^3$ as the boundary of the cotangent bundle of the 2-sphere, $T^*(S^2)$. Similar remarks apply to the lens spaces $L(k,1)$ which may be regarded as the boundary of the 2-plane bundle over S^2 with first Chern class $c_1=k$ and which have $\mathrm{H}_1(L(k,1);\mathbb{Z})=\mathbb{Z}_k$. If the integer k is even they spin-Lorentz bound and if it is odd they do not.

The properties of our invariant $u(\Sigma)$ under disjoint union and connected sum now follow straightforwardly from the behaviour of homology groups under these operations.

Generalized Spinor Structures

One way of introducing spinors on a manifold which does not admit a conventional spinor structure is to introduce a $U(1)$ gauge field with respect to which all spinorial fields are charged, the charges being chosen so that the unremovable ± 1 ambiguity in the definition of conventional spinors is precisely cancelled by the holonomy of the $U(1)$ connection [13]. In other words we pass to a $Spin^c(4) \equiv Spin(4) \times_{\mathbb{Z}_2} U(1)$ structure. For general n it is not always possible to lift the tangent bundle of an orientable manifold, with structural group $SO(n)$ to a $Spin^c(4)$ bundle because the obstruction to lifting to a $Spin(n)$, i.e. the second Stiefel-Whitney class w_2, may not be the reduction of an integral class in $H^2(M;\mathbb{Z})$. However, according to Killingback and Rees [14] (see also Whiston [15]) this cannot happen for a compact orientable 4-manifold. From a topological point of view we may clearly replace $Spin^c(4)$ by its Lorentzian analogue: $SL(2,\mathbb{C}) \times_{\mathbb{Z}_2} U(1)$. Thus from a purely mathematical point of view we could always get around the difficulty of not having a spinor structure by using the simplest generalization of a spinor structure at the cost of introducing an extra and as yet unobserved $U(1)$ gauge field. Another possibility would be to use a non-abelian gauge field as suggested by Back, Freund, and Forger [16] and discussed by Isham and Avis [17]. There is no evidence for a gauge field that is coupled in this way to all fermions. It is also not clear that one could arrange that all the anomalies that would arise from such a coupling would cancel.

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References

1. Morris, M.S., Thorne, K.S., Yurtsever, U.: Phys. Rev. Lett. **61**, 1446–1449 (1988)
2. Novikov, I.D.: Zh. Eksp. Teor. Fiz. **95**, 769 (1989)
3. Frolov, V.P., Novikov, I.G.: Phys. Rev. D **42**, 1057–1065 (1990)
4. Geroch, R.P.: J. Math. Phys. **8**, 782–786 (1968)
5. Milnor, J.: L'Enseignement Math. **9**, 198–203 (1963)
6. Reinhart, B.L.: Topology **2**, 173–177 (1963)
7. Yodzis, P.: Commun. Math. Phys. **26**, 39 (1972); Gen. Relativ. Gravit. **4**, 299 (1973)
8. Sorkin, R.: Phys. Rev. D **33**, 978–982 (1982)
9. Bichteler, K.: J. Math. Phys. **6**, 813–815 (1968)
10. Geroch, R.P.: J. Math. Phys. **9**, 1739–1744 (1968); **11**, 343–347 (1970)
11. Gibbons, G.W.: Nucl. Phys. B **271**, 479 (1986); Sanchez, N., Whiting, B.: Nucl. Phys. B **283**, 605–623 (1987)
12. Kirby, R.: Topology of 4-manifolds. Lecture Notes in Mathematics. Berlin, Heidelberg, New York: Springer
13. Hawking, S.W., Pope, C.N.: Phys. Letts. **73** B, 42–44 (1978)
14. Killingback, T.P., Rees, E.G.: Class. Quantum. Grav. **2**, 433–438 (1985)
15. Whiston, G.S.: Gen. Relativ. Gravit. **6**, 463–475 (1975)
16. Back, A., Freund, P.G.O., Forger, M.: Phys. Letts. **77** B, 181–184 (1978)
17. Avis, S.J., Isham, C.J.: Commun. Math. Phys. **64**, 269–278 (1980)

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