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On a Class of Generalized Poincaré Groups: Inhomogeneous $SL(n, C)$

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Abstract. A certain class of non semi-simple Lie groups $ISL(n, C)$ based on $SL(n, C)$ is investigated. Its Lie algebra and invariants are determined. The connection between $ISL(2, C)$ and the Poincaré group is discussed.

Introduction

In recent time much attention has been paid to the problem of extending relativistically the $SU(6)$ supermultiplet theory which has been proposed independently by many authors [1—3]. Such an extension should have the property that the resulting invariance group contains the Poincaré group $\mathscr{P}$ as a subgroup and $SU(6)$ as a "little group". The latter assumption is motivated by the fact that $SU(6)$ is supposed to describe the spin as well as the $SU(3)$ internal degrees of freedom of the elementary system.

One such relativistic extension** is the "inhomogeneous $SL(6, C)$" group [hereafter denoted $ISL(6, C)$]. It is built in complete analogy with $ISL(2, C)$ which is isomorphic to the covering group of the Poincaré group $\mathscr{P}$. The group $ISL(2, C)$ contains as a subgroup $SL(2, C)$ — the group of 2×2 complex matrices with determinant 1 — which is isomorphic to the covering group of the proper Lorentz group. It contains also an invariant Abelian subgroup, the translation group, which is the additive group of 2×2 Hermitian matrices. Correspondingly the $ISL(6, C)$ group contains as a subgroup $SL(6, C)$, the group of 6×6 complex matrices with determinant 1, which constitutes the "homogeneous part" of $ISL(6, C)$. It also contains as an invariant Abelian subgroup the 36 dimensional additive group of 6×6 Hermitian matrices.

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** The $ISL(6, C)$ group has been proposed independently by B. Sakita [1], L. Michel (private communication), T. Fulton and J. Wess [4], and H. Bacry [5].

Despite difficulties of interpretation, the SU_6 theory has had considerable success, not only in classifying the baryons and mesons, but also in relating form factors*. Moreover, many attractive results have been derived from the $ISL(6, C)$ group [7] and it should be interesting to investigate it from a mathematical point of view.

In order to find the irreducible unitary representations of $ISL(6, C)$ one can either use global methods in analogy with WIGNER's procedure for the Poincaré group, or on can use purely infinitesimal methods, i.e., look for the Hermitian irreducible representations of the Lie algebra. It is the purpose of this paper to give a suitable realization of the Lie algebra of $ISL(6, C)$ or, more generally, of $ISL(n, C)$. Having given the elements of the Lie algebra with their commutation relations, we determine all invariants of the group. This is done by generalizing the Pauli-Lubansky vector W_μ, which for the Poincaré group is the covariant spin operator. In fact all invariants of $ISL(n, C)$ are built from this generalized spin operator and the generalized translation operator. Just as for the Poincaré group, the components of the generalized spin operator generate the little groups. For $ISL(6, C)$ there are 16 distinct little groups, one of which of course is SU_6. Although we mainly consider $ISL(n, C)$ we shall find some results also for $IGL(n, C)$, i.e., the group where $SL(n, C)$ is replaced by the full linear group $GL(n, C)$. The cases $ISL(6, C)$ and $ISL(2, C)$ are also considered in more detail.

1. Definition of the groups *IGL*(*n*, *C*) and *ISL*(*n*, *C*) and their Lie algebras

The group $IGL(n, C)$ is defined as follows: an element of $IGL(n, C)$ is a pair (Λ, b) where Λ is a non-singular $n \times n$ complex matrix and b is a $n \times n$ Hermitian matrix. The multiplication law is

$$(\Lambda_1, b_1)(\Lambda_2, b_2) = (\Lambda_1\Lambda_2, b_1 + \Lambda_1 b_2 \Lambda_1^+) \tag{1}$$

where Λ^+ denotes the Hermitian conjugate of Λ. The group $ISL(n, C)$ is a subgroup of $IGL(n, C)$ and is obtained by restricting the Λ's to unimodular matrices.

It readily follows from Eq. (1) that the matrices b form an invariant Abelian subgroup (under addition). Its order is n^2. The homogeneous parts $GL(n, C)$ and $SL(n, C)$ are subgroups of order $2n^2$ and $2n^2 - 2$, respectively.

Each matrix Λ can be put in an exponential form, namely

$$\Lambda = \exp\left[\sum_{a,b} (\alpha_a^b E_b^a + i\beta_a^b E_b^a)\right] \tag{2}$$

where α_a^b and β_a^b are real coefficients and E_b^a is the $n \times n$ matrix all elements of which are zero except one; this element equals 1 and belongs

* See, for instance, [6]. This paper also contains references to other papers.

to the a^{th} row and the b^{th} column. The matrices E_b^a and $i E_b^a$ generate $GL(n, C)$. The $SL(n, C)$ group is generated by

$$E_b'^{\,a} = E_b^a - \frac{1}{n} \left(\sum_c E_c^c \right) \delta_b^a \tag{3}$$

and by $i E_b'^{\,a}$ which are traceless matrices.

The unitary subgroup $U(n)$ of $GL(n, C)$ is generated by a set of n^2 antihermitian matrices L_α, for instance $E_b^a - E_a^b$ and $i(E_b^a + E_a^b)$. Eq. (2) can be written in the form

$$\Lambda = \exp \left[\sum_\alpha (\lambda^\alpha L_\alpha + i \mu^\alpha L_\alpha) \right] \tag{4}$$

where λ^α and μ^α are real. The matrices L_α obey

$$L_\alpha^+ = - L_\alpha \,. \tag{5}$$

Correspondingly, the unimodular unitary group $SU(n)$ is generated by a set of $n^2 - 1$ antihermitian traceless matrices L_i, for instance $E_b'^{\,a} - E_a'^{\,b}$ and $i(E_b'^{\,a} + E_a'^{\,b})$.

Let us examine the irreducible representations of $SL(n, C)$. Let V_n be an n dimensional complex vector space and ξ a vector in V_n. An infinitesimal transformation

$$\Lambda = 1 + \lambda^i L_i + i \mu^i L_i \tag{6}$$

transforms ξ into

$$\xi'^{\,a} = (\Lambda \xi)^a = \xi^a + \lambda^i (L_i)_b^a \xi^b + i \mu^i (L_i)_b^a \xi^b \,. \tag{7}$$

Such a transformation defines the representation denoted $D(n, 1)$ which is irreducible with respect to the subgroup $SU(n)$. Three other representations of dimension n can be defined, namely

$$D(1, n): \quad \xi' = (\Lambda^+)^{-1} \xi = \xi + \lambda^i L_i \xi - i \mu^i L_i \xi \tag{8}$$

whose vector components are written $\xi_{\dot a}$,

$$D(\overline{n}, 1): \quad \xi' = (\Lambda^T)^{-1} \xi = \xi + \lambda^i L_i^* \xi + i \mu^i L_i^* \xi \tag{9}$$

with vector components ξ_a,

$$D(1, \overline{n}): \quad \xi' = \Lambda^* \xi = \xi + \lambda' L_i^* \xi - i \mu^i L_i^* \xi \tag{10}$$

with vector components $\xi^{\dot a}$. Such a notation makes apparent the $SU(n)$ structure. With respect to this subgroup $D(n, 1)$ and $D(1, n)$ are equivalent to the representation $\{n\}$ of $SU(n)$ and $D(\overline{n}, 1)$ and $D(1, \overline{n})$ are equivalent to the conjugate representation $\{\overline{n}\}$. As it has been shown elsewhere*, all irreducible representations of $SL(n, C)$ can be labelled by two irreducible representations $\{p\}$ and $\{q\}$ of $SU(n)$ in the form $D(p, q)$. Moreover, one has the following rules

$$D(p, q) \otimes D(p', q') = D(p \otimes p', q \otimes q') \,.$$

* It is a simple generalization of what is done in the case of $SL(6, C)$ in [8].

For $SU(n)$, $D(p, q)$ reduces to $\{p\} \otimes \{q\}$, the usual direct product of the irreducible representations $\{p\}$ and $\{q\}$ of $SU(n)$.

In the special case of $SL(2, C)$ the numbers p and q are usually replaced by the spin eigenvalues j and j' with $p = 2j + 1$ and $q = 2j' + 1$. The vector space V_2 is the well-known two-dimensional Weyl spinor space corresponding to the representation $D_{\frac{1}{2}0} \equiv D(2, 1)$. By taking the direct sum $D_{\frac{1}{2}0} \oplus D_{0\frac{1}{2}}$ we get the four-dimensional Dirac spinor space.

It will appear useful to generalize the Dirac representation to the $SL(n, C)$ case. It is built from the sum of the two representations $D(n, 1)$ and $D(1, n)$. From Eq. (6) and definition (8) the generators of $SL(n, C)$ can be written in such a reducible representation in the form of $2n \times 2n$ matrices

$$M_i = \begin{array}{|c|c|} \hline L_i & \\ \hline & L_i \\ \hline \end{array} \qquad N_i = \begin{array}{|c|c|} \hline iL_i & \\ \hline & -iL_i \\ \hline \end{array} \tag{11}$$

As it is well known, the generators of $SL(n, C)$ belong to the adjoint representation, namely $D(n^2 - 1, 1) \oplus D(1, n^2 - 1)$ where $\{n^2 - 1\}$ denotes the adjoint representation of $SU(n)$ according to the rule

$$\{\bar{n}\} \otimes \{n\} = \{1\} \oplus \{n^2 - 1\}\,. \tag{12}$$

It can easily be shown* that the $ISL(n, C)$ group has a very simple $2n$ dimensional representation obtained by adding to the matrices (11) the matrices representing the translation operators, namely

$$P_\alpha = \begin{array}{|c|c|} \hline & \\ \hline L_\alpha & \\ \hline \end{array}$$

where L_α are for instance those of Eq. (4). The P_α's generate obviously an invariant Abelian subgroup and they belong to the $\{\bar{n}\} \otimes \{n\}$ representation of $SU(n)$ and to the $D(\bar{n}, n)$ representation of $SL(n, C)$. It can also readily be verified that the matrices M_i, N_i and P_α generate $ISL(n, C)$. Obviously we could have chosen the representation $D(n, \bar{n})$ instead of $D(\bar{n}, n)$ for the translation operators P_α, a choice which is actually suggested by Eq. (1).

The $2n$-dimensional representation that we have just defined suggests the following choice of generators for $IGL(n, C)$. The generators of the

* This was shown for $n = 2$ and $n = 6$ in [9].

homogeneous group are E^a_b, defined as before, and $E^{\dot a}_{\dot b}$ where the dots mean that we have to find the only non-vanishing element 1 in the b^{th} row and a^{th} column of the right-lower $n \times n$ matrix. Those of the translation group can be denoted $P_{a\dot b}$ whose non-vanishing element belongs to the a^{th} column and b^{th} row of the left-lower $n \times n$ matrix. We get the following commutation rules for $IGL(n, C)$

$$[E^a_b, E^c_d] = \delta^c_b E^a_d - \delta^a_d E^c_b \tag{13a}$$

$$\left[E^{\dot a}_{\dot b}, E^{\dot c}_{\dot d}\right] = -\delta^{\dot c}_{\dot b} E^{\dot a}_{\dot d} + \delta^{\dot a}_{\dot d} E^{\dot c}_{\dot b} \tag{13b}$$

$$\left[E^a_b, E^{\dot c}_{\dot d}\right] = 0 \tag{13c}$$

$$[E^a_b, P_{c\dot d}] = -\delta^a_c P_{b\dot d} \tag{13d}$$

$$\left[E^{\dot a}_{\dot b}, P_{c\dot d}\right] = \delta^{\dot a}_{\dot d} P_{c\dot b} \tag{13e}$$

$$[P_{a\dot b}, P_{c\dot d}] = 0\,. \tag{13f}$$

If we are interested in the subgroup, $ISL(n, C)$, we have to suppose that $\sum_a E^a_a$ and $\sum_a E^{\dot a}_{\dot a}$ are zero, but the commutation relations (13) are still valid.

In $SL(n, C)$ there exists an invariant antisymmetric form of n^{th} order which is the well-known Levi-Civita-Ricci tensor $\varepsilon^{ab\ldots f}$. Moreover, given a set of n vectors belonging to the $D(\bar n, n)$ representation $(X_1)_{a\dot m}, (X_2)_{b\dot n} \ldots (X_n)_{f\dot r}$ we can define the following symmetric form

$$\Delta(X_1, X_2, \ldots X_n) = \varepsilon^{ab\ldots f} \varepsilon^{\dot m\dot n\ldots\dot r} (X_1)_{a\dot m} (X_2)_{b\dot n} \ldots (X_n)_{f\dot r}\,. \tag{14}$$

Such a form will allow us to build the invariants of the group $ISL(n, C)$.

2. The Poincaré group

The Poincaré group is well known to physicists in the form where the transformations are realized on the four-dimensional space-time. Its Lie algebra is spanned by the ten generators $M^{\mu\nu}$, P^μ ($\mu, \nu = 0, 1, 2, 3$) with the commutation relations

$$\begin{aligned} [M^{\mu\nu}, M^{\varrho\lambda}] &= i(g^{\nu\varrho} M^{\mu\lambda} - g^{\nu\lambda} M^{\mu\varrho} - g^{\mu\varrho} M^{\nu\lambda} + g^{\mu\lambda} M^{\nu\varrho}) \\ [M^{\mu\nu}, P^\varrho] &= i(g^{\nu\varrho} P^\mu - g^{\mu\varrho} P^\nu) \end{aligned} \tag{15}$$

where $g^{00} = -g^{11} = -g^{22} = -g^{33} = 1$. It is also well known that this Lie algebra has two invariants

$$\begin{aligned} P^2 &= P^\mu P_\mu \\ W^2 &= W^\mu W_\mu \end{aligned} \tag{16}$$

where $W_\mu = \frac{1}{2} \varepsilon_{\mu\nu\varrho\lambda} M^{\varrho\lambda} P^\nu$ and $\varepsilon_{0123} = 1$. The vector W_μ, which is generally called the Pauli-Lubansky polarization vector, is orthogonal to P^μ:

$$W^\mu P_\mu = 0 . \tag{17}$$

Let us now see what is the correspondence between the elements $M^{\mu\nu}$, P^μ and the elements E^a_b, $E^{\dot a}_{\dot b}$, $P_{a\dot b}$ of the algebra of $ISL(2, C)$ as defined in Section 1. It is easy to verify that we obtain the commutation relations (15) by making the identifications

$$\begin{aligned}
M^{23} &= \frac{1}{2}\left(E^1_2 + E^2_1 + E^{\dot 1}_{\dot 2} + E^{\dot 2}_{\dot 1}\right)\\
M^{31} &= -\frac{i}{2}\left(E^1_2 - E^2_1 + E^{\dot 1}_{\dot 2} + E^{\dot 2}_{\dot 1}\right)\\
M^{12} &= \frac{1}{2}\left(E^1_1 - E^2_2 + E^{\dot 1}_{\dot 1} - E^{\dot 2}_{\dot 2}\right)\\
M^{01} &= \frac{i}{2}\left(E^1_2 + E^2_1 - E^{\dot 1}_{\dot 2} - E^{\dot 2}_{\dot 1}\right)\\
M^{02} &= \frac{1}{2}\left(E^1_2 - E^2_1 + E^{\dot 1}_{\dot 2} - E^{\dot 2}_{\dot 1}\right)\\
M^{03} &= \frac{i}{2}\left(E^1_1 - E^2_2 - E^{\dot 1}_{\dot 1} + E^{\dot 2}_{\dot 2}\right)\\
P^0 &= P_{1\dot 1} + P_{2\dot 2}\\
P^1 &= P_{1\dot 2} + P_{2\dot 1}\\
P^2 &= i(P_{1\dot 2} - P_{2\dot 1})\\
P^3 &= P_{1\dot 1} - P_{2\dot 2} .
\end{aligned} \tag{18}$$

In a unitary representation $M^{\mu\nu}$ and P^μ are Hermitian and therefore we find the following hermiticity properties for E^a_b, $E^{\dot a}_{\dot b}$ and $P_{a\dot b}$:

$$\begin{aligned}
(E^a_b)^+ &= E^{\dot a}_{\dot b}\\
(E^{\dot a}_{\dot b})^+ &= E^a_b\\
(P_{a\dot b})^+ &= P_{b\dot a} .
\end{aligned} \tag{19}$$

Now we want to construct the polarization vector $W_{a\dot b}$ corresponding to W_μ. It should be linear in the generators E^a_b, $E^{\dot a}_{\dot b}$ and in $P_{a\dot b}$. Thus it has the form

$$W_{a\dot b} = \alpha E^c_a P_{c\dot b} + \beta E^{\dot c}_{\dot b} P_{a\dot c} . \tag{20}$$

By using the property that $W_{a\dot b}$ commutes with $P_{c\dot d}$ we find that $\alpha = \beta$ and we choose $\alpha = \beta = 1$. The correspondence between W^μ and $W_{a\dot b}$

is the same as between P^μ and $P_{a\dot{b}}$, i.e.,

$$\begin{aligned} W^0 &= W_{1\dot{1}} + W_{2\dot{2}} \\ W^1 &= W_{1\dot{2}} + W_{2\dot{1}} \\ W^2 &= i(W_{1\dot{2}} - W_{2\dot{1}}) \\ W^3 &= W_{1\dot{1}} - W_{2\dot{2}} \,. \end{aligned} \tag{21}$$

What invariants can be built using the two vectors $P_{a\dot{b}}$ and $W_{a\dot{b}}$? Since they must be invariants with respect to $SL(2, C)$ they must be scalars for this group. The only scalar one can form from two vectors $A_{a\dot{b}}$ and $B_{c\dot{d}}$ is $\Delta(A_{a\dot{b}}, B_{c\dot{d}})$, according to Section 1. One easily verifies that

$$\begin{aligned} \Delta(P_{a\dot{b}}, P_{c\dot{d}}) &= P^\mu P_\mu \\ \Delta(P_{a\dot{b}}, W_{c\dot{d}}) &= P^\mu W_\alpha = 0 \\ \Delta(W_{a\dot{b}}, W_{c\dot{d}}) &= W^\mu W_\mu \end{aligned} \tag{22}$$

so that we get in this way all invariants of $ISL(2, C)$ or $\mathscr{P}$ in a compact form.

The little group belonging to a given momentum vector P^μ is defined as that subgroup of $\mathscr{P}$ which leaves P^μ invariant. As is well known one obtains four distinct little groups, namely SO_3 if P^μ is timelike, the homogeneous Lorentz group $L(1, 2)$ in case of spacelike P^μ, the two-dimensional Euclidean group E_2 when P^μ is lightlike and finally $L(1, 3)$ if P^μ is the null vector. For the group $ISL(2, C)$ we expect, of course, little groups which should be the covering groups of SO_3, $L(1, 2)$ and E_2. In fact the little group belonging to the momentum matrix $\hat{P} \equiv \{P_{a\dot{b}}\}$ is that subgroup of $SL(2, C)$ which satisfies

$$\Lambda \hat{P} \Lambda^+ = \hat{P} \,. \tag{23}$$

Since $\hat{P}$ is Hermitian it can be diagonalized and furthermore one can always assume that the eigenvalues are $\pm m$ or 0 by applying a transformation of $SL(2, C)$. Therefore one sees that the little groups can be classified with three numbers p, q, r, namely the number of eigenvalues $+m$, $-m$ and 0, respectively. Let us call the corresponding groups $SU(p, q; r)$. Then one has the Table

det $\hat{P}$	little group
>0	$SU(2, 0; 0) \approx SU(0, 2; 0) \sim SO_3$
<0	$SU(1, 1; 0) \sim L(1, 2)$
$=0$	$SU(1, 0; 1) \approx SU(0, 1; 1) \sim E_2$
	$SU(0, 0; 2) \approx SL(2, C) \sim L(1, 3)$

Here $\approx$ denotes isomorphic and $\sim$ locally isomorphic.

3. Invariants and little groups of $ISL(n, C)$

The generalization from $ISL(2, C)$ to $ISL(n, C)$ is straight-forward. The vector

$$W_{a\dot{b}} = E_a^c P_{c\dot{b}} + E_{\dot{b}}^{\dot{c}} P_{a\dot{c}} \tag{24}$$

commutes with all translation operators $P_{c\dot{d}}$. Therefore, by forming scalars of $W_{a\dot{b}}$ and $P_{c\dot{d}}$ we obviously obtain invariants of the whole group. According to Section 1 we can construct $n+1$ scalars

$$\begin{aligned} &\Delta(P_{a\dot{b}}, P_{c\dot{d}}, \ldots, P_{g\dot{h}}) \\ &\Delta(W_{a\dot{b}}, P_{c\dot{d}}, \ldots, P_{g\dot{h}}) \\ &\Delta(W_{a\dot{b}}, W_{c\dot{d}}, P_{e\dot{f}}, \ldots, P_{g\dot{h}}) \\ &\vdots \\ &\Delta(W_{a\dot{b}}, W_{c\dot{d}}, \ldots, W_{g\dot{h}}) \end{aligned} \tag{25}$$

involving, respectively, n P's, $(n-1)$ P's and one $W, \ldots$ and so on up to n W's. Due to the antisymmetry of the invariant tensor $\varepsilon^{ab\ldots f}$ and to the constraints $\sum_a E_a^a = 0$, $\sum_a E_{\dot{a}}^{\dot{a}} = 0$ one finds that the second invariant involving one W is identically zero.

The little groups of $ISL(n, C)$ are the subgroups of elements Λ_P which fulfil

$$\Lambda_P \hat{P} \Lambda_P^{+} = \hat{P}\,, \tag{26}$$

where $\hat{P}$ is the matrix $\{P_{a\dot{b}}\}$. Just as in the case $ISL(2, C)$ we may assume that P is diagonal and that its eigenvalues are $\pm m$ or 0. If $\hat{P}$ has p eigenvalues $+m$, q eigenvalues $-m$ and r eigenvalues 0 we denote the corresponding little group by $SU(p, q; r)$. For $p = n$, $q = r = 0$ we get $SU(n)$ as a little group. In this case we can check that the number n of invariants equals the number of generators minus twice the number of commuting operators which are not invariants*.

In the case of $ISL(6, C)$ we get 16 non-isomorphic little groups. Clearly $SU(6)$ corresponds to the case $p = 6, q = r = 0$. This can also be seen in a formal way by replacing $P_{a\dot{b}}$ by $m\delta_{a\dot{b}}$ in the commutation relations for $W_{a\dot{b}}$:

$$[W_{a\dot{b}}, W_{c\dot{d}}] = W_{c\dot{b}} P_{a\dot{d}} - W_{a\dot{d}} P_{c\dot{b}}\,. \tag{27}$$

Then the $W_{a\dot{b}}$ satisfy the commutation relations of $SU(6)$ if dotted indices are understood to be contravariant indices of this group.

A vector belonging to the representation $D(\bar{6}, 6)$ splits into $\{1\} \oplus \{35\}$ with respect to $SU(6)$. Such a decomposition generalizes that of a four vector into a scalar and a spatial vector in Minkowski space. It might be interesting to consider the usual space-time as embedded in a 36-dimen-

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* This formula is implicitly given in Ref. [10], p. 52.

sional space; in such a case, many questions arise: what is the topology of the generalized light cone (which is now of the 6^{th} order but reduces in the "Minkowski subspace" to a quadratic cone)?

Such an investigation could lead to a new definition of the $SL(6, C)$ involving the Minkowski space and the eightfold way description of internal symmetry rather than one based on the covering group which is not very familiar to physicists.

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