a C^*-algebra—are added only as needed. Usually the reader is provided with counterexamples to demonstrate why these hypotheses are required. To offset the burden of maintaining complete generality the authors have been very careful to keep the group case highlighted throughout. Major results are often restated in the group case. There is even an entire chapter devoted to the theory of representations of compact group. Because of this blend of styles between that of a research encyclopedia and that of a textbook, these volumes will make an excellent reference for the student wishing to begin studying representation theory of Banach algebras, as well as for the expert who wishes to check the details of the Mackey normal subgroup analysis in the nonseparable case.

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BULLETIN (New Series) OF THE
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Combinatorial search, by Martin Aigner. Wiley-Teubner Series in Computer Science, 1988, 368 pp., $44.95. ISBN-0-471-92142-4 (Wiley) ISBN-3-519-02109-9 (Teubner)

Popular combinatorial search problems involve such matters as "twenty questions," weighing to find a counterfeit coin, or locating a word in a dictionary. All these problems can be described using a simple model. Given a search domain consisting of a finite number of points with one point being the target point, the problem is to locate the target point at minimum cost by performing a sequence of tests. A test is a partitioning of the search domain into subsets with the value of the test indicating which subset contains the target point.

Various generalizations of the simple model are possible. There may be a probability distribution on the number of points. A test may partition the search domain into three or more subsets, or certain tests may not be available. Also, the cost of a test may depend on which test was performed just before the given test.

It is convenient to describe the combinatorial search by an ordered tree (a decision tree) and let the leaves of the tree denote the points in the search domain. If every edge is associated with a digit, the path from the root to a leaf is associated with a sequence of digits which is a code of the leaf. An optimum search tree is then an optimum code.

In the decision tree formulation, the root of the tree is said to be at level zero, its children at level one, etc. So the number of arcs leading from the root to a node is the path length or level number of that node, and the level number of a leaf is the number of tests needed to locate the leaf. The basic question is to find a decision tree to minimize the maximum path length (the worst case analysis) or the average path length (the average case analysis). If the leaves have different weights and the objective is to find the average path length, then we have the Huffman's tree, or the optimum binary code. For the optimum alphabetic code, the book describes the Hu-Tucker algorithm briefly. If we consider all leaves as nodes of a graph and permissible tests are those partitionings which leave the components connected, then in Huffman's case, the graph is a complete graph and in the alphabetic case, the graph is a chain.

Turning to the group testing problem in Chapter 2, the author describes the class of weighing problems, i.e., given a set of good and defective coins and a balanced scale, we want to locate the defective coin (which is lighter) in a minimum number of weighings. Thus, we can partition the coins into three groups and weigh two groups, obtaining a $\lceil \log_3 n \rceil$ bound for the number of tests. If the defective coin is either lighter or heavier, we have the bound $\lceil \log_3(2n+2) \rceil$.

In the spring scale model, we know the exact weight of a good coin, and we are allowed to weigh any subset of coins. The problem becomes more complicated when there is more than one defective coin.

In Chapter 3, the book considers the graph searching problem. Again we partition the graph into A and its complement and the test tells if an unknown edge has at least one end in A. Other possible tests would reveal if both ends of the edge are in A, exactly one end is in A or neither end is in A. The search for an unknown edge can be generalized into a special subgraph such as a triangle, etc. Chapter 4 describes the sorting problem, mainly a subset of the material of Volume IV of Knuth. This includes the kth largest element problem as well as sorting networks. Chapter 5 deals with the poset problem. Given a partially ordered set such as two chains, we want to use the minimum number of comparisons to find a linear ordering compatible with the given partial order. Chapter 6 should be called the feasibility problem of polyhedra (i.e., deciding if a point satisfies a set of linear inequalities). This chapter also includes the longest increasing subsequence problem and other miscellaneous topics.

The book contains many figures, numerical examples, exercises and answers. It is a very nice textbook as well as a good reference book. Although the book is published in 1988, it contains recent references some from 1987 and 1988.

The writing is formal and rigorous as many books in mathematics are. It is a book which should be on the bookshelf of all people interested in combinatorics.

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Minimal flows and their extensions, by Joseph Auslander. North-Holland Mathematical Studies, vol. 153, North-Holland, Amsterdam, New York, Oxford and Tokyo, 1988, xi + 265 pp., $86.75. ISBN 0-444-70453-1

Abstract topological dynamics deals with actions of groups on compact spaces. Such an action is called a *flow*. At present the emphasis of research is on *minimal flows*, i.e., actions for which no proper closed invariant subsets exist.

The abstract axiomatic approach to the subject began with Gottschalk and Hedlund's pioneering book of 1955 **[G-H]**, where the search for a suitable framework for the (then) new subject is apparent.

The introduction of the enveloping semigroup of a flow by R. Ellis (1960) **[E,1]** and the proof that it actually is a group when the flow is distal, were the next important achievements.

The action of the group T on the compact space X is *distal* if (assuming the existence of a metric on X) $\operatorname{Inf}\{d(tx,ty): t \in T\} > 0$ for every pair of distinct points $x, y \in X$. This notion which was first introduced by D. Hilbert, was very central to the development of the subject. A stronger condition on a flow is that the group T acts *isometrically* on X. For a while it was not clear whether these two conditions are not equivalent. Then in 1963 H. Furstenberg realized that Kakutani-Anzai's constructions of skew products on the torus yield examples of minimal distal but not isometric flows. A simple example of this type is the transformation $(x,y) \mapsto (x + \alpha, x + y) x, y \in \mathbb{R}/\mathbb{Z}$, α irrational. (At the same time and independently another example was found in **[A-G-H]**.) H. Furstenberg then took the next major step in the theory of topological dynamics, a step which gave the subject its present character. Recognizing the above example to be an *isometric extension* of an isometric flow, (the rotation by α on the circle) and observing that such extensions preserve distallity, he defined the class of Quasi-Isometric flows to be the class of minimal flows which can be