8. F. Klein, *On Riemann's theory of algebraic functions and their integrals*, Dover, New York, 1963.

9. M. Schiffer and D. C. Spencer, *Functionals of finite Riemann surfaces*, Addison-Wesley, Reading, Mass., 1957.

10. C. L. Siegel, *Topics in function theory*, Wiley, New York, 1969.

11. E. T. Whittaker, *A treatise on the analytical dynamics of particles and rigid bodies*, Dover, New York, 1944.

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Module categories of analytic groups, by Andy R. Magid, Cambridge Tracts in Math., vol. 81, Cambridge Univ. Press, New York, 1982, x + 134 pp., $29.50.

The relationship between a group G and the collection of its finite-dimensional linear representations (or the category Mod(G) of finite-dimensional G-modules) is often subtle. For compact Lie groups, there are classical duality results affirming that the group is recoverable from a knowledge of its representations and how they tensor. For example, in case G is abelian, Pontryagin duality gives an isomorphism between G and $\hat{\hat{G}}$. Here the dual group $\hat{G}$ consists of the 1-dimensional representations of G (complex-valued characters), the product of characters corresponding to the tensor product of associated representations.

Tannaka duality [**5**] does something similar for arbitrary compact Lie groups. The role of $\hat{G}$ is played by the collection of all finite-dimensional representations of G, whose "representations" are in turn identified with elements of G. In Chevalley's formulation [**1**], one forms the Hopf algebra $R(G)$ of **C**-valued "representative functions" (matrix coordinate functions for representations of G), with a coproduct reflecting the product in G. Because G is compact, $R(G)$ is finitely generated, hence gives rise to a complex linear algebraic group $\overline{G}$. The points of $\overline{G}$ can be thought of as algebra homomorphisms $R(G) \to \mathbf{C}$, by identifying $R(G)$ with functions on $\overline{G}$. Duality means that G is realized as the group of real points of $\overline{G}$. In this formulation, $R(G)$ plays the role of a dual group, encapsulating the structure of Mod(G) as a category with tensor products.

In a long series of joint papers (1957–1969), G. Hochschild and G. D. Mostow explored the Hopf algebra of representative functions of an arbitrary complex analytic group (cf. [**3**]). In case G is semisimple, its finite-dimensional representation theory is essentially that of its compact real form; so $R(G)$ is finitely generated and gives G the structure of an algebraic group. But in general the story is far more complicated. In particular, distinct groups may give rise to the "same" category Mod(G). This happens in a fairly transparent way when G fails to have a faithful finite-dimensional (analytic) representation, so one should take as one's starting point a faithfully representable group. But even then Mod(G) fails to determine G uniquely, though it does determine the dimension of G. Hochschild and Mostow spelled things out in detail through a careful study of $R(G)$. This work has subsequently been refined and extended by Magid, who presents in the monograph under review a thorough exposition of the question. As one might expect from the preceding remarks, the subject matter is more algebraic than analytic (analytic groups without analysis) and involves Hopf algebras more than categories.

Here is a brief outline of what Magid does. G always denotes a connected complex analytic group, and representations are always assumed to be analytic. He starts with a survey of elementary examples: five types of solvable analytic groups of dimension at most 3, which reappear from time to time in order to illustrate key points. He points out distinct 3-dimensional groups whose categories Mod(G) (with tensor product) are equivalent. Next he introduces the Hopf algebra $R(G)$ and shows how to characterize it just in terms of Mod(G): a certain group of automorphisms of Mod(G) has $R(G)$ as a naturally occurring algebra of functions.

Assume further that G is faithfully representable. In general $R(G)$ is not a finitely generated algebra over **C**. But as a direct limit of finitely generated Hopf subalgebras, it yields a "pro-affine algebraic group" which has a normal subgroup isomorphic to G, complemented by an inverse limit of algebraic tori. Algebraically, $R(G)$ looks like the group ring of a certain group of characters of G (measuring how far G is from being algebraic), with coefficients in a finitely generated **C**-subalgebra D which is stable under right G-action and which has maximal ideals in 1-1 correspondence with elements of G. The author refers to an algebra like D as a "left algebraic group structure" on G, giving G at least the structure of algebraic variety if not of algebraic group. This is further linked to the structure of the Zariski-closure of G in a given linear embedding, which is explored in detail.

The book is attractively laid out, with just a sprinkling of typographical errors, such as "Zuriski-dense" on p. 75.

The author does his best to keep the logical pathway free of brambles, by carefully introducing and then summarizing each chapter, as well as noting the sources of the main results. He includes an appendix on commutative analytic groups, sketching proofs of the key facts needed. Even so, the reader must struggle at times to keep the overall goal in sight, as the technical machinery builds up. (For an initial overview geared more to Lie algebras, the author's paper [**4**] might be helpful.) Moreover, the reader is assumed to have a fairly broad background in analytic groups, at the level of the concluding chapters of Hochschild's book [**2**]. For example, one basic and highly nontrivial theorem quoted here is the existence of a faithful analytic representation for any (complex!) semisimple analytic group. The proof of this theorem in [**2**] involves, among other things, comparison with a compact real form. All of this background might be taken temporarily on trust; Magid himself avoids direct use of real forms and develops his themes in a largely self-contained fashion.

The author has written a generally clear and concise account of the Hochschild-Mostow theory of representative functions on complex analytic groups, incorporating a number of his own refinements and simplifications. The finer points will not be of interest to all readers, but the main line of development should appeal to anyone who is curious about what lies beyond Tannaka duality.

REFERENCES

1. C. Chevalley, *Theory of Lie groups*. I, Princeton Univ. Press, Princeton, N. J., 1946.
2. G. Hochschild, *The structure of Lie groups*, Holden-Day, San Francisco, Calif., 1965.
3. G. Hochschild, G. D. Mostow, *Complex analytic groups and Hopf algebras*, Amer. J. Math. **91** (1969), 1141–1151.
4. A. R. Magid, *Lie algebras with the same modules*, Illinois J. Math. **25** (1981), 611–621.
5. T. Tannaka, *Über den Dualitätssatz der nichtkommutativen topologischen Gruppen*, Tôhoku Math. J. **45** (1938), 1–12.

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Convex sets and their applications, by Steven R. Lay, Wiley, New York, 1982, xvi + 244 pp., $29.50.

Steven Lay teaches at an undergraduate institution and he wrote this book with his students in mind. Because of its title, the book invites comparison with the well-known book *Convex sets* by Lay's teacher, F. A. Valentine [**9**]. But Lay's intended audience calls for a different kind of book. He works not in a linear topological space, but in R^n. He motivates and clarifies his material with numerous diagrams and an occasional apt analogy. He follows each section with a carefully graded set of problems. And, as the title implies, he offers some applications; perhaps the best example is a chapter on optimization. In summary, Lay aims to do for convex sets what the authors of this review tried to do for convex functions [**7**].

Lay says in his preface that "there is no text at this level which has convex sets and their applications as its unifying theme"—a bit of an overstatement, we think. There are two fine books by Russian authors: *Convex figures* by Yaglom and Boltyanskii [**10**] and *Convex figures and polyhedra* by Lyusternik [**9**], though it could be argued that they are not textbooks in the American tradition. Benson's *Euclidean geometry and convexity* [**1**] is definitely a textbook but is is oriented toward plane and solid geometry. Kelly and Weiss cover much the same ground as Lay in *Geometry and convexity* [**5**] but their book is more topological and probably more difficult for undergraduates. And of course, there is a host of advanced books, of which Grünbaum's *Convex polytopes* [**3**], Eggleston's *Convexity* [**2**], Rockafellar's *Convex analysis* [**8**] and the aforementioned book of Valentine are worthy of note. There are, then, other books that are developed around the theorem of convex sets. But all of