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Optical Fields in a Multilayer Microsphere with a Quasi-periodic Pascal Sequence

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Abstract

We studied numerically the frequency spectrum of photons in a multilayered microsphere coated by a quasiperiodic (Pascal) dielectric stack. It is found that the transmittancy spectrum of such a stack consists of quasiband gaps and narrow resonances caused by re-reflection of optical waves. When the number (Pascal order) of layers increases, the band gaps and resonances split, and the structure of the frequency spectrum acquires a fractal form. Some parts of the spectrum show the self-similarity in the different frequency scales.

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Keywords: Coated microsphere, quasiperiodic multilayered stack, Pascal layers, numerical calculations.

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1 Introduction

The developments in the technologies of manufacture of nanostructures and microspheres, it gives the possibility of studying systems of multilayers with all kinds of arrangements, which can derive in a great quantity in photonic and optoelectronic applications. [1], [2], [3], [4], [5].

The design of photonic crystals with periodic structures gives the possibility to obtain complete and absolute photonic band gap for optical radiation. Moreover, the location of light in disordered systems offers the possibility of a wide range of applications in optoelectronic applications [6], [7], [8].

In the quasiperiodic systems are observed properties that the periodic and random systems do not have, In [9] there are studied the electronic conditions of GaAs and A1As Fibonacci heterostructures grown along the (001) direction. One finds wide bands with different spatial location in ranges of energy. In some works, systems are studied by structures that follow a Singer's quasiperiodic succession, in [10], [11] there finds direct evidence of the location of conditions and of selfsimilarity in a system artificial compounds of piezoelectric.

It is known that in the systems of microspheres with layers $\lambda/4$ the light is caught by more time therefore, such a system can be regarded as a $3D$ photonic crystal. In such a microsphere, only the spectrum of narrow resonances (eigenfrequencies) can be observed as peaks in the frequency spectrum of radiation. Therefore, it is of great interest to investigate how these features are modified in a spherical stack with a structure quasiperiodic.

In this paper, we investigate the optical properties of a layered microsphere with a dielectric stack, in which optical layers are constructed following the Pascal sequence. We investigate the coefficient of transmission T as function of the order of Pascal and as function of the spherical number m.

This paper is organized as follows. In Section 2, we define the succession of Pascal and expose the basic ideas to calculate with the method of the transfer matrix, the coefficient of transmission T of our system dielectric microsphere coated by a multilayered stack. In Section 3, we outline the scheme of applying the STM technique for calculating the frequency transmittancy spectrum of a Pascal stack. In Section 4, we summarize our results.

2 Basic Equations

it is well-known that the Pascal (triangle) numbers are formed as follows. In the zero line is 1. The next line is $1,1$, and the second line is formed by adding $0+1=1$, as the first term of the line, add $1+2=3$ as the second term of the line, the third term of the line is adding $2+1=3$ and the fourth term is sum $1+0=1$. The higher items are constructed in the same way. The Pascal numbers can also be formed by the binomial coefficients, as follows: In the zero line is 1. In line one placed the coefficients of $(a+b)^1 = a+b$ which are $1,1$. In second line are placed the numerical coefficients of $(a+b)^2 = a^2+2ab+b^2$, which are $1,2,1$, and so on.

In order to study the optical properties of a spherical Pascal stack, let us first formulate the transfer matrix method. We exploit a spherical multilayered geometry and use the

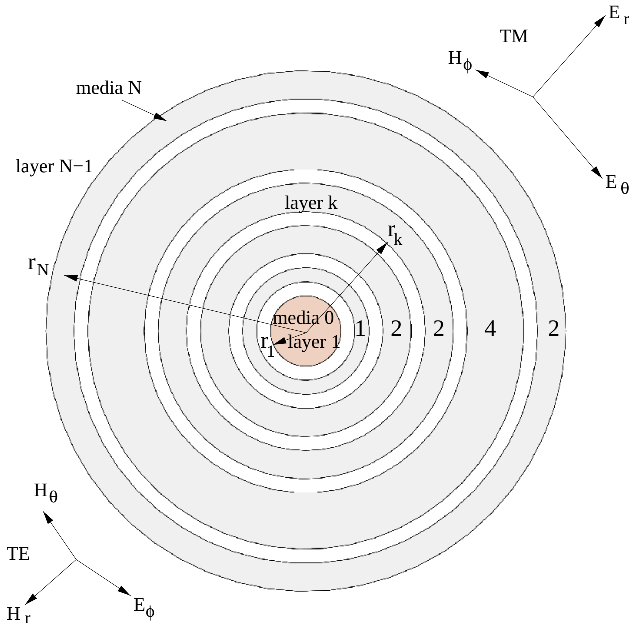

Figure 1. Geometry of a multilayered microsphere. A stack of quasiperiodic multilayers is deposited on the surface of a microsphere.

formalism developed in references [12] and [13]. In the case of multilayer microspheres, Maxwell's equations are:

$$\nabla\times\overrightarrow{H} = i\omega\varepsilon_0\varepsilon(\omega)\overrightarrow{E},\ \ \nabla\times\overrightarrow{E} = -i\omega\overrightarrow{B}, \tag{2.1}$$

where $\overrightarrow{E}$ and $\overrightarrow{B}$ are electric and magnetic fields, and $\varepsilon(\omega)$ is a dielectric permittivity of a layer. We use the complex exponential multiplier in the form $\exp(i\omega t)$. These equations (2.1) in spherical coordinates, are reduced to the Helmholtz equation for a scalar function called the Debye potential $\Pi(\rho,\theta,\varphi)$ [14], [15]. The equation for the radial part Debye-potential $\Pi = \Pi(r)$ can be solved in terms of the spherical Hankel functions. It is possible for each layer to obtain a matrix representation, denoted by a subscript. With the right conditions between layers (see [12]) is possible to obtain a relationship for the transmission matrix between the inner and outer layers in the spherical stack. Using the Sommerfeld's radiation conditions is easy to obtain the coefficient R and T, Eq.(15) in Ref. [12]. We use this equation to calculate the reflectance, transmittance for structure of the spherical stack. In this paper we exploit such a technique for a quasiperiodic (Pascal) spherical stack.

Our $1D$ quasiperiodic (QP) structures is the spherical stack, where the Pascal sequence is formed by blocks L which in turn are formed by two blocks of different materials. In each block that is $L = L(B,C)$, the width of the layer B does not change, while the width of the layer C changes according to the following sequence of Pascal: $1-2-2-4-2-4-4$, and so on. The numbers 1, 2, 2, 4, etc, are the quantity of the odd numbers in row 0, 1, 2, 3, etc. We say that when L is L_1 layer is formed by materials B and C of the same width. L_2 is formed by layers where the width of the layer C is twice the width of the layer B. For

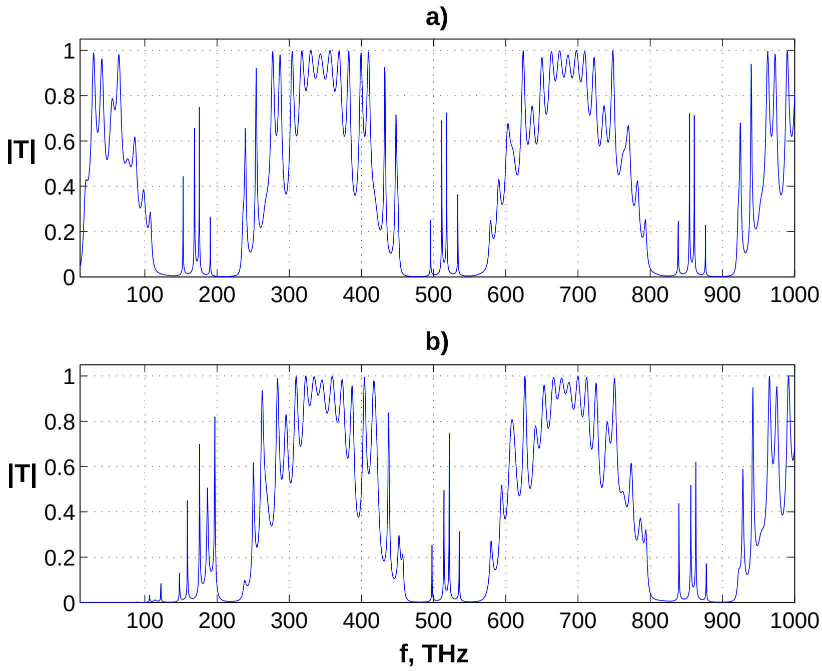

Figure 2. Frequency spectrum of the transmittance coefficient $|T|$ for the cases (a) $m = 1$, and (b) $m = 6$. The microsphere is coated by a Pascal stack with $N = 18$ (9 two-layers blocks, Pascal order R_8).

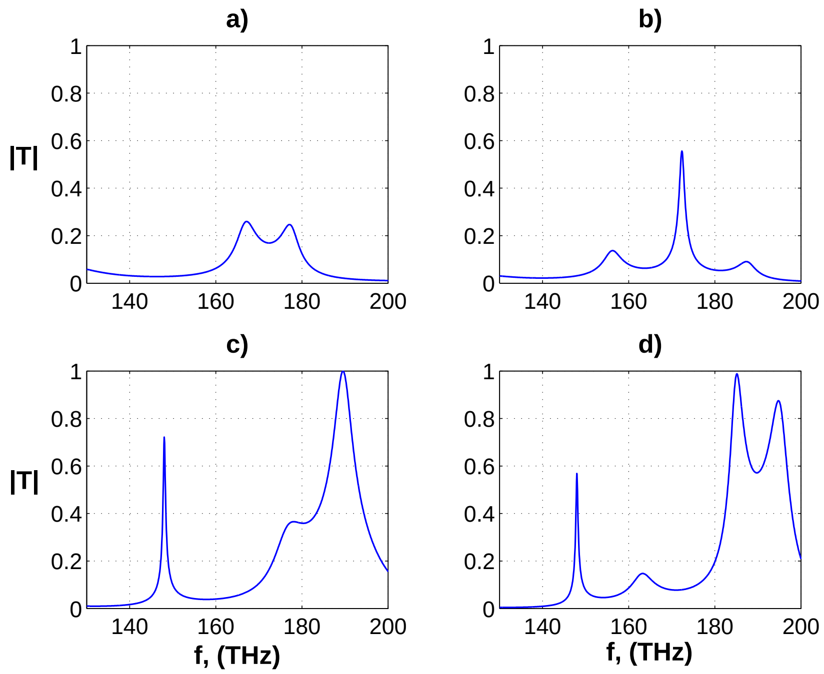

Figure 3. Frequency spectrum of the transmittence coefficient $|T|$ for (a),(b) $m = 1$, and (c),(d) $m = 6$. Pascal order is in (a),(c) R_5, and (b),(d) R_6, that corresponds to $N = 12$ (6 two-layers blocks one more layer of SiO_2) and $N = 14$ (7 two-layers blocks) in the stack accordingly.

example, an L_4 implies that the thickness of the layer C of L is 4 times thicker than the layer B. Thus, we can form a sequence of Pascal as follows: $L_1L_2L_2L_4L_2L_4\cdots$, see Fig. 1. We define the order of the sequence of Pascal as follows: It is said that the order of Pascal is R_0 if the sequence is L_1, the order is R_1 if the sequence is L_1L_2, the order is R_2 if the sequence is $L_1L_2L_2$. For example, R_9 has 9 two-layers blocks, which are $L_1L_2L_2L_4L_2L_4L_4L_8L_2$.

The study of the transmission coefficients in the spherical stack is too difficult problem to solve it analytically. Therefore, in what follows we present the numerical investigation of the frequency spectrum of the transmittance coefficient T.

3 Numerical Results

The following parameters have been used in our calculations: the geometry of the system is $A\{L_1(B,C)L_2(B,C)L_2(B,C)L_4(B,C)L_4(B,C)...\}D$, where the letters A, B, C, D indicate the materials of layers in the spherical stack, respectively. The bottom microsphere has a refraction index $n_A = 1.5 + 2\cdot10^{-4}i$ (A, glass, radius 1000nm). The refractive index of the materials B is $n_B = 3.58 + 9\cdot10^{-4}i$ (Si, width 122nm), the refractive index of the materials C is $n_C = 1.46 + 10^{-3}i$ (SiO_2, and the thickness of the layer changes according to the Pascal rule mentioned above) and $n_D = 1$ (D, surrounding space). For L-block layers B is constructed as $\lambda/4$ layers. To consider the realistic layers case, we have added a small

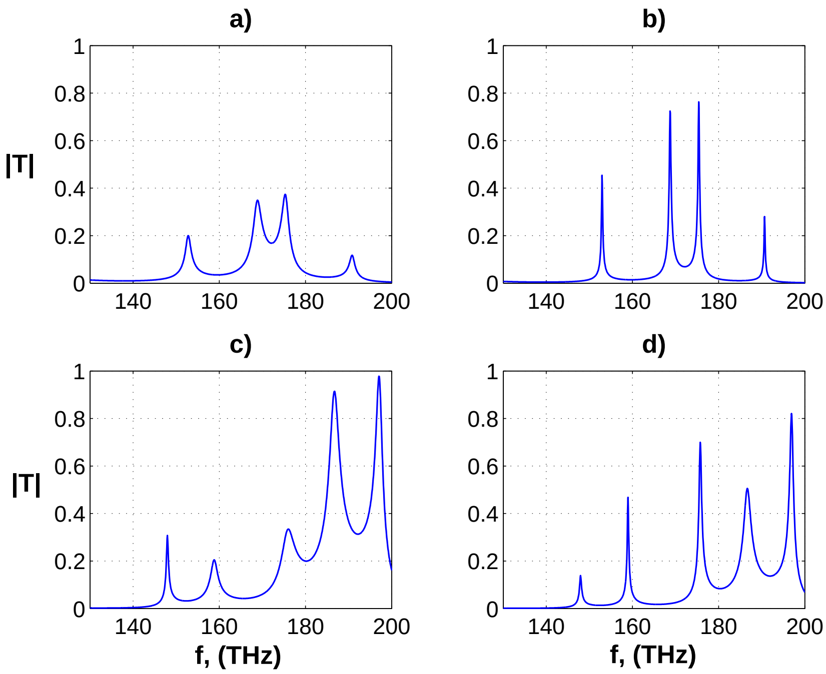

Figure 4. The same as in Fig.3, except Pascal order is in (a), (c) R_7 and (b), (d) R_8, that corresponds to $N = 16$ (8 two-layers blocks) and $N = 18$ (9 two-layers blocks) in the stack accordingly.

imaginary part to each n_i , that corresponds to a material dissipation. We notice that even in a material lossless case in such a system (open system), there are losses due to leakage of the field into the surrounding space [16].

In a number of works, it was found that in a plane optical stack, constructed following the Pascal sequence, the transmission coefficient T has a rich structure as a function of the frequency of light and, in fact, is multifractal [17], [18], [19]. Re-reflections of light from the layers interfaces in a Fibonacci lattice leading to narrow resonances, separated by numerous pseudo band gaps, which reflects the multifractal nature of such a structure. Therefore, we first study how such features are modified in the spherical Fibonacci stack due to radial dependence of the transfer matrix. It is also of interest to calculate the frequency spectrum of the transmittance coefficient T for different values of spherical quantum number m.

We summarize our results in Figs.2 - 6. In the Fig.2 we present the transmission spectrum for a spherical multilayer sequence. We have used the frequency range [10 – 1000] THz ([30000 – 300] nm) and 18-layers stack (9 two-layers blocks, the Pascal order R_8), spherical quantum number $m = 1$ and 7. One can see that for $m = 1$ the frequency spectrum T (Fig.2(a)) has a nearly perfect mirror symmetry for some center frequencies in the bands allowed: the mirror symmetry at $f = 343$ THz and at $f = 686$ THz. The same symmetry can be seen for some frequency in the bandgap: at $f = 172$ THz, $f = 514$ THz and at $f = 858$ THz. However, as m increases, the symmetric distribution of the transmittance coefficient is

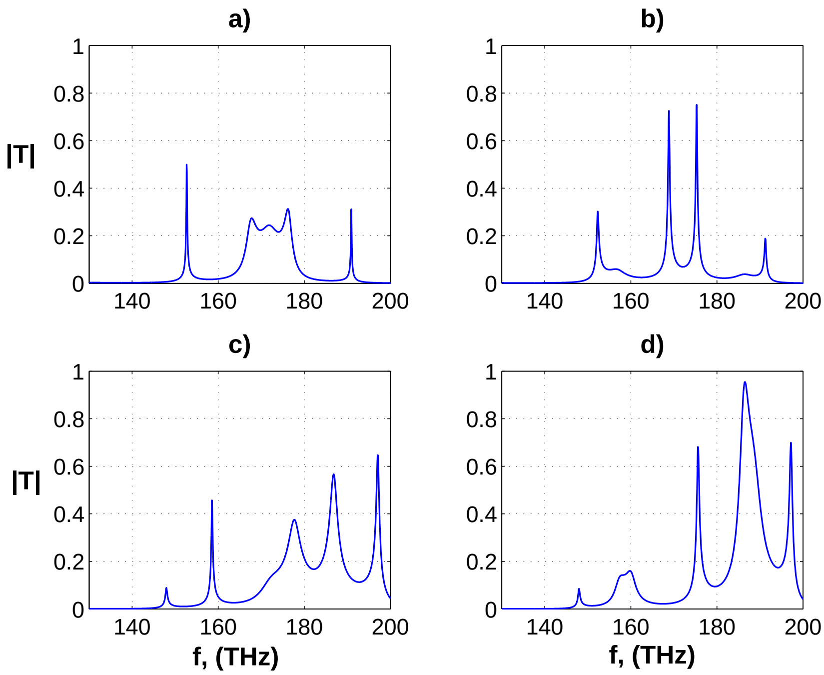

Figure 5. The same as in Fig.3, except Fibonacci order is in (a), (c) R_9 and (b), (d) R_{10}, that corresponds to $N = 10$ (20 two-layers blocks) and $N = 11$ (22 two-layers blocks) in the stack accordingly.

broken, see the above frequencies in Fig.2 (b), where perfect symmetry is no longer either a broken. A zone of small transmittance with $|T| << 1$ emerges (corresponding whispering gallery mode -WGM) in the area of low frequencies ~ 200 THz. In such a regime, the bandgaps structure is deformed or even destroyed.

The structure of the transmittancy spectrum becomes quite irregular with increase of the Pascal order R_j. To see more details, we pay further attention to a narrower frequency range $(130-200)$ THz or $(2307-1500)$ nm. Results are shown in Figs.3 - 5.

Fig.3 shows that a small Pascal order R_6 Fig.3 (a) the structure of the transmittance coefficient $|T|$ is soft or depends rather smoothly on the light frequency f.

With further increase of the Pascal order R_j, the structure of $|T|$ becomes more indented.A new maxima (and minima) reshape the initially smooth form of the transmittancy spectrum to a well expressed fractal structure, see Fig.3 (b), Fig.4 (a), Fig.4 (b), Fig.5 (a) and Fig.5 (b). All these figures correspond to a spherical number $m = 1$, and Pascal order R_6, R_7, R_8, R_9 and R_{10}, respectively. Similar behavior is seen in the spectra of the same figures, but with spherical number $m = 6$. You can see from these spectra the appearance of new highs and lows, but the spectrum already does not acquire a symmetrical form with respect to the frequency resonances.

In the case of the Fibonacci sequence [12] it was shown that when the number of two-layers blocks in the stack (Fibonacci order F_N) increases, even smooth parts of the spectrum acquire a fractal shape. In this paper, we show that for Pascal number case such selfsimi-

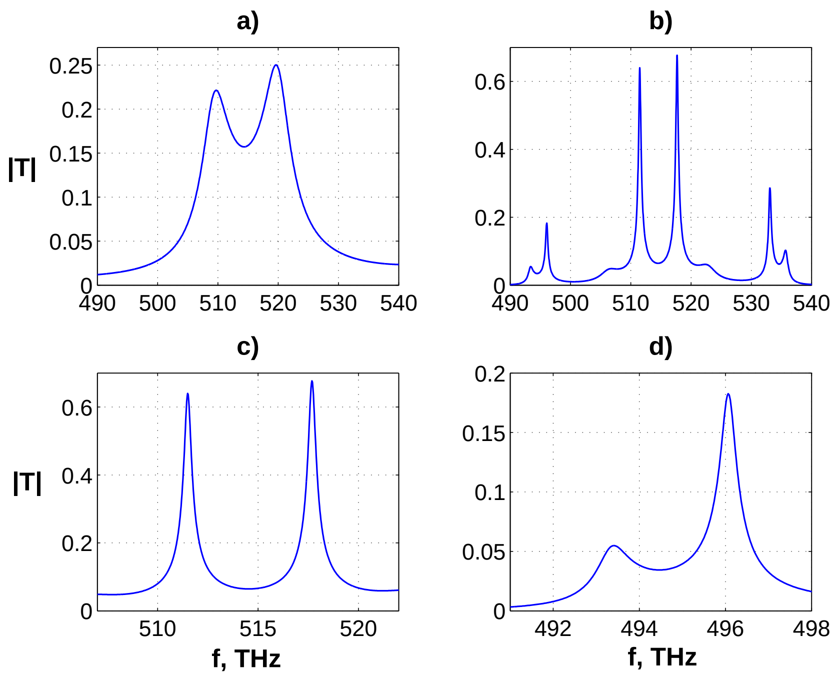

Figure 6. In the cases (a) show the transmittence coefficient $|T|$ for R_5 and $m = 1$, that corresponds to $N = 12$ (6 two-layers blocks). In cases (b), (c) and (d) show $|T|$ for R_{12} and $m = 1$, that corresponds to $N = 22$ (12 two-layers blocks.

larity still exists but it is not so clear expressed. In order to check this, in Fig.6 we compare the scaled transmittancy spectrum R_5 and spectrum R_{12}.

Note that the scale is the same in Fig.6 (a) and Fig.6 (b), but in the Fig.6 (a) the Pascal order is R_5 and in the Fig.6 (b) is R_{12}.

Further we present the extension in the Fig.6 (a) to show the details of the transmittance spectrum in different frequency scale changes. Notice that in the Fig.6 (c) and (d) the spectrum is similar to that in the Fig.6 (a) but for different frequencies. This result allows us to affirm that the self-similarity in the spectra in Fig.6 (a) and (b) still can be observed.

4 Conclusion

We have studied the photonfrequency spectrum in a multilayered microsphere coated by a quasiperiodic dielectric stack, constructed following quasiperiodic (Pascal) sequence. It is found that the frequency spectrum of the transmittance coefficient of such a stack consists of quasiband gaps and narrow resonances in the bandgap. When the number of layers in the stack (Pascal order) increases, the structure of the transmittance spectrum becomes more complicated and the peaks in the forbidden band become narrower due to re-reflection of light from total quasiperiodic structure. When spherical quantum number m increases, the symmetric distribution of the transmittance coefficient is broken. The self-similarity in transmission spectra still exists, however for large Pascal number orders it has less expressed shape than in the Fibonacci number case.

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