sets. That same statement could be made, of course, about the entire book. We are among those who are applauding.

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Counterexamples in topological vector spaces, by S. M. Khaleelulla, Lecture Notes in Math., vol. 936, Springer-Verlag, Berlin and New York, 1982, xxi + 179 pp., $10.70.

Here is a rule-of-thumb test to identity latent mathematicians: Make an assertion. If the young person tries to prove it, (s)he fails the test; if (s)he tries to find a counterexample, you have a future mathematician on your hands.

Examples are more important than theorems. If you teach me the rules of a game and attempt to develop a theory, I will interrupt to say "Let's play it once". A course in groups containing pure theory would allow the conjecture "All groups are commutative" to stand unchallenged—besides failing to educate the students.

The role of examples is educational: the derivative of a specific function, a group with 5 members; but we shall be concerned with those which are always thought of as counter: a nowhere differentiable function, a nonmeasurable set.

Is the earliest known counterexample the book of Job? (Assertion: Holiness brings good fortune.)

What is the role of counterexamples in mathematics? (Are there any in Euclid?) I attempt to list the roles in decreasing order of importance; the "big" examples fall early in my list:

1. To refute widely held beliefs. (A nowhere differentiable continuous function, a series whose sum is discontinuous.)

2. To show the need to work in a more general setting. (A nonsequential limit point.)

3. To show the inadequacy of a definition. (Space-filling curve: what does dimension mean?).

4. To open a new field. (C. Neumann: a Fredholm integral equation of the second kind.)

5. To show that a theory is nontrivial. (A noncountable set.)

6. To show that a method of proof fails. (A region allowing no solution to the Dirichlet problem contradicting the argument: place a charge on the boundary.)

7. (Similar to 6). To show that a mathematical model is inadequate for a physical situation. (A bumble bee cannot fly.)

8. To show the failure of a proposed program. (a. Solution of Hilbert's 10th problem. b. Godel's supreme counterexample to "All arithmetic truths are provable.")

9. To complete a classification. (The monster finite groups.)

10. To show that a classification is incomplete. (Weyl algebra; the Wedderburn-Artin theorem fails to classify simple Noetherian rings.)

11. To tidy up a theory. ([3]: "It would have been...unacceptable...not to know (something)...Roy's example removes all doubt.")

12. To answer a long standing question. (And win a goose. Enflo's separable Banach space with no basis.) To answer a very interesting question.

13. To show that two concepts are not the same. (A barrelled normed space of first category.)

14. Teaching aids: To show examples for study. To show that hypotheses are needed. (A metric space of first category.)

15. To win arguments. (A friend with an imperfect memory of Euclid's proof asserted that $\prod\{p\colon p \leqslant q\} + 1$ is always prime. In another context, it is often asserted that the importance of one's work is measured by the number of citations. Surely Baire is a counterexample. In all the standard texts I can find no citation of an article by him. I can't even find his first name or initial!)

16. Collectors' items. To complete a large table showing whether p and q and r...implies s for every conceivable combination. The motivation is like Hilary's famous explanation of why he climbed Mount Everest.

On the negative side, the presence of too many counterexamples is a serious flaw. [**2**]: "As in the rest of Thin Sets one finds mainly pathology...Körner's example...was...a fatal illness...the past ten years have produced a spate of counterexamples...it is time to do something else...I would rather see some positive results".

There are many ways to present a list. The most stark way [**7**] is to give a table and a reference for each entry. The advantage is that many results can be presented in a short space, in this case 121 mapping theorems in 16 pages. The opposite extreme is to write a text with an ordinary index. To make the search easier [**8, 9**], present a set of tables with a heading P and two lists of properties which do or do not imply P. This makes for a more self-contained presentation but fewer results which, also, are limited by the scope of the book. A compromise [**R, 1, 5, 6**] is to present a list or tables with just enough accompanying text to justify the entries. The best list of those mentioned is [**6**] with a well-designed computer printout which enables a quick location of a space with properties p_i: but not q_j, i, $j = 1, 2, \ldots$.

The disadvantage of the compromise, especially in the case of [**R**], is that the supporting text is extremely condensed and unmotivated—hence really inaccessible. A teacher who wished to present a typical entry would require a year of preparation to plan the introduction of preparatory material. This is compounded in [**R**] by the lack of references. As one example: "A Fréchet space which is not distinguished", the excellent index tells us what the words mean. The construction is self-contained and very difficult with no reference to author or other source. This is the last item in [**4**] where also a reference is given—it would have been better for this information to be in [**R**]. (The easier construction of a nondistinguished l.c. space is in [**9**].) The reviewer is listed twice on p. 65 in the disguise of the letter W. Alas his chance for immortality in connection with W-barrelled spaces has been annulled by Steve Saxon's result (not in [**R**]) that W-barrelled is equivalent to second category [**9**, #5-2-301].

The index should be emended: Echelon space 61; Normal topology 51.

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R. The subject of this review.

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Convexity theory and its applications in functional analysis, by L. Asimow and A. J. Ellis, Academic Press, London, v + 266 pp., $56.00. ISBN 0-1206-5340-0

This book focuses on the role of compact convex sets in functional analysis. We will begin this review by trying to indicate why this role has been an important one and by giving a brief description of the historical evolution of research in this area. We will then turn to commenting directly on the contents and contribution of the book under review.

One reason for the central role of compact convex sets in functional analysis is their ubiquity—as evidenced by the Banach-Alaoglu theorem that the unit ball of the dual space of a Banach space is weak*-compact. Compact convex sets play a key role, for example, in the fields of function algebras, group