to the subject and is, moreover, indispensible for a lot of important mathematics. It is true that one should figure out what abstract notions mean in a more concrete context, as in the case of the Baer sum. But one expects in a Springer Lecture Note that something more than that has happened after 66 pages. Another complaint concerns the title. It suggests that one will learn a lot about representation theory, not just the fact that Schur muliplier occur when one tries to lift a projective representation to a linear one. It would have made more sense to put "isoclinism" in the title. If you want to learn about isoclinism and its connections with the multiplier, I suggest reading instead the survey article by one of the authors [2]. It has a swifter pace so that one can see more easily where things are going. In the same proceedings one may also read Wiegold's paper on the Schur multiplier. On page 204 of the book one should modify the definition of unicentral to save what follows. The correct requirement is that $\pi Z(G)$ equals $Z(Q)$ for all central extensions $\pi\colon G \to Q$.

REFERENCES

1. R. Baer, *Erweiterungen von Gruppen und ihren Isomorphismen*, Math. Z. **38** (1934), 375–416.

2. F. R. Beyl, *Isoclinisms of group extensions and the Schur multiplicator*, Groups — St. Andrews 1981 (C. M. Campbell and E. F. Robertson, eds.), London Math. Soc. Lecture Note Ser., Vol. 71, Cambridge, 1982, pp. 169–185.

3. R. L. Griess, Jr., *Schur multipliers of the known finite simple groups*. II, The Santa Cruz Conference on Finite Groups, Proc. Sympos. Pure Math., vol. 37, Amer. Math. Soc., Providence, R.I., 1980, pp. 279–282.

4. H. Hopf, *Fundamentalgruppe und zweite Bettische Gruppe*, Comment. Math. Helv. **14** (1942), 257–309.

5. S. Mac Lane, *Origins of the cohomology of groups*, Topology and Algebra, L'Enseignement Math. Mono. 26 Kundig, Genève, 1978, pp. 191–219.

6. J. Milnor, *Introduction to algebraic K-theory*, Ann. of Math. Stud. No. 72, Princeton Univ. Press, Princeton, N.J., 1971.

7. I. Schur, *Über die Darstellung der endlichen Gruppen durch gebrochene lineare Substitutionen*, J. Reine Angew. Math. **127** (1904), 20–50.

8. ______, *Untersuchungen über die Darstellung der endlichen Gruppen durch gebrochene lineare Substitutionen*, J. Reine Angew. Math. **132** (1907), 85–137.

9. ______, *Über die Darstellungen der symmetrischen und alternierenden Gruppen durch gebrochene lineare Substitutionen*, J. Reine Angew. Math. **139** (1911), 155–250.

10. C. Soulé, *K_2 et le groupe de Brauer d'apres A. S. Merkurjev et A. A. Suslin*, Sem. Bourbaki, Vol. 1982/83, Exp. 601.

11. J. Wiegold, *The Schur multiplier: an elementary approach*, Groups — St. Andrews 1981, *op. cit.*, pp. 137–154.

WILBERD VAN DER KALLEN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 10, Number 2, April 1984

0273-0979/84 $1.00 + $.25 per page

Introduction to number theory, by Loo Keng Hua, Springer-Verlag, Berlin, 1982, xviii + 572 pp., $46.00. ISBN 0-3871-0818-1

The book under review is a translation from the Chinese of a book first published in 1956. (The Chinese edition was reviewed by K. Mahler in *Mathematical Reviews*.) Some of the chapters in this translation have been supplemented by notes updating some of the results of these individual chapters. The notes were provided by Yuan Wang and the translation is by Peter Shiu.

Number theory enjoys, today, an interest and vigor comparable to any discipline in mathematics. Although some of the results are centuries (and even millenia) old, and some have a specialized and seemingly isolated character, interest in them is as broad and sustained as ever.

One need only mention such ancient topics as perfect numbers, unique factorization, and distribution of primes, which are found in Euclid; representations of integers by quadratic forms—a subject persued by the Babylonians; the solutions to diophantine equations, whose origins are lost in history.

How are we to account for this vigor, which has been sustained over the centuries? This is a difficult question to answer and some are tempted to ascribe it to a quasi-theological basis. At least, Kronecker felt that the integers occupied a special place in the history of ideas.

Whatever the philosophical reason, it is a fact that over the years there has been repeatedly, a sort of catalysis between number theory and other branches of mathematics, sometimes borrowing and sometimes giving, both enriching and being enriched. Consider for example the problem of constructing regular polygons. This geometric question was solved by Gauss with the help of "Gaussian sums" whose properties relied on some number-theoretic results, such as the existence of a primitive root. In this case, the study of Gaussian sums and their variants became a part of number theory. In other cases, the number-theoretic problem gave birth to new developments in other fields. As an example, the study of diophantine equations has provided an important stimulus in algebraic geometry. In still other cases, number-theoretic pursuits have given rise to new disciplines. For example H. Bohr's studies of the Riemann zeta function led him to the concept of almost periodic functions. (Some would question whether the zeta function is properly a part of number theory.)

In some instances the development has remained a part of number theory with no apparent intellectual justification. It is difficult to understand, for example, why the transcendence of π should be a theorem in number theory. Indeed A. Weil, in one of his essays on number theory, questions whether analytic number theory is number theory at all. We can do no better than to quote from "Alice...": "When I use a word, it means just what I choose it to mean—no more and no less."

In addition to its influence in other branches of mathematics, number theory has had "applications" of various sorts some rather unexpected. Hua notes a couple of examples. The first is a recent encryption scheme of Adleman, Rivest, and Shamir whose decoding requires the factorization of large integers. The other is the detection of a logical error in a computer, found by Rosser, Schoenfeld, and Yohe, while calculating zeros of the zeta function. We could add to these applications the use of finite fields in coding theory and the use of diophantine approximation in the problem of approximate quadrature, a topic to which Hua himself has contributed.

Hua espouses the point of view we have remarked upon, and so has written a comprehensive book. It is fairly long—twenty chapters in about 575 pages, but the reader should not be discouraged, for many of the chapters can be read independently of one another.

Apart from classical material, the book covers a wide variety of topics. We shall illustrate, with two topics, the way in which number-theoretic problems have influenced developments in other realms of mathematics.

The first concerns the theory of trigonometric—sometimes called exponential—sums. These sums play a crucial role in numerous applications to number-theoretic and other problems.

Let $f(x)$ be a real valued function defined on some real domain and let $A > 0$ be any real number. An exponential sum has the form

$$E = E(f, y, A) = \sum_{0 \leqslant n \leqslant A} e^{2\pi i f(n) y},$$

where the sum is extended over integral values n. It is evident that $E = O(A)$. If, however, values of $f(x)$ exhibit some regularity, then one might hope that the individual terms would damp each other. Thus the central problem is to determine conditions on f and y which produce this cancellation resulting in $E = o(A)$, and more precisely to specify how much "smaller" than A the sum is. Much work has been devoted to this fundamental problem. If for example p is a prime, $y = 1/p$, and $A = p$, and $f(x)$ is a primitive polynomial $\in \mathbf{Z}[x]$, then the sum can be related to a geometric question; using the Riemann hypothesis for curves over finite fields, A. Weil had shown the remarkable result that

$$E = O(\sqrt{p}).$$

Such a sharp result cannot be expected (nor indeed is it true) in general. In any event, the search for good estimates has stimulated work in other areas of mathematics.

Deriving and then using an appropriate estimate, Hua gives a proof of the fact that if $n(p)$ is the least quadratic nonresidue mod p, then

$$n(p) < p^{1/2\sqrt{3}} \log^2 p.$$

We add parenthetically that the existence of a "small" quadratic nonresidue has played a role in estimating the number of steps required to factorize a number.

The second example we give arises from additive number theory and can, with justification, be said to have stimulated the general theory of modular and related forms.

Let it be required to determine $r(n)$—the number of ways of writing the integer $n > 0$ in the form $n = a_1 + a_2 + \cdots + a_s$—where the a_i belong to sets $A_i \subset \mathbf{Z}$. It is easily seen that if $f_i(x) = \sum_{a_i \in A_i} x^{a_i}$, then

$$F(x) = \sum_{n=0}^{\infty} r(n) x^n = \prod_i f_i(x) \qquad (r(0) = 1).$$

If now the functions $f_i(x)$ have favorable properties, then we can infer properties of $r(n)$. Perhaps the oldest example, due to Euler, is the case when

$A_i = \{0, i, 2i, \ldots, mi, \ldots\}$ and s is unrestricted. We then get

$$f_i(x) = (1 - x^i)^{-1}, \qquad F(x) = \prod_{i=1}^{\infty} (1 - x^i)^{-1},$$

and then $r(n) = p(n) =$ number of unrestricted partitions of n. The function $F(x)$ has modular properties i.e. if $x = e^{2\pi i \tau}$, then the mapping

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad a, b, c, d \in \mathbf{Z}, \quad ad - bc = 1,$$

transforms $F(x)$ in an especially nice way. The set of such transformations forms the unimodular group G, and functions behaving well under G or its subgroups play an important role in number theory, and their study comprises a very active area of research. There have been recently discovered connections between modular forms and characters of the sporadic simple groups. At any rate, using these modular properties on $F(x)$, Hua gives a proof of a theorem of Hardy and Ramanujan, viz.,

$$\lim_{n \to \infty} \frac{\log p(n)}{\sqrt{n}} = \sqrt{\frac{2\pi}{3}} = 2\sqrt{\zeta(2)}\,.$$

Many readers will know that this is a weak form of Hardy's and Ramanujan's asymptotic formula for $p(n)$, which was perfected to a rapidly convergent series by Rademacher.

If $A_i = \{0, 1, 4, \ldots, m^2, \ldots\}$ then

$$\sum_{n=1}^{\infty} r(n) x^n = \left(\sum_{-\infty}^{\infty} x^{m^2} \right)^s = (f(x))^s,$$

and here $r(n)$ is the number of representations of n as a sum of s squares. The function $f(e^{2\pi i \tau})$ is a theta function whose modular properties were observed by Jacobi who also studied the case $s = 4$. In this book Hua analyzes the case $s = 3$ and derives an explicit and detailed formula for $r_3(n)$.

These two examples illustrate the close interaction of number theory with some other parts of mathematics.

One of the most striking features of the book is its economy of style, incorporating an astonishing number of results and topics. It achieves this efficiency, in part, by avoiding the temptation of stating and proving results in their greatest generality. To be sure, proofs of some theorems are left as exercises which supplement the, unfortunately, relatively few exercises to be found in the book.

As an example of this characteristic we single out especially the chapter on algebraic number theory. In the compass of 50 pages, the author takes us from the very beginnings of the subject through ideals, class numbers etc., and ends with applications to Mersenne primes and diophantine equations. This is achieved with no sacrifice of lucidity.

What is true of this chapter holds to a greater or lesser extent for most chapters. The interested mathematician may approach the material with minimal prior knowledge. The language is classical and the reader will not be impeded by the necessity of having a large mathematical vocabulary. On the other hand, the reader will be amply rewarded with beautiful results of considerable depth and can come away with a sense of satisfaction.

In one of his letters to Sophie Germain, Gauss, referring to number theory, wrote that "the enchanting charms of this sublime science are not revealed except to those who have the courage to delve deeply into them." This book provides an admirable vehicle for so delving.

RAYMOND G. AYOUB

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 10, Number 2, April 1984

0273-0979/84 $1.00 + $.25 per page

The analysis of linear partial differential operators. I, by Lars Hörmander, Grundlehren der mathematischen Wissenschaften, Vol. 256, Springer-Verlag, Berlin, 1983, ix + 391 pp., $39.00. ISBN 3-5401-2104-8

The analysis of linear partial differential operators. II, by Lars Hörmander, Grundlehren der mathematischen Wissenschaften, Vol. 257, Springer-Verlag, Berlin, 1983, viii + 389 pp., $ 49.50. ISBN 3-5401-2139-0

Since the second World War the theory of linear partial differential equations has undergone two major revolutions. The first was the advent, in the late forties, of a formalized theory of "generalized functions". Its starting point was the use of test-functions. The idea was not entirely new; it had been introduced earlier in the theory of Radon measures (in particular, on locally compact groups [**Weil 1940**]) and had something to do with the old quantum mechanics: one could not always assign a value at a point to certain "functions", such as Dirac's, but one could "test" them on suitable sets, or "against" suitable functions. In the most important case the test-functions are smooth (i.e., C^∞) and vanish identically off some compact set. The corresponding generalized functions were called "distributions" in [**Schwartz 1948**]. Distribution theory assimilated many ideas and discoveries of the preceding decades (by Heaviside, Hadamard, Sobolev, Bochner and others). To these it added new ones, of which the most successful were perhaps the now-called Schwartz spaces $\mathscr{S}$, $\mathscr{S}'$ and the theory of Fourier transform of tempered distributions—although again the link between slow (or tempered) growth, the Fourier transform and localization, and, beyond, causality, was not absolutely new, and certainly not to physicists. Schwartz gave a strong functional analysis slant to the theory,