3. D. Colton and R. Kress, *Integral equation methods in scattering theory*, John Wiley, N. Y., 1983.

4. C. L. Dolph, *The integral equation method in scattering theory*, Problems in Analysis (R. C. Gunning, ed.), Princeton Univ. Press, N. J., 1970, pp. 201–227.

5. D. S. Jones, *Acoustic and electromagnetic waves*, Clarendon Press, Oxford, 1986.

6. J. B. Keller, *Rays, waves and asymptotics*, Bull. Amer. Math. Soc. **84** (1978), 727–750.

7. R. E. Kleinman and G. F. Roach, *Boundary integral equations for the three dimensional Helmholtz equation*, SIAM Review **16** (1974), 214–236.

8. P. D. Lax and R. S. Phillips, *Scattering theory*, Academic Press, N. Y., 1967.

9. R. G. Newton, *Scattering theory of waves and particles*, McGraw-Hill, N. Y., 1966.

10. M. Reed and B. Simon, *Scattering theory*, Academic Press, N. Y., 1979.

11. B. D. Sleeman, *The inverse problem of acoustic scattering*, IMA J. Appl. Math. **29** (1982), 113–142.

12. C. H. Wilcox, *Scattering theory for the d'Alembert equation in exterior domains*, Lecture Notes in Math., vol. 442, Springer-Verlag, Berlin and New York, 1975.

DAVID L. COLTON

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 2, April 1987

0273-0979/87 $1.00 + $.25 per page

Eigenvalues in Riemannian geometry, by Isaac Chavel, with a chapter by Burton Randol and an appendix by Jozef Dodziuk, Academic Press, Inc. (Harcourt Brace Jovanovich, Publishers), Orlando, San Diego, New York, London, Toronto, Montreal, Sydney, Tokyo, 1984, xiv + 362 pp., $62.00. ISBN 0-12-170640-0

The recent explosion of activity studying the relation between geometric and analytic properties of spaces has fused many areas of mathematics, such as the traditionally disparate fields differential geometry, partial differential equations, topology, mathematical physics, and number theory. One of the most popular topics in this study is the search for properties of the spectrum of the Laplace operator of a manifold in terms of its geometric invariants. Until recent decades there have been few significant developments, owing to the need for expertise in many fields. Eigenvalue problems are directly related to many geometric problems as well as to the disciplines mentioned above. Moreover, the techniques that have been developed in studying the Laplacian and its spectrum are equally important as the theorems about eigenvalues. This versatility factor coupled with the recent undeniable success of geometric analysis is responsible for the sudden blossoming of this classical area of mathematics.

The most fundamental object of study is the Laplace-Beltrami operator. Being invariantly defined, it is the simplest geometric elliptic operator which appears everywhere in geometry. It is the principal part of the expression for scalar curvature of a conformal factor in a metric as well as the mean curvature and stability form of a hypersurface. More importantly it is the linearization of the many nonlinear operators in geometry such as the Gauss curvature operator, the mean curvature operator and the Monge-Ampère operator. It is essential, therefore, to the understanding of nonlinear phenomena as well as the operator to study in the first model problems.

The early results were summarized fifteen years ago by Berger, Gauduchon, and Mazets in *Le spectre d'une variété riemannienne*,[1] which at that time well represented the literature on the "geometry of the Laplace operator." Growing out of a set of lectures from the late sixties, the purpose of these notes was to acquaint graduate students or newcomers to the field of eigenvalues on manifolds with the basic background and results. One of the high points was the asymptotic expansion for the heat kernel which enabled one to read off geometric invariants of the manifold such as the volume and integral of scalar curvature from the "high-end" behavior of the spectrum. There were also a few scattered results on the first nonzero eigenvalue estimates. However, with the currently vigorous activity, it is high time for another overview of newer results and, perhaps more importantly, of the techniques from the field. To this end, Isaac Chavel offers his *Eigenvalues in Riemannian geometry*.

From his very particular vantage point, Chavel gives his summation of what is known about the interplay of the spectrum of the Laplacian and geometry in the book *Eigenvalues in Riemannian geometry*. The principal focus of this book is on the relationship between the lower spectrum (as opposed to asymptotics of the spectrum) and the geometry of the manifold. A substantial portion of this theory was developed in the last two decades and hence Chavel's book can be viewed as an organized (and desperately needed) update on the field. As well as being a modern and extensive list of theorems about eigenvalues, the book also covers sufficient (yet minimal) background material in geometry and elliptic PDEs so that it can be used as a graduate text.

The book serves as an excellent reference for areas and techniques which the author favors. However, beginners may find the presentation directionless and consisting of an assembly of isolated theorems. It would have been beneficial to the readers if more geometric results via eigenvalues were discussed. Undoubtedly, efforts to present another viewpoint on the subject will be made.

PETER LI

[1] An updated bibliography has been compiled by P. Bérard and M. Berger, and published as an appendix in *Spectral geometry*: *Direct and inverse problems* by P. Bérard.

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 2, April 1987

0273-0979/87 $1.00 + $.25 per page

Equilibrium capillary surfaces, by Robert Finn, Grundlehren der mathematischen Wissenschaften, vol. 284, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1986, xi + 245 pp., $57.00. ISBN 0-387-96174-7

Robert Finn in the preface to his book writes:

"Capillarity phenomena are all about us; anyone who has seen a drop of dew on a plant leaf or the spray from a waterfall has observed them. Apart