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STEPHEN A. SAXON
UNIVERSITY OF FLORIDA

Encyclopaedia of mathematical sciences, vol. 7, Several complex variables I (A. G. Vitushkin, ed.), Springer-Verlag, New York, Berlin, Heidelberg, 1990, 241 pp., $59.00. ISBN 0-387-17004-9

The present volume is the first of four devoted to the theory of functions of several complex variables by the Soviet encyclopaedia. This section of the encyclopaedia appears under the general editorship of A. G. Vitushkin; each of the four volumes consists of several articles written, with two exceptions by leading Soviet experts. The whole project is massive, totalling about a thousand printed pages, but for the most part the articles are not detailed expositions of their subject, being instead summary outlines of their subjects with rather full commentary but generally without proofs. These four volumes are convincing evidence of the great development seen by multidimensional function theory in the postwar era.

The first volume, the volume under review, is devoted to mainly analytic topics as opposed, say, to the theory of coherent sheaves or the relations of function theory with algebraic geometry. For these subjects, see subsequent volumes. In this volume, we find an introductory essay entitled "Remarkable Facts of Complex Analysis" by Vitushkin, which gives a brief overview of the contents of all four of the volumes. This is followed by articles by G. M. Khenkin on integral formulas in complex analysis, by E. M. Chirka on complex analytic sets, by Vitushkin on the geometry of hypersurfaces and by P. Dolbeault, on the theory of residues in several variables.

Vitushkin's introductory article is written in a style that is accessible to a broad variety of mathematicians. At the beginning

he assumes no particular knowledge of several complex variables, and he discusses in a descriptive way some of the basic aspects of the subject such as the Hartogs continuation phenomenon: The simplest manifestation of this idea is that functions holomorphic outside a ball in C^N, $N \geq 2$, continue holomorphically through the ball. Along the way Vitushkin mentions certain outstanding problems in the subject, e.g., the corona problem: Given bounded holomorphic functions f_j, $j = 1, \ldots, k$, on the unit ball in C^N with $\sum_{j=1}^{k} |f_j| \geq \delta > 0$ everywhere on the ball, do there exist functions g_j, $j = 1, \ldots, k$, with $\sum_{j=1}^{k} f_j g_j = 1$? That the desired functions exist in the case that $N = 1$ is a celebrated theorem of Lennart Carleson [2], but so far in C^N, with $N \geq 2$, no one has been able to prove the result on the ball or on any other domain. Counterexamples are known in the case of smoothly bounded domains in C^N that are strongly pseudoconvex except at a single boundary point [6]. This introductory essay serves to inform the reader of the content of the four volumes that follow, and it could also serve to inform a mathematician whose expertise is in a direction other than complex analysis of some of the directions currently important in function theory.

   Khenkin's is the longest of the articles in the volume and is devoted to the theory of integral formulas in multidimensional complex analysis. This is a much richer theory than the corresponding theory of the Cauchy integral in one variable. The richness grows in part from the wider variety of analytic objects one naturally tries to represent by integral formulas and in part from the vast number of quite different formulas that are available. In classical one-dimensional complex analysis the only integral formula used was the Cauchy integral formula, which represents holomorphic functions. In the higher-dimensional theory, integral formulas are used to represent not only functions but also differential forms of arbitrary bidegree, sections of vector bundles, and so on. Moreover, instead of a single integral formula, many different formulas are available, so it is probably better not to think in terms of particular integral formulas but rather to think of the general method of integral formulas, as Khenkin suggests in the title of his article. The subject has evolved to the point where the method is well established and available as a flexible tool for use in attacking problems of many kinds. This is not to suggest that the method is especially simple; the formulas are complicated in many cases.

Often the formulas in Khenkin's article do not fit on a single printed line of text. In spite of the complexity, there are general methods in the theory. One of these is the method of the Bergman kernel, which constructs integral formulas by abstract Hilbert space methods. This method is elegant and conceptually straightforward, but a price is to be paid for the conceptual simplicity: The formulas obtained by this method do not provide refined estimates in any simple way. An alternative method for constructing integral formulas derives from the systematic application of Stokes's theorem. This leads to the Cauchy-Fantappiè formula, which must be one of the most audacious formulas in all of mathematics; it gives the impression of creating something from nothing. Most of the integral formulas in complex analysis, including many of those obtained by the method of the Bergman kernel, derive from the Cauchy-Fantappiè formula by repeated uses of Stokes's formula. Moreover, it is sufficiently explicit to permit good estimates for the solutions of function-theoretically significant problems. In Khenkin's article, one finds many of these explicit formulas together with a fair sampling of their applications.

Chirka's paper is devoted to the theory of analytic sets, considered from a generally geometric point of view. The theory of analytic sets is the study of the zero sets of holomorphic functions. In one dimension, these sets are discrete subsets of planar domains or of Riemann surfaces, and their is little to say about their structure or geometry. In the higher-dimensional case, the zero sets are much more complicated. In the simplest case, they are complex manifolds, but generally they have singular points, points near which they are not locally manifolds. Substantial books have been devoted to the study of these sets and their globalizations, the complex analytic spaces. We stress the geometric nature of the theory presented by Chirka to differentiate it from the more algebraic development of the theory of analytic sets that can be given and that is presented in the books of Gunning and Rossi [5] and Abhyankar [1]. The topics discussed here go well beyond the usual elementary treatments found in many textbooks and include, e.g., an outline of multiplicity theory and intersection theory. There is also a development of the metric aspects of the theory of analytic sets. This theory is based above all on an inequality of Wirtinger, from which it follows that analytic varieties have locally finite volume. For nonsingular varieties, this is clear, but at points of the singular set, it is not at all clear. The local finiteness of volume gives

the possibility of integrating differential forms over analytic sets, and this integration process is an essential tool in many geometric considerations. At a somewhat deeper level, one finds in this article a discussion of the work that has been done to characterize holomorphic chains among the currents on a complex manifold $\mathcal{M}$. Given a current of appropriate dimension, when is it of the form $\sum_j n_j[V_j]$ in which the n_j are integers and $[V_j]$ denotes the current of integration over the subvariety V_j of $\mathcal{M}$? This is not an easy problem, but definitive answers have been given in recent years in the work of King, Harvey, and Shiffman. In the final part of his article, Chirka addresses the boundary theory of analytic varieties and suggests this as a promising direction for further research, a sentiment with which the reviewer heartily concurs. The deepest work obtained in this general direction so far is that of Harvey and Lawson, which characterizes those odd-dimensional submanifolds of C^N that are boundaries of complex varieties. In the case of one-dimensional varieties, results in this direction are found already in Wermer's work of the late 1950s; the higher-dimensional case appeared in the early 1970s. A fuller treatment of the theory of analytic sets from the point of view espoused by Chirka is given in his book [4].

The next article, by Vitushkin, is of a somewhat different character from the others in the volume in that it contains detailed proofs, sometimes rather elaborate, of some of the important results it discusses. The general subject is the geometry and mapping theory of real hypersurfaces in C^N, a direction that has flowered in the last twenty years. There are certain broad subdivisions within the subject. One of these is the division between smooth and real-analytic hypersurfaces. In the case of real-analytic hypersurfaces one has available techniques that lead to refined results, analogues of which do not exist in the smooth case. Another of the natural divisions is that between the strongly pseudoconvex hypersurfaces and those that are not strongly pseudoconvex. (A smooth hypersurface is *strongly pseudoconvex* if at each of its points, it is possible to choose local holomorphic coordinates in such a way that with respect to these coordinates, the hypersurface is strictly convex.) A third division that appears among the strongly pseudoconvex hypersurfaces is that between *spherical* and *nonspherical* hypersurfaces. The spherical hypersurfaces are the ones that are locally equivalent to the standard unit sphere in C^N. One of the principal problems considered in Vitushkin's article is the problem

of local classification: Given two real hypersurfaces, $\mathcal{M}$ and $\mathcal{M}'$ and given points $p \in \mathcal{M}$ and $p' \in \mathcal{M}'$, under what conditions does there exist an isomorphism of a neighborhood of p in $\mathcal{M}$ onto a neighborhood of p' in $\mathcal{M}'$? *Isomorphism* here is to be understood in the sense of isomorphism of CR-manifolds. In the real-analytic case, the maps in question are those that extend holomorphically to a neighborhood in the ambient C^N of the point p. Although results on this problem were found by Poincaré and E. Cartan, the contemporary basis for the work lies in a fundamental paper by Chern and Moser [3]. Vitushkin gives a detailed exposition of the results obtained in this classification; the exposition incorporates certain improvements and extensions obtained in his Moscow seminar. This theory provides a normal form for embedded hypersurfaces, which renders possible the study of a variety of local and global questions. For example, using it, one can study the groups of both the local and the global automorphisms of the surface, and, as a result of this study, it is now known that the automorphism group of a nonspherical, strictly convex hypersurface is finite. One of the most striking results in the study of embedded hypersurfaces is a theorem about the analytic continuation of biholomorphic maps, which goes back to work of Pinchuk:

Theorem. *Let D and D' be strictly pseudoconvex domains in C^N with nonspherical real-analytic boundaries. Let $p \in bD$ and $p' \in bD'$. If φ is a biholomorphic map from a neighborhood U of p in C^N onto a neighborhood of p' that carries $bD \cap U$ into bD', then φ continues analytically along every path in bD. If, in addition, D is simply connected, then φ gives rise to a covering map from D onto D'.*

This paper of Vitushkin is an updated version of the earlier paper [8].

The final article in the volume is by Pierre Dolbeault and is dedicated to the theory of residues. This theory is based in good measure on topology; the material Dolbeault presents might well be called the *cohomological* theory of residues. The article is only about twenty pages long, but it gives a resumé of residue theory as developed initially by Leray and elaborated by subsequent mathematicians. In addition to the Leray theory, one finds here the theory of the Grothendieck residue symbol and some of the work of Griffiths, which relates residue theory to algebraic geometry.

The theory of residues as presented here is in many ways a natural development in the higher-dimensional case of the Cauchy theory of residues in one variable. Much of it proceeds in quite straightforward ways, but, at certain critical junctures, the theory as it now stands makes essential appeal to the very deep work of Hironaka on the resolution of singularities. In each instance this appeal is natural, but the question does arise: Would it be possible to construct residue theory without recourse to the resolution of singularities? It seems that an attempt to do so would be a worthy undertaking. Another view, in much more detailed presentation, of residue theory is given in the recent work of Tsikh [7].

In sum, the volume under review is the first quarter of an important work that surveys an active branch of modern mathematics. Some of the individual articles are reminiscent in style of the early volumes of the first *Ergebnisse* series and will probably prove to be equally useful as a reference; all contain substantial lists of references. Except for Vitushkin's introduction, it is unlikely that these articles will be of much use to anyone without a substantial background in complex analysis, but for the appropriate reader, they will be valuable sources of information about modern complex analysis.

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EDGAR LEE STOUT
THE UNIVERSITY OF WASHINGTON