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Fibrewise topology, by I. M. James. Cambridge University Press, Cambridge, New York, New Rochelle, Melbourne, Sydney, 1989, x+198 pp., $49.50. ISBN 0-521-36090-0

Fibrewise topology can be thought of as the topology of continuous families of spaces or maps. The objects then of this category are space-valued functions $E(b)$ of a parameter b which varies in a topological space B. For instance, if we assign to every linear map $b : R^m \to R^n$ its image, then $E(b) = \mathrm{im}(b)$ is such a function, $B = \mathscr{L}(R^m, R^n)$. This example has obvious continuity properties, but in general it is not so clear what continuity of $E(b)$ should mean. One standard procedure is to form the set $E = \{(b, x)|b \in B, x \in E(b)\}$ and topologize this set in such a way that the projection $p : E \to B$, $p(b, x) = b$ is continuous. This makes good sense if all $E(b)$ are contained in one big space $\mathscr{X}$, so that $E \subset B \times \mathscr{X}$. Even if B is not topologized to begin with, the fact or choice of having all $E(b)$ contained in one space $\mathscr{X}$ often suggests or induces an appropriate topology on B. (Example: If M is a compact smooth manifold, let B be the set of all submanifolds of R^n, $n \leq \infty$, which are diffeomorphic to M, and put $E(b) = b$.)

The book under review takes the result of this procedure as the starting point, i.e. a continuous map $p : E \to B$. This is now called a *fibrewise topological space over* B—and $E(b) = p^{-1}(b)$ can be thought of as a continuous family of spaces, $b \in B$. The parameter space B is called the *base* space, $E(b) = p^{-1}(b)$ is the *fibre* over b, and E itself may be called a fibrewise topological space over B—if the projection is understood from the context. A morphism between two objects is a continuous family $\varphi(b) : E(B) \to E'(b)$ of continuous maps, i.e. a continuous map $\varphi : E \to E'$, such that $p'\varphi = p$; it is called a *fibrewise continuous map* over B. The book is very consistent in its fibrewise thinking and fibrewise language. As the author remarks, "the effect is somewhat monotonous $\cdots$ but experience shows that to compromise on this point is liable to cause confusion." One can agree with this cautious, careful attitude, but the reviewer feels that the functional (parametric) point of view could have been used simultaneously, perhaps informally, to facilitate the understanding or to stimulate the reader's own ideas, and his cooperation. For instance, the induced fibrewise pullback is then described simply as composing $B' \to B$ with the function E. Or, if B is itself a space of spaces of a certain kind (projective space, space of 2-point subsets, space of free transitive G-sets where G is a fixed finite group, etc.), then it is clear that $E(b) = b$ is a universal kind of family.

The section headings of the book are as follows: Fibrewise (F.) topological spaces, F. separation conditions, F. compact spaces, Tied filters, F. quotient spaces, F. pointed topological spaces, Relation with equivariant topology, F. compactification, The F. mapping-space, F. compactly-generated spaces, Naturality and naturalization, F. uniform structures, F. uniform topology, The Cauchy condition, F. Completion, Functoriality, F. compactness and precompactness, F. homotopy, F. pointed homotopy, F. cofibrations, F. pointed cofibrations, F. non-degenerate spaces, F. fibrations, Relation with equivariant homotopy theory, Fibre bundles, Numerable coverings, F. connectedness. From these one might think that the book merely carries over classical topology to the fibrewise setting. In detail, however, it looks quite different. Even innocent-looking notions like base points, discreteness, normality and local-compactness require judicious attention in the fibrewise setting if they are to behave as expected; sometimes they give rise to unsolved problems. In general, the choices made and the solutions found by the author are really satisfactory. But as in classical topology, the notions evolve before they attain their optimal form as the author himself indicates: In quite a few footnotes he points out that he found it appropriate to modify definitions which he had introduced in earlier work on fibrewise topology. As a detail in this spirit I mention a notion of sliceability which is due to Ch. Ehresmann (I think). It assumes not only a local section of $p : E \to B$ through every point of E (as the author does) but *continuously* so, i.e. it assumes an open neighborhood $U \subset E \times B$ of the graph of p and a continuous map $\sigma : U \to E$ such that $p\sigma(x, b) = b$ and $\sigma(x, p(x)) = x$ for all $(x, b) \in U$. It is more of an all-in-one construction, and it seems to capture the topological essence of submersions.

The author predicts "a growing interest in research on fibrewise topology." Not only is this to be expected, indeed, we also have already a large store of fibrewise methods and results in topology *and* several other fields such as differential and algebraic geometry, differential equations, singularities, deformations, and others, which make the book most welcome. The book is written in a topological mode, it is true, but it is accessible and suitable for a wider readership, being clear and careful in style, emphasizing the search for the "right" notion and "right" proof. The specialist on the other hand may still find interesting homotopy theory (fibrewise) which is new to him, in the last part of the book.

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The logarithmic integral I, by Paul Koosis. Cambridge Studies In Advanced Mathematics, vol. 12, Cambridge University Press, Cambridge, New York, New Rochelle, Melbourne, Sydney, 1988, xvi + 606 pp., $89.50. ISBN 0-521-30906-9.

One of the most important and distinctive features usually associated with the class of analytic functions is what is commonly referred to as the unique continuation property. Put in its simplest form it says: if $f(z)$ is defined and analytic on an open set Ω in the complex plane and if either

(1) $f(z) = 0$ on a set with a limit point in Ω, or
(2) $f^{(n)}(z_0) = 0$, $n = 0, 1, 2, \ldots$, at some $z_0 \in \Omega$,

then $f(z) \equiv 0$ on Ω. In short, an analytic function is completely determined by its behavior on a rather small portion of its domain of definition. Koosis's book, *The logarithmic integral*, (LI), is in large part concerned with extensions and applications of this basic fact.

By 1892—at the age of twenty-one—Émile Borel had become convinced that it must be possible to extend the uniqueness property to much larger, more general, nonanalytic classes of functions defined, for example, on sets without interior points. To some, however, it seemed highly unlikely that such a program could be carried out in any meaningful way and Poincaré had even constructed certain examples to strengthen the negative point of view. Nevertheless, Borel persisted in his conviction and at his thesis defense of 1894—at which Poincaré was the rapporteur—he