With the exception of the Chapman theorem, the results italicized in §4 are only available in raw research-paper form. There is a real need, for instance, for an exposition of Taylor's example which does not require the reader go through "$J(X)IV$" by Adams. This material can be given a reasonably self-contained treatment, as can the other examples I have cited of major results in the subject. And all the results cited were available to experts, at least in preprint, before the book was completed. One must conclude that the primary purposes of this book are the education of students and the outlining of the literature. It is not a definitive exposition of the subject, and it is not addressed to the larger topology community.

A striking feature of the book is its huge bibliography which shows that the authors know the kind of history I tried to sketch in §§1 and 2, as well as the recent literature. However, the history is relegated to "who did what when" notes at the end of each chapter (very accurate notes, by the way). Perhaps a broad sweep of history is not to the authors' taste, but I think the subject calls for it. After all, shape theory is hardly elegant mathematics, not even when well written, as in this book. It is technical mathematics, and technical mathematics needs all the justification available.

In summary, this is a book which presents its subject well, but in a rather narrow framework. It is accessible to students at a relatively early stage of their studies, and will direct them to, but not guide them through, the more advanced topics. Those who use elementary shape theory in their work will find this book a convenient reference source.

ROSS GEOGHEGAN

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Riesz spaces II, by A. C. Zaanen, North-Holland Mathematical Library, Vol. 30, North-Holland Publishing Company, Amsterdam, 1983, xi + 720 pp., ISBN 0-4448-6626-4

The analytic theory of Riesz spaces, which is the study of linear mappings between Riesz spaces, was initiated by F. Riesz in his 1928 address to the International Congress of Mathematicians held at Bologna. In his address, Riesz emphasized the important role played in analysis by partial order and indicated how classical results concerning functions of bounded variation were related to their order structure. His ideas led to the foundation of the theory of vector lattices, or Riesz spaces as they are known nowadays, with fundamental contributions from H. Freudenthal and L.Kantorovitch in the middle thirties. Freudenthal's contribution was the abstract spectral theorem which bears his name, a theorem whose formal resemblance to the spectral theorem for selfadjoint operators immediately suggests its importance and whose content provides a unified approach to both the classical spectral theorem and the Radon-Nikodym theorem of classical measure theory within the abstract framework of partial order. On the other hand, Kantorovitch was concerned with the algebraic and convergence properties of Riesz spaces which were, once again, suggested by classical measure theory and set down the foundations for the study of linear operator theory on Riesz spaces.

The theme of unification through abstraction and the motivation drawn from classical measure theory that so clearly dominated the theory of Riesz spaces at its inception have continued to play major roles in the subsequent development of the subject. These features are readily apparent in the book under review, which forms the second volume of a detailed and comprehensive account of the theory of Riesz spaces. The author is, of course, well known for his work in integration theory and functional analysis and for his contribution to the study of integral equations, Orlicz spaces, and Riesz spaces over a period which spans more than four decades. The author therefore brings impeccable credentials to the task of presenting a subject, to which he has contributed and significantly influenced, from a perspective which not only indicates its roots, its motivations and its current state, but which also points to some of the lines of its future development.

The first volume, *Riesz spaces* I, appeared in 1971 and was written jointly with W. A. J. Luxemburg. It was concerned more with the algebraic foundations of the theory of vector lattices and dealt with questions of band structure and projection properties, prime ideals, the Freudenthal theorem, and representation theory. *Riesz spaces* II begins with Chapter 11, which is a further development of material concerning prime ideals in the setting of distributive lattices as well as in Riesz spaces. The general flavour of the material of this chapter is entirely algebraic, and those readers interested primarily in the analytic theory, which is the main content of *Riesz spaces* II, can immediately proceed to Chapter 12. Here is introduced the basic object of study: the space $L_b(L, M)$ of all order bounded linear maps from the Riesz space L to the Dedekind complete Riesz space M. The structure of $L_b(L, M)$ is analysed in some detail beginning with the important property that $L_b(L, M)$ is itself a Dedekind complete Riesz space. Various extension theorems for positive linear operators, which go back to Kantorovitch, are derived from a suitable version of the Hahn-Banach theorem. While the basic material of this chapter is fairly well known, the discussion is supplemented by relatively recent results which characterize those Riesz spaces L on which every positive linear operator is order continuous.

Even as recently as 1978, P. R. Halmos and V. S. Sunder indicated that it was still possible to ask interesting questions about kernel operators. In their monograph entitled *Bounded integral operators on L^2-spaces* (Springer-Verlag, Berlin and New York, 1978), they considered the theme suggested by its title. However, the desirability of considering kernel operators defined on more general Riesz spaces of functions was soon evident and in Chapter 13 of *Riesz spaces* II, the basic theory of order bounded operators is immediately used as a general framework in which to investigate the structure of kernel operators between (order) ideals of measurable functions. The centrepiece of this chapter is the following intrinsic characterization of kernel operators due to A. V. Buhvalov: if L, M are (order) ideals of measurable functions on (possibly different) σ-finite measure spaces, and if L satisfies some rather natural conditions, then a linear mapping from L to M is a kernel operator if and only if it maps order bounded sequences which converge to zero on every set of finite measure to sequences which converge to zero almost everywhere. The proof which is presented deserves comment, as it is entirely elementary and is based on an interesting interplay of classical measure theory on the one hand and of lattice properties of order bounded operators on the other. As such, it illustrates very well the nature of the subject at hand. The first ingredient is that the order bounded kernel operators from L to M form a band in the Dedekind complete Riesz space $L_b(L, M)$, and this band is precisely that generated by the finite rank operators, facts which rest on the classical Radon-Nikodym theorem. The second ingredient is that any order-bounded operator which satisfies the Buhvalov condition and which is disjoint from all finite rank operators is necessarily the zero operator, which depends upon a variant of the classical Egorov theorem. The result now follows almost immediately by combining the ingredients, since any operator in $L_b(L, M)$ can be expressed uniquely as the sum of its projections onto the band of kernel operators and its disjoint complement. The ideas here can be traced back to a much earlier paper of H. Nakano on bilinear forms. The remainder of Chapter 13 very efficiently and effectively exploits the Buhvalov criterion to yield kernel representation theorems for various classes of continuous linear operators on familiar spaces. Included here are characterizations of Carleman and Hille-Tamarkin operators as well as the theorem of N. Dunford that each continuous linear operator from an L_1-space to an L_p-space, $1 < p \leqslant \infty$, is an (absolute) kernel operator.

Much of the modern theory of normed Riesz spaces finds its origin in a series of Notes on Banach function spaces written by the present author in collaboration with W. A. J. Luxemburg and which appeared in the Proceedings of the Netherlands Academy of Science in the early sixties. These papers were a natural outgrowth of the theory of function norms and not only provided a clear and firm basis for the general theory, but systematically developed, refined, and unified the existing literature, in particular the important contributions of H. Nakano, I. Amemiya, T. Ogasawara, and their collaborators. Chapters 14–16 find their roots in this series of Notes and reflect very clearly the motivating influences of the theory of normed Köthe spaces, which now begins to emerge as a beautiful illustration of the general theory which it inspired. These chapters contain a detailed account of the roles played in the general setting of normed Riesz spaces by properties which are the abstract core of the familiar theorems from integration theory which bear the names of Riesz-Fischer, Fatou, Lebesgue, and Beppo-Levi, and the relation of these properties to norm completeness, perfectness, and reflexivity is patiently and clearly developed.

The basic duality and representation theorems for abstract L and M spaces, even abstract L_p-spaces, are given in Chapter 17. Included for the first time in this chapter is the much more recent characterization of those Banach lattices for which the band of singular functionals is an abstract L-space, in terms of an intrinsic condition called the semi-M-property, a property which is enjoyed by all abstract M-spaces and all Orlicz spaces, but not, in general, by all Banach function spaces.

A very detailed and systematic study of order-bounded compact operators on Banach lattices is contained in Chapter 18. As in the case with the chapter on kernel operators, the results presented here are of very recent origin and have a very special flavour. Indeed, as the author remarks in his preface, the rather considerable time interval between the appearance of *Riesz spaces* I and *Riesz spaces* II is due to the fact that important parts of the theory of kernel operators on the one hand and the theory of positive compact operators on the other were put into an elegant and more or less final form only in recent years. As an example, it was not known until 1976 that, even in a familiar space such as $L_2[0, 1]$, any positive operator (pointwise) dominated by a compact operator was itself compact. While compactness properties of kernel operators in various settings have been well understood for some time, initial results concerning compact operators on Banach lattices were somewhat sporadic and negative from the point of view of lattice structure. For example, the order bounded compact operators on $L_2[0, 1]$ do not form even a sublattice, much less a band, and there are positive compact operators outside the band generated by the finite rank operators and consequently not representable as kernel operators. However, as in the case of kernel operators, the situation improves remarkably if one considers initially not order-bounded operators on an L_2-space, but rather order-bounded compact operators from a space of type $C(K)$ (or an abstract M-space) to a space of type $L_1(\mu)$ (or an abstract L-space). That the collection of such operators does form a band is the heart of the approach described in Chapter 18, an approach which is elementary, which is purely order-theoretic (a key element is the Freudenthal theorem), and which has interesting consequences. The point, then, is to reduce the study of order-bounded compact maps on spaces such as L_2 to the study of compact maps from abstract M-spaces to abstract L-spaces. This leads to the introduction of a class of mappings, known as AMAL-compact maps from a Banach lattice L to a Banach lattice M, of which M is assumed to have order-continuous norm. This class of mappings consists of those order-bounded operators which map order intervals of L to sets whose images are relatively compact in each abstract L-space quotient of M. The principal structure theorem involved then asserts that the order-bounded AMAL-compact maps from L to M form a band in the Dedekind complete Riesz space of order bounded operators from L to M. Besides the compact domination result in L_2-spaces alluded to earlier, this structural theorem has a number of further consequences of varying depth and interest. As order-bounded kernel operators are AMAL-compact, it provides a basis for a unified treatment of existing compactness criteria for kernel operators. Further, the AMAL-compact operators on an abstract L-space coincide precisely with the (so-called) Dunford-Pettis operators which consequently form a band. Yet another interesting consequence is that a positive operator on an arbitrary Banach lattice which is dominated by a compact operator always has a compact cube, but does not necessarily have compact square. As is the case throughout the book, instructive examples are given to show that the results presented are best possible and that the hypotheses are minimal.

The theory of Orlicz spaces has, of course, played an important role in the development of the general theory of normed Riesz spaces and, in the first part of Chapter 19, the properties of these spaces now appear as illustrations of much that has gone before. The second part of this chapter is devoted to the classical eigenvalue theorems of Perron, Frobenius and Jentzsch. While there is some overlap here with existing books, particularly that of H. H. Schaefer (*Banach lattices and positive operators*, Springer-Verlag, 1974), the methods presented are new. For example, the Ando-Krieger spectral radius theorem is based on a certain order continuity property of the spectral radius.

An orthomorphism is an order-bounded band-preserving linear map. In familiar spaces orthomorphisms coincide with multiplication operators and, consequently, the theory of orthomorphisms is very closely related to the abstract study of both scalar and vector-valued Radon-Nikodym theorems. The last decade has seen a fairly intensive study of orthomorphisms from several points of view, and the final chapter presents an elementary, coherent and very smooth synthesis of the important basic elements of the theory. A feature of the presentation here is the simultaneous development of the theory of *f*-algebras, which provides the algebraic underpinnings of the theory of orthomorphisms.

In *Riesz spaces* II, there is some overlap, but not too much, with several books treating various aspects of the theory that have appeared since 1971. These books, together with the themes they develop, are listed in the author's preface, and it will suffice here to note that what overlap exists occurs mainly in what might be considered standard material and even then, new insights are frequently offered.

The author has insisted throughout on elementary techniques, and this preference for direct analysis, bypassing representation theorems, often reveals a great deal more about the situation at hand. Throughout the book it leads to simplification of existing techniques, to unity of method, and to a very highly polished presentation. In particular, the prerequisites needed to read the book are kept to an absolute minimum, the principal requirement being, of course, familiarity with basic measure theory.

The expository style adopted by the author is patient, thorough and detailed with meticulous attention being given to points of historical detail. Particular care has been taken to make the major themes self-contained. The flavour of the subject and the perspective it brings to related areas of analysis are illustrated with effect by the many examples throughout the text, and there is a great deal of supplementary material in the form of numerous exercises, many of which appear with detailed hints. Thus, as much of the material presented is very close to final form, the book will not only serve the specialist as a valuable reference, but will also provide ready access to the key ideas of the theory to the interested nonspecialist and to the student. The book stands as a very substantial contribution to the subject of its title.

PETER G. DODDS