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Relativistic Quantum Theory without Quantized Fields

I. Particles in the Minkowski Space

M. B. Mensky
State Committee of Standards, Moscow, USSR

Abstract. Peculiarities of symmetrical quantum systems are considered with the aid of the Mackey's induced representations theory. The four-dimensional coordinate representation of the relativistic quantum mechanics suggested by Stueckelberg in 1941 is rederived, reinterpreted and generalized for an arbitrary spin. Then it is applied to introduce the causal propagator as a particle-antiparticle transition amplitude without consideration of a field equation. Finally the theory of relativistic quantum particles interaction is reformulated without an appeal to the concept of quantized fields.

1. Introduction

The present paper is the first one in the series devoted to reformulation of the relativistic quantum theory of particle interactions in terms of elementary particle states with no appeal to the concept of quantized field. The new formulation is based essentially upon the four-dimensional coordinate representation of relativistic quantum mechanics suggested by Stueckelberg at 1941. The symmetry properties of Minkowski space-time and group-theoretical methods are used in the present paper. The next one will deal with the quantum particle theory in the de Sitter space-time. The same methods appear applicable in this case because the de Sitter space possesses a sufficiently large symmetry group.

Relativistic wave functions have been considered by Stueckelberg [1] with the four-dimensional normalization integral

$$\|\psi\|^2 = \int d^4x |\psi(x)|^2 . \tag{1}$$

The theory based on such functions has met difficulties in interpretation and was forgotten. Yet some authors were discussing the unusual relativistic position operator in the last years [2–5], which proved [2] to correspond to the representation, considered by Stueckelberg in [1]. Let us call this operator the Stueckelberg position operator and the corresponding representation—the Stueckelberg one. The equivalent concept of localization was used in an other connection in [6].

The Stueckelberg position operator satisfies all plausible conditions (transformation properties, orthogonality of eigenvectors and so on). However this operator acts in the space of functions normalized according to four-dimensional integral (1) and its eigenvectors (localized states) are orthogonal in respect to the corresponding scalar product. This space is not a space of real states for a particle with definite mass and spin. This leads to difficulties in interpretation.

It was shown in [7, 8], that the Stueckelberg coordinate representation as well as its generalization for any spin arise naturally as a representation of the Poincaré group induced from the Lorentz subgroup. A connection was found between this representation and the one describing states with the definite mass and spin. In the present paper we shall find on this basis a probability amplitude for propagation the particle from one point to the other, introduce a conception of causal propagator, calculate the causal propagator for any spin, and finally construct the quantum particle theory with no appeal to the concept of quantized field.

As to the Stueckelberg coordinate representation, it acquires physical sense in terms of virtual states arising as separate interfering alternatives for any real process to occure.

The final scheme we arrive at is identical with the space-time interpretation of the quantum field theory suggested in classical works by Feynman [9]. Yet the rather simple Feynman rules needed much more complicated concepts and methods of quantum field theory for their substantiation. It will be demonstrated below that the Feynman scheme may be formulated as a closed one if the Stueckelberg coordinate representation is used.

This result proves to have some significance apart from reinterpretation of the well known series for the S-matrix. Propagators arise in this approach as probability amplitudes rather than Green's functions, and because of this the present approach may have nontrivial applications for example in the case when some classical field is present besides interacting quantum particles.

For the readers' convenience the results of [7, 8] are summed up in the next section, i.e. the Stueckelberg coordinate representation and the Wigner linear momentum representation are derived with the aid of the inducing method (Mackey's theory) in the group theory. The subsequent sections describe the scheme for the construction of relativistic quantum theory of interacting particles. Some more details have to be published in the book [10].

2. Poincaré Invariance and Induced Representations

Given a space $\mathscr{X}$ on which a group G acts transitively as a transformation group. Then $\mathscr{X}$ is called a homogeneous space of the group and it may be realized as the quotient space G/K of the group with respect to a subgroup K, the latter being the stabilizer for some (arbitrarily chosen) $x_0 \in \mathscr{X}$ i.e.

$$K = \{k \in G : kx_0 = x_0\} .$$

Explicitly, having chosen x_0 (and hence K), a point $x \in \mathscr{X}$ corresponds to the right coset gK where $gx_0 = x$. It is convenient to choose in each coset a representative $x_G \in G$. We then have an injection $x \to x_G$ from the homogeneous space into the group. Once such a choice has been made there arises the *system of factors* $(g, x)_K$ by

$$gx_G = (gx)_G (g, x)_K .$$

Any linear representation Δ of the subgroup K (acting, say, in a space $\mathscr{L}$) may be lifted to a linear representation U of G acting in a space $\mathscr{H}$. This *induced representation*, denoted by $\Delta(K)\uparrow G$ is defined as follows [11, 12][1]. A vector in $\mathscr{H}$ is a function on $\mathscr{X}$ with values in $\mathscr{L}$. Denoting it by φ the transformation law is

$$(U(g)\varphi)(x) = \Delta(k)\varphi(g^{-1}x) \quad \text{with} \quad k = (g^{-1}, x)_K^{-1} . \tag{2}$$

Equivalently one may consider φ as a function on the group restricted by the structural condition

$$\varphi(gk) = \Delta(k^{-1})\varphi(g) . \tag{3}$$

The transformation law is then simply

$$(U(g)\varphi)(g') = \varphi(g^{-1}g') . \tag{4}$$

If then representation Δ has an invariant sesquilinear form $\langle \varphi, \varphi' \rangle$ and dx is an invariant measure on $\mathscr{X}$ then $\Delta(K)\uparrow G$ has the invariant sesquilinear form

$$(\varphi, \varphi') = \int dx \langle \varphi(x), \varphi'(x) \rangle . \tag{5}$$

One more thing necessary for our purposes is *intertwining of induced representations*. Let $U_\Delta = \Delta(K)\uparrow G$ and $U_\Lambda = \Lambda(H)\uparrow G$ be two induced representations. The operator $T: \mathscr{H}_\Delta \to \mathscr{H}_\Lambda$ is called intertwining operator (this being expressed by $T \in [U_\Delta, U_\Lambda]$ if

$$TU_\Delta(g) = U_\Lambda(g) T$$

holds for all $g \in G$. It may be shown [11, 12] that an arbitrary intertwining operator for these representations has the form

$$(T\varphi)(g) = \int_{G/K} dx\, t(g^{-1}x_G)\varphi(x) , \tag{6}$$

where $t(g)$ is a linear operator from $\mathscr{L}_\Delta$ to $\mathscr{L}_\Lambda$ and the mapping $g \to t(g)$ satisfies the structural condition

$$t(hgk) = \Lambda(h) t(g) \Delta(k) \tag{7}$$

for all $g \in G$, $h \in H$, $k \in K$.

We shall be concerned with two kinds of induced representations of the Poincare group P. The first is the set of unitary, irreducible representations [13] describing the states of a particle with mass m and spin j and which will be denoted

[1] It may be noted [see (8)] that this induction process provides a natural quantization method for a classical system possessing sufficient symmetry, e.g. when the configuration space is a homogeneous space of the symmetry group.

by U_{mj}. For their description in terms of induced representations see also [14]. The homogeneous space is here the hyperboloid of 4-velocities v

$$(v, v) = (v^0)^2 - \boldsymbol{v}^2, \qquad v^0 > 0 .$$

Picking as the reference point x_0 the point $e_0 = (1, 0, 0, 0)$ the stabilizer group consists of the space-time translation group T and the 3-dimensional rotation group R. Then

$$U_{mj} = \Delta_{mj}(TR) \uparrow P ,$$

where Δ_{mj} is given by

$$\Delta_{mj}(a_T r) = e^{\mathrm{i} m a^0} \Delta_j(r)$$

with $a = (a^0, a^1, a^2, a^3)$, $a_T \in T, r \in R$ and Δ_j the $2j+1$-dimensional unitary irreducible representation of R. With the invariant measure $dv = v_0^{-1} d^3 v$ on the velocity hyperboloid and the scalar product defined according to (5) U_{mj} is unitary.

The other kind of representation we shall consider results if one takes Minkowski space as the homogeneous space of P from which the induction process starts[2]. Choosing the origin as the reference point x_0 the stabilizer group is the homogeneous Lorentz group L. This leads to induced representations $U_D = D(L) \uparrow P$. Here D is some representation of the homogeneous Lorentz group, usually suggested as finite dimensional. The space $\mathscr{H}_D$ consists of functions ψ on Minkowski space with values in $\mathscr{L}_D$($\mathscr{L}_D$ being the carrier space of D). We have the transformation law

$$(U_D(a_T)\psi)(x) = \psi(x - a) , \tag{8}$$

$$(U_D(l)\psi)(x) = D(l)\psi(l^{-1}x) \qquad \Lambda \in L . \tag{9}$$

There exists an Hermitean but indefinite form $\langle \psi, \psi' \rangle_D$ in $\mathscr{L}_D$ invariant under $D(L)$. Correspondingly we have in the carrier space $\mathscr{H}_D$ of U_D the invariant (indefinite) form

$$(\psi, \psi')_D = \int d^4x \langle \psi(x), \psi'(x) \rangle_D = \int d^4x \bar{\psi}(x) \psi'(x) , \quad \bar{F} = F^+ \Gamma . \tag{10}$$

The space $\mathscr{H}_D$ may be considered as an adaptation of the Stueckelberg representation to the case of arbitrary spin. Vectors in this space cannot be regarded as physical states of a particle. The most significant reason is that they may have a limited extension in time. The improper vector $\psi_x(x') = \delta^4(x - x')F$, $F \in \mathscr{L}_D$ corresponds to 4-dimensional localization in the space-time point x. One may interpret such vectors as related to a virtual event encountered in the interaction of the particle with others (see below).

The operator $J_{mj}: \mathscr{H}_{mj} \to \mathscr{H}_D$ establishing relations between spaces of real states of a particle and localized (Stueckelberg) states has to maintain the symmetry properties. It means that J_{mj} intertwines corresponding representations: $J_{mj} \in [U_{mj}, U_D]$. With the aid of the theorem on intertwining of the induced

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2 It has been brought to my attention that this construction has also been discussed in [15].

representations [Eqs. (6) and (7)] one obtains

$$J_{mj}\varphi(x)=\psi_{mj}(x)=(m/(4\pi^{3/2}))\int dv e^{-\mathrm{im}(v,x)}D(v_L)J_j\varphi(v)\,, \tag{11}$$

where $J_j\in[\Delta_j, D\downarrow R]$. This means that the operator $J_j:\mathscr{L}_j\to\mathscr{L}_D$ intertwines $\Delta_j(R)$ with the representation $r\mapsto D(r)$ of the rotation group R. v_L is the Lorentz transformation (boost) bringing e_0 to v.

The form $(\psi_{mj}, \psi'_{mj})_D$ does not exist. Thus $J_{mj}\mathscr{H}_{mj}$ forms a subspace of generalized (unnormalizable) vectors in $\mathscr{H}_D$. But the form $(\psi, \psi_{mj})_D$ is well defined provided ψ is a normalizable vector in $\mathscr{H}_D$. This yields the invariant form $A(\psi,\varphi)=(\psi, J_{mj}\varphi)_D$ with arguments $\psi\in\mathscr{H}_D$, $\varphi\in\mathscr{H}_{mj}$. This form may be naturally interpreted as a *probability amplitude for the real state φ to convert into the localized (Stueckelberg) state ψ*. Analogously $A(\varphi,\psi)=(J_{mj}\varphi,\psi)_D=(\psi_{mj},\psi)_D$ will be interpreted in the following as an *amplitude for the localized state ψ to convert into the real state φ*.

3. Amplitude of Particle Transition in Space-Time

Connection between the localized state space $\mathscr{H}_D$ and the particle state space $\mathscr{H}_{mj}$ may be established in the opposite direction. It is an intertwining operator $K_{mj}\in[U_D, U_{mj}]$ that is needed for the purpose. The same method gives for it

$$K_{mj}\psi(v)=\varphi(v)=(m/(4\pi^{3/2}))\int dx e^{\mathrm{im}(v,x)}K_i D(v_L^{-1})\psi(x)\,, \tag{12}$$

where $K_j\in[D\downarrow R, \Delta_j]$. The operator K_{mj} appears to extract the part corresponding to the given m, j from a localized state of a particle.

An important physical conclusion may be drawn if one projects $\mathscr{H}_D$ onto the physical space $\mathscr{H}_{mj}$ and rewrights the result again in the coordinate representation. The composition of these two operations is described by the generalized projector $P_{mj}=J_{mj}K_{mj}$ for which one has

$$P_{mj}\psi(x)=\int dx' P_{mj}(x-x')\psi(x')\,, \tag{13}$$

where

$$P_{mj}(x-x')=(m^2/(16\pi^3))\int dv e^{-\mathrm{im}(v,x-x')}P_j(v)\,, \tag{14}$$

$$P_j(v)=D(v_L)P_jD(v_L^{-1})\,, \tag{15}$$

$$P_j=J_jK_j\,. \tag{16}$$

If the operators J_j and K_j correspond to each other in a certain sense then $K_jJ_j=\mathbb{1}$ and consequently P_j is a projector. It has sense as a projector onto the definite spin j [of all spins described by the representation $D(L)$] in the rest system. A projector on this spin in an arbitrary reference frame $P_j(v)$ arises as a result of boosting.

It may be shown that the operator $P_j(v)$ is a *polynomial* in 4-velocity components v^μ, $\mu=0,1,2,3$, provided D is a *finite-dimensional* representation. Therefore the kernel $P_{mj}(x-x')$ can be transformed to the form

$$P_{mj}(x-x')=P_j((i/m)\partial/\partial x)P_m(x-x')\,, \tag{17}$$

where

$$P_m(x-x')=(m^2/(16\pi^3))\int dv e^{-\mathrm{i}m(v,x-x')} \tag{18}$$

is the negative-frequency part of the Pauli-Jordan function.

Let us postulate that virtual localized states, arising in the course of interactions, may be converted one into another, these conversions occuring through the intermediate real states. This means that the localized state $\psi' \in \mathscr{H}_D$ converts into some real state $\varphi \in \mathscr{H}_{mj}$ and further into the localized state ψ. In order to calculate the probability amplitude of such a process, one must multiply the amplitudes of the transitions $\psi' \to \varphi$ and $\varphi \to \psi$, and add the products, resulting from all possible alternative intermediate states φ. The latter means that φ runs through some basis in $\mathscr{H}_{mj}$. These calculations may be carried out with the aid of amplitudes $A(\varphi, \psi')$ and $A(\psi, \varphi)$ found in the preceding Section. It is obvious, that the resulting amplitude may be expressed by means of the projector P_{mj} as follows:

$$A_{mj}(\psi, \psi')=(\psi, P_{mj}\psi')_D\,. \tag{19}$$

For point-localized states $\psi_x, \psi_{x'}$ this formula gives

$$A_{mj}(\psi_x, \psi_{x'})=\langle F, P_{mj}(x-x')F'\rangle_D=\bar{F}P_{mj}(x-x')F'\,. \tag{20}$$

This is the reason for the kernel $P_{mj}(x-x')$ to be called the *amplitude of propagation of the particle* (not causal propagation however, which will be considered below).

4. Antiparticle and the Causal Propagator

One of the main principles of the relativistic quantum theory in the present formulation concerns the character of the particle and antiparticle propagation. Following the idea of Stueckelberg and Feynman [9] we shall postulate that the particle and antiparticle differ by the sign of mass and propagate in the mutually opposite directions of the time axis. In fact there appears a new object in the theory which could be called a particle-antiparticle complex. Let us make these statements more precise.

An elementary particle was defined in Section 2 by the induced representation $\Delta_{mj}(K)\uparrow P$, m being supposed positive. Let us define an *antiparticle* in the same way but replacing m with $(-m)$. All quantities characterizing a particle will be marked by the superscript "plus" while those of an antiparticle will be marked by the sign "minus". For example

$$\Delta_{mj}^{(\pm)}(a_T r)=e^{\pm\,\mathrm{i}m(e_0,a)}\,\Delta_j(r)\,. \tag{21}$$

There exists a natural correspondence between particle and antiparticle states described by the *charge conjugation* operation $I_C\colon \mathscr{H}_{mj}^{(\pm)} \to \mathscr{H}_{mj}^{(\mp)}$ as follows

$$I_C\varphi^{(\pm)}(v)=C_j\overset{*}{\varphi}{}^{(\pm)}(v)\,, \tag{22}$$

where $C_j \in [\overset{*}{\Delta}_j, \Delta_j]$ and $\overset{*}{C}_j C_j = \mathbb{1}$. The coordinate representation is the same for particles and antiparticles, and a charge conjugation $I_C : \mathscr{H}_D \to \mathscr{H}_D$ has the following form in it:

$$I_C \psi(x) = C \overset{*}{\psi}(x) , \tag{23}$$

where $C \in [\overset{*}{D}, D]$, $C \overset{*}{J}{}_j^{(+)} = J_j^{(-)} C_j$ and $\overset{*}{C} C = \mathbb{1}$.

The considerations of Section 5 which may also be conducted for the case of an antiparticle lead to the particle and antiparticle transition amplitudes:

$$P_{mj}^{(\pm)}(x-x') = P_j^{(\pm)}((i/m)\partial/\partial x) P_m^{(\pm)}(x-x') , \tag{24}$$

where

$$P_m^{(\pm)}(x-x') = (m^2/(16\pi^3)) \int dv\, e^{\mp\, \mathrm{im}(v, x - x')} \tag{25}$$

are the negative- and positive-frequency parts of the Pauli-Jordan function correspondingly, and matrices $P_j^{(\pm)}(v)$ are obtained by "boosting" (17) from the properly chosen matrices $P_j^{(\pm)}$.

Let us define the *causal propagator* $P_{mj}^c(x-x')$ as a probability amplitude for the particle or (alternatively) antiparticle transition from the point x' to x. Taking into account that a particle extends to the future while an antiparticle goes to the past one must put

$$P_{mj}^c(x-x') = \theta(x-x') P_{mj}^{(+)}(x-x') + \theta(x'-x) P_{mj}^{(-)}(x-x') , \tag{26}$$

where

$$\theta(x-x') = \begin{cases} 1 & \text{if} \quad x^0 > x'^0 \\ 0 & \text{if} \quad x^0 < x'^0 , \end{cases} \tag{27}$$

One may write this in covariant form as

$$P_{mj}^c(x-x') = P_j((i/m(\partial/\partial x) P_m^c(x-x') , \quad P_j(v) = P_j^{(+)}(v) = P_j^{(-)}(-v) , \tag{28}$$

where P_m^c is the scalar causal propagator of Stueckelberg. It is important, that $P_m^c(x-x')$ turns out to be a *Green's function for the Klein-Gordon equation:*

$$(\square + m^2) P_m^c(x-x') = -i\delta(x-x') . \tag{29}$$

The *probability amplitude for the causal transition* between two point-localized states is expressed through the causal propagator as

$$A_{mj}^c(\psi_x, \psi_{x'}) = \langle F, P_{mj}^c(x-x') F' \rangle_D = \bar{F} P_{mj}^c(x-x') F' . \tag{30}$$

Remark 1. The formula (17) defines the polynomial $P_j(v)$ only on the hyperboloid of four-velocities. The extension of the polynomial onto the whole linear momentum space R^4 is ambiguous, because it admits adding of an arbitrary polynomial $Q(k)$, $k \in R^4$, multiplied by $(k, k) - 1$. Consequently the differential operator $P_j(i\partial/m)$ may be altered by an additional term, containing the Klein-

Gordon operator as a factor. This leads to an additional quasilocal term $Q(\partial)\delta(x-x')$ in the causal propagator $P^c_{mj}(x-x')$. The requirement for the *propagator to be a Green's function* of some equation diminishes this ambiguity.

Definition of the causal propagator proves to be unambiguous for the spin $j \leqq 1$ described by one of the simplest representations D of the Lorentz group. In turn the requirement for the propagator to be a Green's function may be formulated as the requirement for the corresponding integral operator to have the differential operator as an inverse. In such a form this requirement may be justified in the framework of the present approach, based on transition amplitudes. This line of investigation leads to the *local formulation of the theory* [10].

Remark 2. One can easily derive the following integral relation

$$i \int_\Sigma d\sigma^\mu(x'') P^c_{mj}(x-x'') \frac{\overleftrightarrow{\partial}}{\partial x''^\mu} P^c_{mj}(x''-x') = P^c_{mj}(x-x'), \tag{31}$$

where integration goes over any closed hypersurface surrounding the point x', the other point x being outside. This relation means that the amplitude of transition from x' to x is equal to the sum of amplitudes of transition through all possible points of the surface Σ. The relation looks like the *Einstein-Smoluchowski condition* in the functional integration theory [16, 17]. It may serve as the basis for definition of the functional integral of a certain kind. The latter might enable one to generalize a causal propagator to the case of an arbitrary external classical field.

The amplitude of a causal propagation $A^c_{mj}(\psi_x, \psi_{x'})$ is linear in the $\psi_{x'}$ and antilinear in ψ_x. This corresponds to its interpretation as an amplitude of the transition $\psi_{x'} \to \psi_x$. With the aid of the charge conjugation one may go over to the amplitudes for the *causal production and annihilation* of the pairs of localized states:

$$A^p_{mj}(\psi_x, \psi_{x'}) = \bar{F} P^p_{mj}(x-x') \bar{F}'^T; \quad A^a_{mj}(\psi_x, \psi_{x'}) = F^T P^a_{mj}(x-x') F',$$

where the superscript T denotes the matrix transposition and the following notations are used:

$$P^p_{mj}(x-x') = P^c_{mj}(x-x') C \Gamma^T; \quad P^a_{mj}(x-x') = C^+ \Gamma P^c_{mj}(x-x'). \tag{32}$$

The ambiguity in interpretation of the causal propagation is a consequence of the fact, that the coordinate representation is unique for the particle and antiparticle. There is an analogous ambiguity in interpretation of interactions. One may accept any possible interpretation, while ensuring, that interpretations of propagation and interactions correspond each other to give Lorentz-invariant convolutions (see the next section).

It appears that the symmetry property

$$[P_j(-v)]^T (C^+ \Gamma)^T = (-1)^{2j} C^+ \Gamma P_j(v)$$

takes place for any finite-dimensional $D(L)$. Consequently the amplitudes of production and annihilation of the pairs of localized states are *symmetrical for an* *integer spin and skewsymmetrical for a half-integer one:*

$$[P_{mj}^{p,a}(x'-x)]^T = (-1)^{2j} P_{mj}^{p,a}(x-x')\,. \tag{33}$$

This symmetry is important for the spin-statistics relation.

5. Local Interaction and Amplitude for Arbitrary Process

The causal propagator, describing transitions of localized states through the intermediate real ones, was defined in the preceding section. Let us assume that besides the causal propagation, the direct transitions between point-localized states also exist. Let the direct transitions not change the point of localization. Call such transitions *local interactions.* The point-localized state $\psi_x(x') = \delta(x-x')F$ is characterized by the localization point $x \in \mathscr{X}$ and the polarization vector $F \in \mathscr{L}_D$. If the local interaction occurs at the point x and includes the particles, corresponding to the representations $D_1, \ldots, D_n$, then it can be defined by the *local interaction amplitude* $\gamma_x(F_1, \ldots, F_n)$. The mapping γ_x of the product $\mathscr{L}_{D_1} \times \ldots \times \mathscr{L}_{D_n}$ into the set of complex numbers is linear in the vectors, describing the particles before transition, and antilinear in the particles after transition.

The mapping γ_x is determined by the matrix $\gamma_x^{i\ldots}{}_{\ldots k}$ as

$$\gamma_x(F_1, \ldots, F_n) = \gamma_x^{i\ldots}{}_{\ldots k} (\bar{F}_1)_i \ldots \ldots (F_n)^k\,. \tag{34}$$

The requirement for the local interaction to be invariant under the Poincare transformations, preserving the point x, yields that $\gamma_x(F_1, \ldots, F_n)$ is invariant under the Lorentz transformations of $F_1, \ldots, F_n$. This means that the matrix $\gamma_x^{i\ldots}{}_{\ldots k}$ is proportional to some generalized Clebsh-Gordan coefficient, the factor being a *coupling constant.* Invariance under translations yields that the coupling constant does not depend on the space-time point: $\gamma_x = \gamma$.

Just as in the case of the causal propagator, the local interaction amplitude may be reinterpreted if one of the functions ψ_x is treated as the complex conjugate of the wave function of the charge conjugated particle. Then production of the particle and annihilation of the corresponding antiparticle would be interchanged. Any interpretation may equally be used, but accordance with the interpretation of causal propagators should be ensured. It is convenient to choose γ to be linear in all its arguments, and interprete it as an amplitude of annihilation of many-particle localized state into a vacuum. Then causal propagators in the form $P_{mj}^{p}(x-x')$ ought to be used, describing the pairs of localized states production.

Let us summarize the previous considerations. Some elementary processes with particles were considered, and their probability amplitudes found or suggested. Those are the following transitions:

i) Conversion of the real state ψ_{mj} into the point-localized one ψ_x *(localization)* with the amplitude $\bar{F}\psi_{mj}(x)$ and the conversion of the localized state ψ_x into the real one ψ_{mj} *(materialization)* with the amplitude[3] $\bar{\psi}_{mj}(x)F$.

[3] In this interpretation a direction of the transition is defined in respect to the "*proper time*", which is opposite to the ordinary time in the case of an antiparticle (the Stueckelberg-Feynman conception). Consequently if ψ_{mj} is a real state of the *antiparticle*, then localisation $\bar{F}\psi_{mj}(x)$ and materialization $\bar{\psi}_{mj}(x)F$ are actually correspondingly production and annihilation of the pair of the point-localized and the real states.

ii) Transition of one localized state $\psi_{x'}$ into another ψ_x through one of the real states of the particle (if $x > x'$) or antiparticle (if $x < x'$), both alternatives forming a single process called *causal propagation*, with the amplitude[4] $\bar{F} P^c_{mj}(x-x')F'$.

iii) Direct transition between the many-particle states localized at a single point *(local interaction)* with the amplitude[5] $\gamma(F_1, \ldots, F_n)$.

One needs one more suggestion to complete the particle theory, that the real processes may occur only through sequences of elementary acts of the mentioned three types. An amplitude of any real process may be reduced to the sum of products of the elementary amplitudes, each product corresponding to an alternative realization of the process through the elementary ones.

Let us make more precise, what concepts and suggestions form the basis of the present formulation of the particle theory. The space of *real states* of the particle (antiparticle) is defined as the carrier space of the irreducible unitary representation of the Poincare group with the positive-definite (correspondingly negative-definite) energy. The space of *localized states* of both the particle and the corresponding antiparticle is defined as the carrier space of the representation, induced from the finite-dimensional representation of the Lorentz subgroup. This space contains a subset of *point-localized states*, playing the most important role in the theory. The whole theory is strictly regulated by the requirement for probability amplitudes to be covariant under the Poincaré group. The rule of summing amplitudes of alternatives is supposed. Besides these rather general suppositions only one specific postulate is suggested. It states that the quantum *transitions of only three kinds or any sequences of them are admitted.* These three elementary transitions are: i) localization and materialization; ii) causal propagation and iii) local interaction. The amplitudes for the elementary transitions can be derived from the qualitative definition of them as given above, and from Poincaré covariance. The amplitude for any composite process can be calculated with the aid of the rule of summing up alternatives.

Some additional explanations are necessary on the calculation procedure. The concrete version of the particle theory is fixed by the list of particles and interactions, admitted by the theory. The particle is determined by the triple (m, j, D), and the interaction is determined by the amplitude γ. Then for any reaction all alternative sequences of elementary transitions, resulting in this reaction, are to be enumerated and corresponding amplitudes summed up. The amplitude of each alternative is the product of elementary amplitudes.

Enumeration of alternatives can be carried out with the aid of the *Feynman diagrams* in an obvious manner.

Remark 3. If the transition between real states occurs for a *finite time interval*, then its amplitude corresponds to the diagrams with external lines, ending on some spacelike surfaces. Such line corresponds to the integral

$$\int d\sigma^\mu P^c_{mj}(x'-x)\overleftrightarrow{\partial}_\mu \psi(x) \quad \text{or} \quad \int d\sigma^\mu \bar{\psi}(x)\overleftrightarrow{\partial}_\mu P^c_{mj}(x-x') .$$

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[4] Two other possible descriptions of the same process are production and annihilation of the pair of localized states with amplitudes $\bar{F} P^p_{mj}(x-x')\bar{F}'^T$ and $F^T P^a_{mj}(x-x')F'$ correspondingly.

[5] Other possible interpretations of the same process are obtainable if production of some localized states are substituted by annihilation or vice versa. The interpretations of local interactions and causal propagators ought to be chosen consistently.

The amplitude described previously (one for an infinite time interval) may be obtained if some of the surfaces tend to the infinite past, while the others tend to the infinite future. This procedure clarifies an interpretation of external lines, because it takes into account an relation of the proper and ordinary (coordinate) time.

6. Concluding Remarks

The above consideration demonstrates that the relativistic quantum theory may be given the form, essentially similar to the form of the nonrelativistic theory. The similarity becomes actually more startling, if the nonrelativistic quantum mechanics is constructed by means of the induced representations of the Galilei group [10]. Yet there is nothing surprising in this similarity, because even the traditional quantum field theory usually gets its final interpretation in terms of the perturbation theory. The point is that each order of the perturbation theory includes just a finite number of particles in the intermediate states, so that the *infinite number of degrees of freedom are effectively reduced to a finite number of them.* The present approach, explicitly using virtual localized states, enables one always to remain in the framework of the theory with a finite number of degrees of freedom. It might be said that the perturbational form of the *quantum field theory* is reformulated as a *quantum particle theory.*

Certainly, all the usual difficulties such as divergencies, appear in the present approach as well. Yet this reinterpretation seems to be useful for the generalization of the particle theory to the case of a curved space-time. Such generalization appears immediately possible for the de Sitter space-time, provided the latter possesses a 10-parameter symmetry group. It is more remarkable that the generalization onto any Riemannian space-time seems to be possible, if one takes the set of all parallel transfers as a background symmetry [18, 19].

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